Plan

Wrap-up Overview → Lecture 1 slides

Proof Techniques

- ✓ Proof by cases
- ✓ Disprove by counterexample
- ▪ Proof by contrapositive
- ▪ Proof by contradiction

Perceptron Algorithm

10-607 Computational Foundations for Machine Learning

Proof Techniques & Perceptron Algorithm

Instructor: Pat Virtue

Proof Techniques

Proof by cases

Disproof by counterexample

- One example is not sufficient to prove
- One counterexample is sufficient to disprove

Proof by contrapositive

Previous Poll

Given model m: {A: True, B: False}

Does m satisfy the following sentence:

$(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$

i. Yes

ii. No

iii. Not enough information

iv. Syntax error in sentence

Previous Poll

Truth table for $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$

How many rows do we need (excluding a header)?

i. 2

ii. 4

iii. 6

iv. 8

v. 16

Previous Poll

Truth table for (A⇒B)⇔(¬B⇒¬A)

How many columns should we have?

i. 2

ii. 3

iii. 4

iv. 5

v. 6

7

A	B	A ⇒ B	¬B ⇒ ¬A	⇔
T	T			
T	F			
F	T			
F	F			

Exercise

Truth table for $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$

Exercise

Truth table for (A⇒B)⇔(¬B⇒¬A)

A	B	$\neg A$	$\neg \mathrm{B}$	$A \Rightarrow B$	$\neg B \Rightarrow \neg A$	$(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$
F	F	T	T	T	T	T
F	T	T	F	T	T	T
T	F	F	T	F	F	T
T	T	F	F	T	T	T

Proof Techniques

Proof by cases

Disproof by counterexample

- One example is not sufficient to prove
- One counterexample is sufficient to disprove

Proof by contrapositive

- Law of contrapositive

  $$(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$$

- Prove $(\neg B \Rightarrow \neg A)$ → Conclude $(A \Rightarrow B)$

Proof by Contrapositive

Proposition: If a, b $\in \mathbb{Z}$ s.t. a+b is even, then a and b have the same *parity*

→ whiteboard

Proof Techniques

Proof by cases

Disproof by counterexample

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Proof by contrapositive

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Proof by contradiction

Proof by Contradiction

Template

Goal: Proof X

① ¬X by Assumption

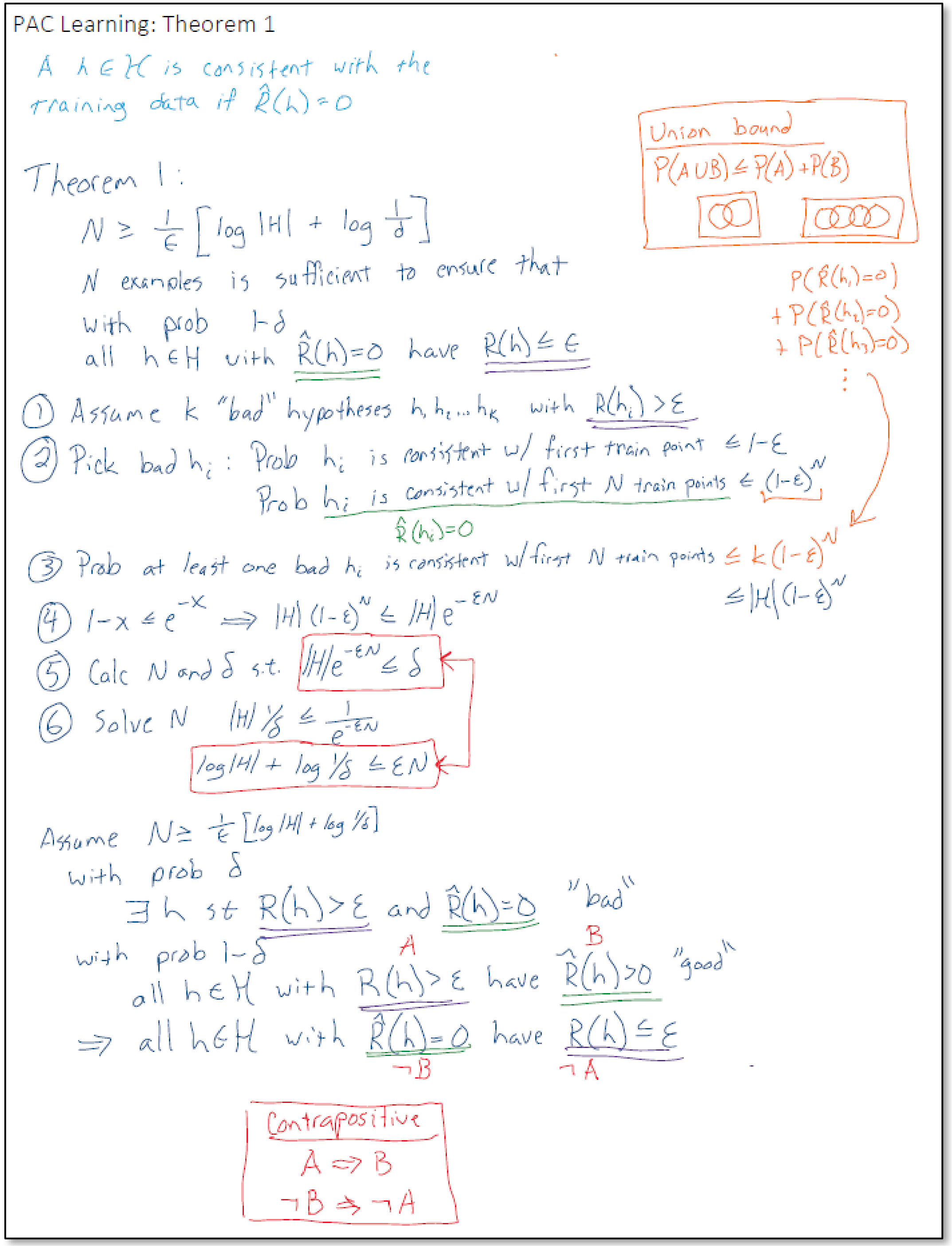
⋮

⑤ logical contradiction

⑥ X

Example from Intro ML

Note: Just an example. Out of scope.



Plan

Wrap-up Overview → Lecture 1 slides

Proof Techniques

- ✓ Proof by cases
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Perceptron Algorithm

- Prep: ML tasks, data, notation
- Prep: Geometry of linear models

ML Data, Tasks, Notation

Notation alert!

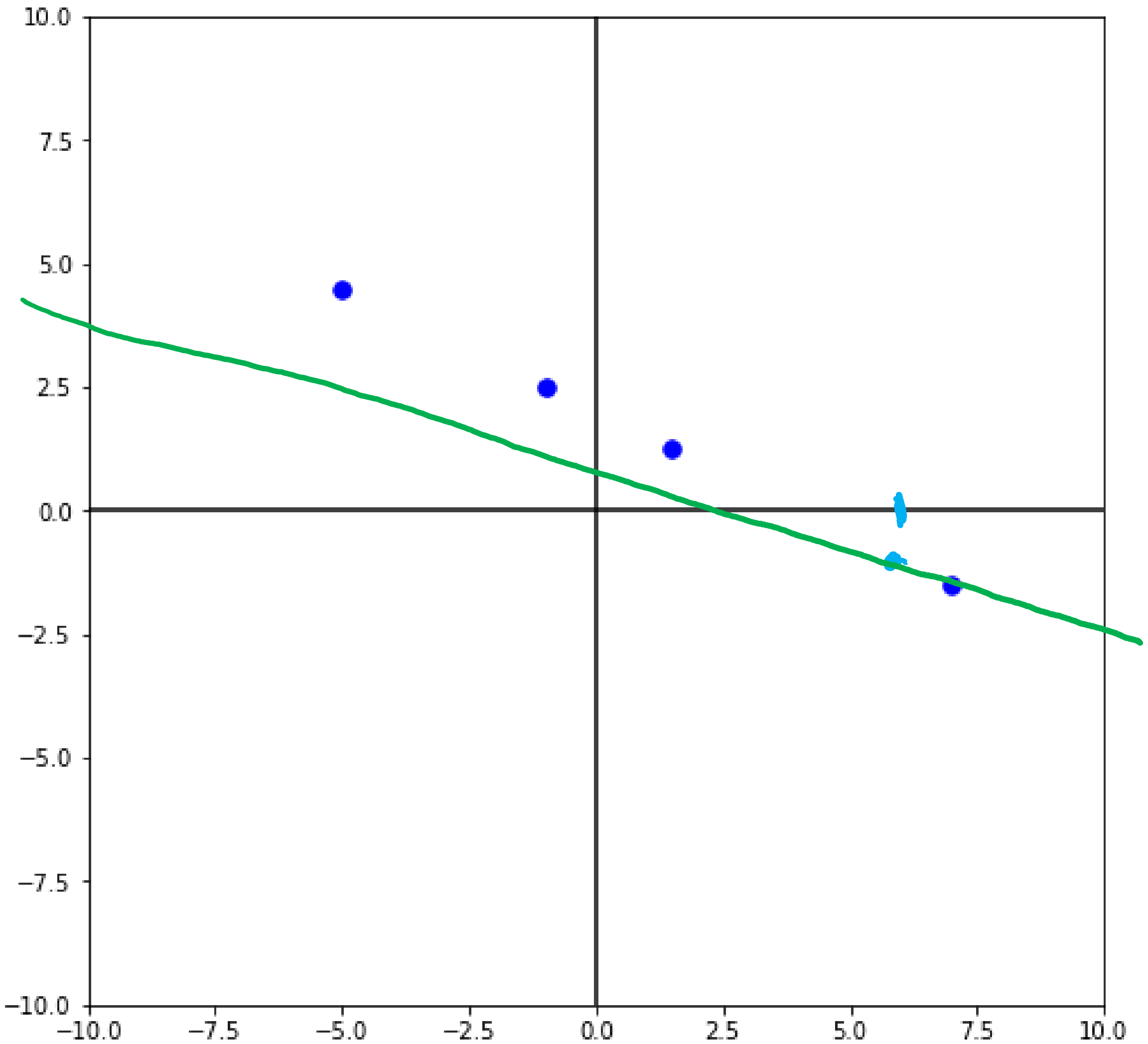
Regression

$$\mathcal{D} = \{(x^{(i)}, \quad y^{(i)})\}_{i=1}^{4}$$
$$= \{(-1, \quad 2.5),$$
$$(\ \ 7, \ -1.5),$$
$$(-5, \quad 4.5),$$
$$(1.5, \quad 1.25)\}$$

$y = f(x)$

$= w x + b$

parameters

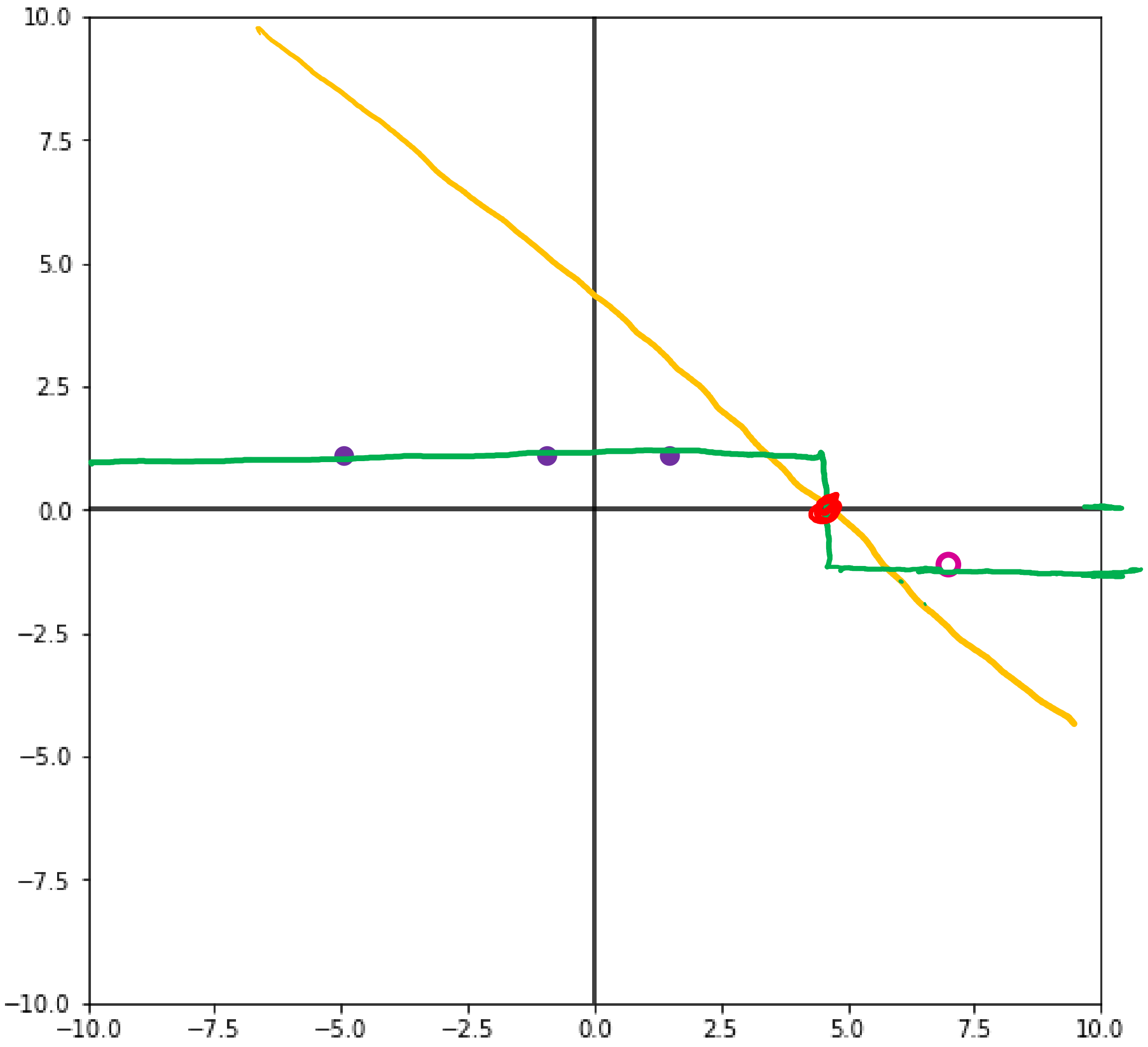


ML Data, Tasks, Notation

Classification

$$\mathcal{D} = \{(x^{(i)}, \quad y^{(i)})\}_{i=1}^{4}$$
$$= \{(-1, \quad 1),$$
$$(\quad 7, \quad -1),$$
$$(-5, \quad 1),$$
$$(1.5, \quad 1)\}$$

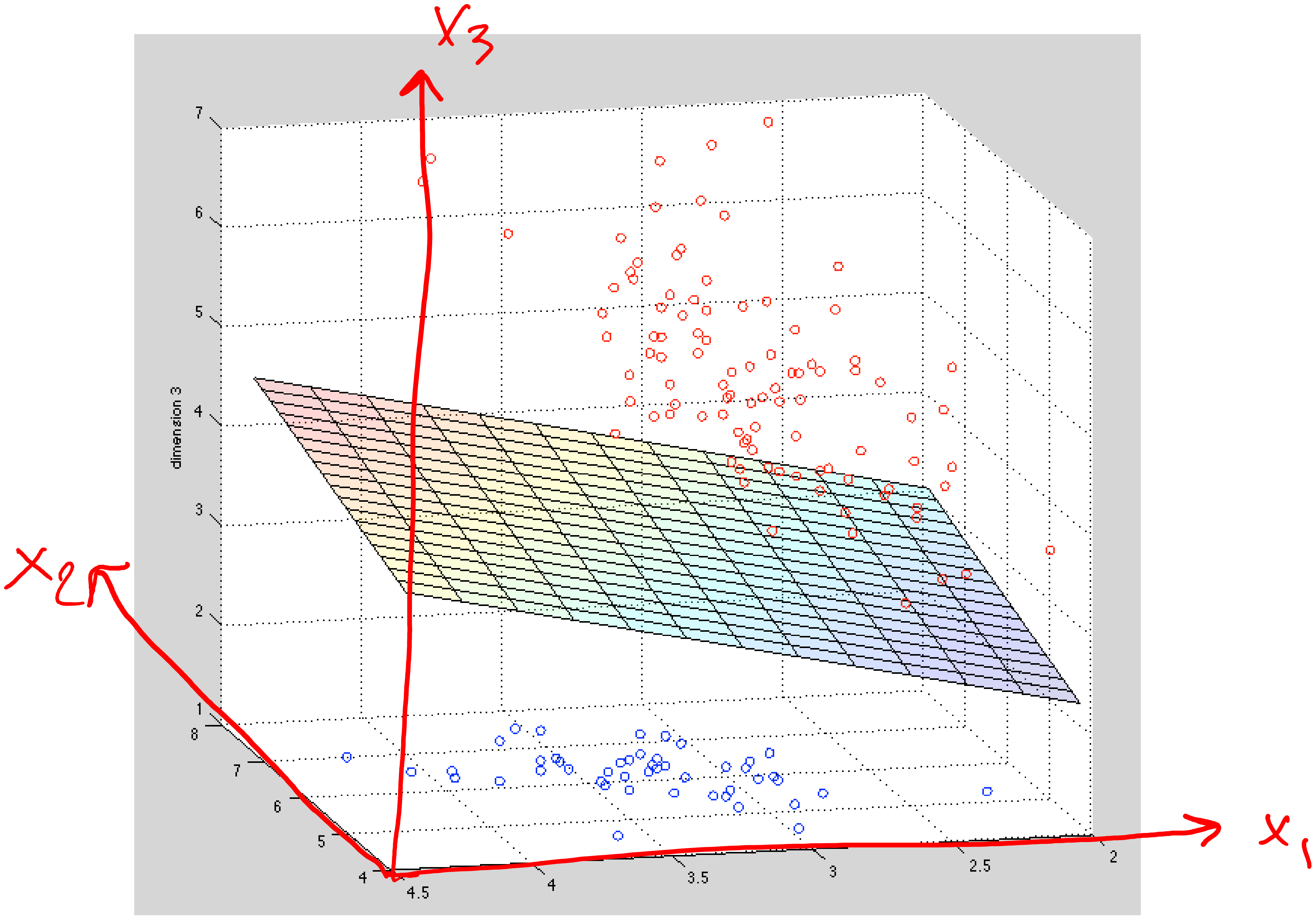
$z = wx + b$



$y = \text{sign}(wx + b)$

$$\text{sign}(z) = \begin{cases} +1 & \text{if } z \geq 0 \\ -1 & \text{o.w.} \end{cases}$$

Perceptron



Exercise

Geometry

Draw a picture of the region corresponding to:

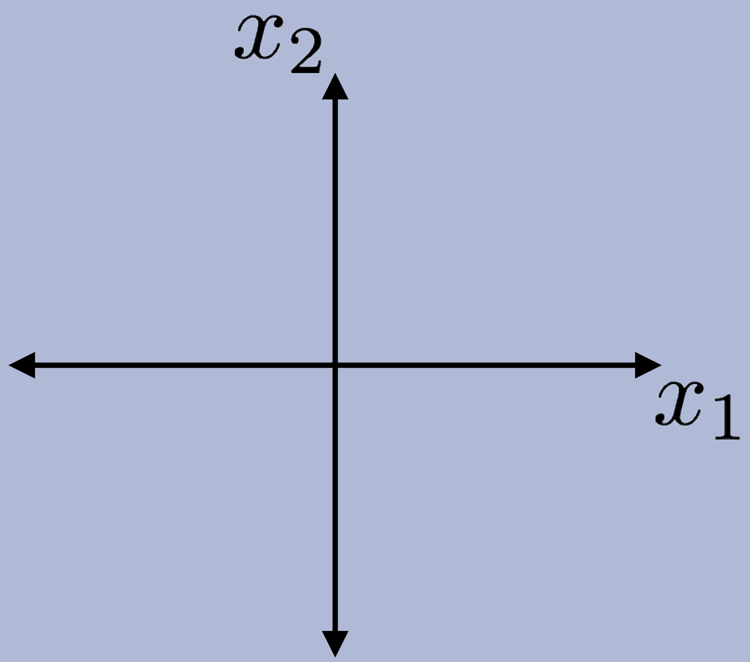
$$w_1 x_1 + w_2 x_2 + b > 0$$

where $w_1 = 2, w_2 = 3, b = 6$

Draw the vector

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Answer Here:

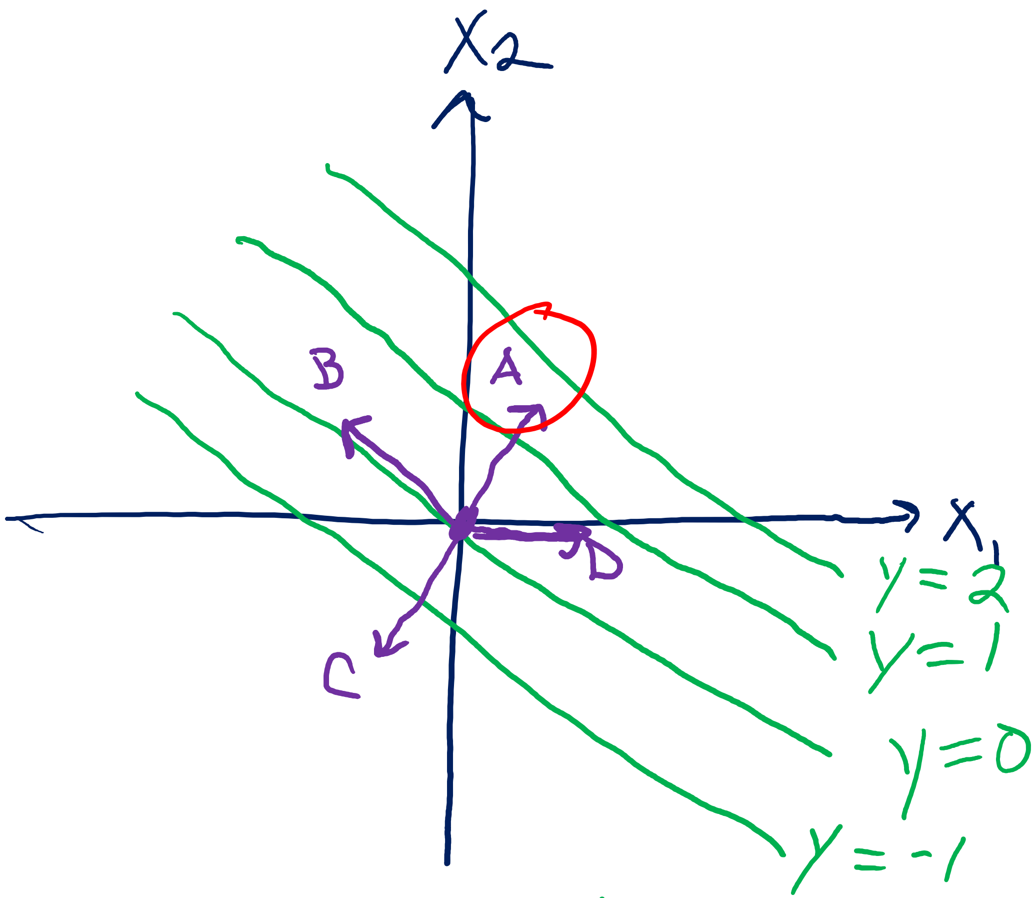


Slide credit: CMU MLD Matt Gormley

Poll 1

Which is the correct vector $\boldsymbol{w}$?

A.
B.
C.
D.
E. I don't know



$$y = \vec{w}^T \vec{x} = w_1 x_1 + w_2 x_2$$