Correction

$a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$

Propositional logic inference rules

- *modus ponens*: from premesis p and $p \Rightarrow q$, conclude q
- $\wedge$ introduction: if we separately prove p and q, then that constitutes a proof of $p \wedge q$.
- $\wedge$ elimination: from $p \wedge q$ we can conclude either of p and q separately.
- $\vee$ introduction: from p we can conclude $p \vee q$ for any q.
- $\vee$ elimination (also called proof by cases): if we know $p \vee q$ (the cases) and we have both $p \vee r$ and $p \vee r$ (the case-specific proofs), then we can conclude r.
- T introduction: we can conclude T from no assumptions.
- F elimination: from F we can conclude an arbitrary formula p.
- Associativity: both $\wedge$ and $\vee$ are associative: it doesn't matter how we parenthesize an expression like $a \wedge b \wedge c \wedge d$. (So in fact we often just leave the parentheses out in such cases. But when having $\vee$ and $\wedge$ together, it's a good idea to keep the parentheses.)
- Distributivity: $\wedge$ and $\vee$ distribute over one another; for example $a \wedge (b \vee c)$ is equivalent to $(a \vee b) \wedge (a \vee c)$.
- Commutativity: both $\wedge$ and $\vee$ are commutative (symmetric in the order of their arguments), so we can re-order their arguments however we please. For example, $a \wedge b \wedge c$ is equivalent to $c \wedge b \wedge a$.

Plan

Wrap-up Overview → Lecture 1 slides

Proof Techniques

- Proof by cases
- Disprove by counterexample
- Proof by contrapositive

10-607 Computational Foundations for Machine Learning

Proof Techniques

Instructor: Pat Virtue

Proof Techniques

Proof by Cases

Proof by Cases

Goal is to prove χ

1	$\phi \vee \psi$	Assume
2	Case 1: ϕ	
	a)	
	b)	
	c) $\phi \Rightarrow \chi$	
3	Case 2: ψ	
	a)	
	b)	
	c)	
	d) $\psi \Rightarrow \chi$	
4	χ	by $\vee$ elimination

Proof by Cases

Notation Alert!

Proposition: Let a, b $\in \mathbb{Z}$. If ab is even, then either a is even or b is even (or both).

Case 1: e e

$2 \cdot 2 = 4$

case 2: e o

$2 \cdot 3 = 6$

case 3: o e

$3 \cdot 2 = 6$

case 4: o o

$3 \cdot 3 = 9$

Proof by Cases

Proposition: Let a, b $\in \mathbb{Z}$. If ab is even, then either a is even or b is even (or both).

→ whiteboard

Proof Techniques

Proof by cases

Disproof by counterexample

- One example is not sufficient to prove
- One counterexample is sufficient to disprove

Disprove by counterexample

Proposition: Let a, b ∈ ℤ. If a is odd and b is odd, then a+b is odd

Let $a = 3$

Let $b = 3$

$a + b = 3 + 3 = 6$

$a + b$ not odd

Poll 1

Given model m: {A: True, B: False}

Does m satisfy the following sentence:

F ¬F ¬T

$(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$

T F T ⇒ F

F

i. Yes

ii. No

iii. Not enough information

iv. Syntax error in sentence

Poll 2

Truth table for $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$

How many rows do we need (excluding a header)?

i. 2

ii. 4

iii. 6

iv. 8

v. 16

Poll 3

Truth table for $(A\Rightarrow B)\Leftrightarrow(\neg B\Rightarrow\neg A)$

$\neg B$

$\neg A$

How many columns should we have?

i. 2

ii. 3 ←

iii. 4

iv. 5 ←

v. 6

7 ←

A	B	$A\Rightarrow B$	$\neg B \Rightarrow \neg A$	$\Leftrightarrow$
T	T			
T	F			
F	T			
F	F			