Warm-up Exercise

Propositional logic inference rules

- *modus ponens*: from premesis p and $p \Rightarrow q$, conclude q
- $\wedge$ introduction: if we separately prove p and q, then that constitutes a proof of $p \wedge q$.
- $\wedge$ elimination: from $p \wedge q$ we can conclude either of p and q separately.
- $\vee$ introduction: from p we can conclude $p \vee q$ for any q.
- $\vee$ elimination (also called proof by cases): if we know $p \vee q$ (the cases) and we have both $p \vee r$ and $p \vee r$ (the case-specific proofs), then we can conclude r.
- T introduction: we can conclude T from no assumptions.
- F elimination: from F we can conclude an arbitrary formula p.
- Associativity: both $\wedge$ and $\vee$ are associative: it doesn't matter how we parenthesize an expression like $a \wedge b \wedge c \wedge d$. (So in fact we often just leave the parentheses out in such cases. But when having $\vee$ and $\wedge$ together, it's a good idea to keep the parentheses.)
- Distributivity: $\wedge$ and $\vee$ distribute over one another; for example, $a \vee (b \wedge c)$ is equivalent to $(a \vee b) \wedge (a \vee c)$ and $a \wedge (b \vee c)$ is equivalent to $(a \wedge b) \vee (a \wedge c)$.
- Commutativity: both $\wedge$ and $\vee$ are commutative (symmetric in the order of their arguments), so we can re-order their arguments however we please. For example, $a \wedge b \wedge c$ is equivalent to $c \wedge b \wedge a$.

Use the propositional logic inference rules provided to prove:

$$(a \wedge b) \Rightarrow (b \wedge a)$$

However, you cannot use the commutativity rule.

Write your proof in two-column format, i.e., give an explicit justification for each statement based on previous statements