

CARNEGIE MELLON UNIVERSITY 10-607

HOMEWORK 1

DUE: Wednesday, Nov. 3, 2021

<https://www.cs.cmu.edu/~10607>

INSTRUCTIONS

- **Format:** Use the provided LaTeX template to write your answers in the appropriate locations within the *.tex files and then compile a pdf for submission. We try to mark these areas with STUDENT SOLUTION HERE comments. Make sure that you don't change the size or location of any of the answer boxes and that your answers are within the dedicated regions for each question/part. If you do not follow this format, we may deduct points.

You may also type you answer or write by hand on the digital or printed pdf. Illegible handwriting will lead to lost points. However, we suggest that try to do at least some of your work directly in LaTeX.

- **How to submit:** Submit to Gradescope a pdf with your answers. Again, make sure your answer boxes are aligned with the original pdf template.
- **Policy:** See the course website for homework policies, including late policy, and academic integrity policies.

Name	
Andrew ID	
Hours to complete all components (nearest hour)	

1 Proof Techniques [36 pts]

1. [6 pts] For any two finite sets A and B , prove that $\neg(A \cup B) = \neg A \cap \neg B$.

Each line of your proof should have a justification, however, you don't need to explicitly write this justification.

Hint: Recall that two propositions p and q can be considered equivalent, if $p \Rightarrow q$ and $q \Rightarrow p$. It is sufficient to prove that any element of $\neg(A \cup B)$.

Proof

3. [6 pts] Let $n \in \mathbb{Z}$. Use proof by cases to prove that n^2 leaves a remainder of 0 or 1 when divided by 3.

Each line of your proof should have a justification, however, you don't need to explicitly write this justification.

Hint: There are three possibilities for n . $n = 3k$, $n = 3k + 1$, $n = 3k + 2$

Proof

4. **[6 pts]** Let r be a rational number and let a be an irrational number. Use proof by contradiction to prove that $r + a$ is irrational.

Provide a 2-column proof with a written justification for each step.

Proof	

5. [6 pts] Prove by contrapositive: Let $x \in \mathbb{Z}$. If $x^2 - 6x + 5$ is even, then x is odd.

Each line of your proof should have a justification, however, you don't need to explicitly write this justification.

Hint: To prove by contrapositive, assume x is even. For propositions p and q , if $\neg q \Rightarrow \neg p$, $p \Rightarrow q$.

Proof

6. [6 pts] Suppose we have a binary classifier which predicts $\hat{y} = 1$ if $f(x) > \theta$ and $\hat{y} = 0$ if $f(\mathbf{x}) \leq \theta$, where $\mathbf{x} \in \mathbb{R}^M$ is a sample input from your data, $\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^N$ and f is a function in $\mathbb{R}^M \rightarrow \mathbb{R}$.

The term *recall* is defined as the proportion of the number of true positives ($y = 1$ and $\hat{y} = 1$) to the number of data points that marked positive in our data ($y = 1$).

Prove that recall for this classifier increases monotonically as we decrease θ .

Each line of your proof should have a justification, however, you don't need to explicitly write this justification.

Proof

2 Collaboration Policy

After you have completed all other components of this assignment, report your answers to the following collaboration questions.

1. Did you receive any help whatsoever from anyone in solving this assignment? If so, include full details including names of people who helped you and the exact nature of help you received.

2. Did you give any help whatsoever to anyone in solving this assignment? If so, include full details including names of people you helped and the exact nature of help you offered.

3. Did you find or come across code that implements any part of this assignment? If so, include full details including the source of the code and how you used it in the assignment.