

Computational Foundations for ML

10-607

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AND is associative

1.	assume $(a \wedge b) \wedge c$	assumption
2.	c	\wedge -elim from 1
3.	$a \wedge b$	" "
4.	a	" "
5.	b	" "
6.	$b \wedge c$	\wedge -intro 5,2
7.	$a \wedge (b \wedge c)$	" 4,6 conclusion
	$(a \wedge b) \wedge c$	\rightarrow lemma

Exercise

Show that AND is commutative

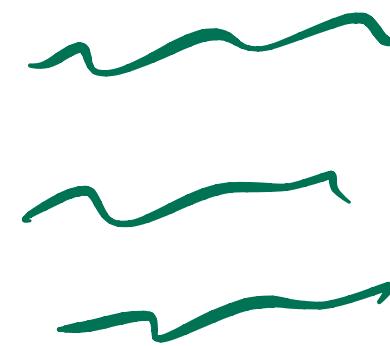
assume

$$a \wedge b$$

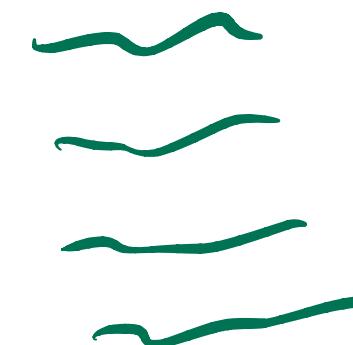
show

$$b \wedge a$$

Assume something



Assume something else



→ intro (lemma¹)



→ intro (second lemma²)



lemma 1
available here

lemma 2
available

1. Assume $PB \wedge J \rightarrow \text{Sandwich}$
2. Assume PB
3. Assume J .
4. Conclude $PB \wedge J$ $\wedge\text{-intro } 2, 3$
5. Conclude Sandwich $\text{m.p. } 1, 4$
6. Conclude $J \rightarrow \text{Sandwich}$ $\rightarrow\text{intro, } 3-5$
7. Conclude $PB \rightarrow (J \rightarrow \text{Sandwich})$ $\rightarrow\text{intro, } 2-6$
8. Conclude $[PB \wedge J \rightarrow \text{Sandwich}]$
 $\Rightarrow [PB \rightarrow (J \rightarrow \text{Sandwich})]$

$$\neg x \equiv x \rightarrow F$$

from $\neg\neg\phi$ conclude ϕ

DNE

$$\phi \vee \neg\phi$$

$$\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$$

$$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$$

excluded middle

} De Morgan

$$((\phi \rightarrow \psi) \rightarrow \phi) \rightarrow \phi$$

Peirce

Resolution

$$a \vee b \vee \neg c \vee \textcircled{d} \vee \neg e$$

$$b \vee \neg d \vee f$$

$$a \vee d \vee \neg c \quad \vee \neg e$$

$$\cancel{\psi} \vee f$$

$$\begin{array}{c} \phi \vee \psi \quad \neg \phi \vee \chi \\ \hline \psi \vee \chi \end{array}$$

$$\begin{aligned} & (\phi \vee \psi) \wedge (\neg \phi \vee \chi) \\ & \Rightarrow (\psi \vee \chi) \end{aligned}$$

Types \mathbb{N} int char

$+$: $\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$

$(3, \text{spot}) : \mathbb{N} \times \text{Dof}$

$+ (3, 7) \mapsto 10$

(int | char) \vdash

$f : \text{int} \rightarrow \text{dof}$

$g : \text{char} \rightarrow \text{dof}$

$\mathbb{N}(7) = T$
 $\text{int}(3.2) = F$

$\lambda x, y, z. \ (x+y) \times z$

$(f \mid g) \vdash : \text{dof} \rightarrow \text{int} \mid \text{char}$

any none

type \mathcal{N} $o : \mathcal{N}$

$=$ $a = b$ (T or F)
 $a \neq b$

$$\phi = \phi$$

$$\phi = \psi \quad = \quad \psi = \phi$$

$$\phi = \psi \quad \psi = \chi$$

$$\phi = \chi$$

$$\frac{(3 + a) \times 5 \geq 10}{a = b}$$

$$(3 + b) \times 5 \geq 10$$

$s : \mathcal{N} \rightarrow \mathcal{N}$ $\phi = \psi \quad = \quad s(\phi) = s(\psi)$

$$s(\phi) \neq 0$$

induction

P a predicate

$$P(0)$$

$$P(x) \rightarrow P(s(x))$$

fresh term

$$P(\phi)$$

$$+ : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$$

$$\phi + 0 = \phi$$

$$\phi + s(\psi) = s(\phi + \psi)$$

$$\begin{aligned} 3+2 &= 3+s(1) \\ &= s(3+1) \\ &= s(3+s(0)) \\ &= ss(3+0) \\ &= ss3 \\ &= s \end{aligned}$$

$$P(a) \leftrightarrow a + 0 = 0 + a$$

$$P(0) \leftrightarrow 0 + 0 = 0 + 0 \quad \checkmark \quad \text{reflexive}$$

assume $P(x)$, prove $P(s(x))$

Given $x + 0 = 0 + x$

$$\begin{aligned} s(x+0) &= s(0+x) \\ s(x) &= s(0+x) \\ s(x)+0 &= s(0+x) \\ &= 0+s(x) \end{aligned}$$