

Announcements



HW1 & Quiz 1

- Graded
- Regrade requests
- Hours, performance

HW2

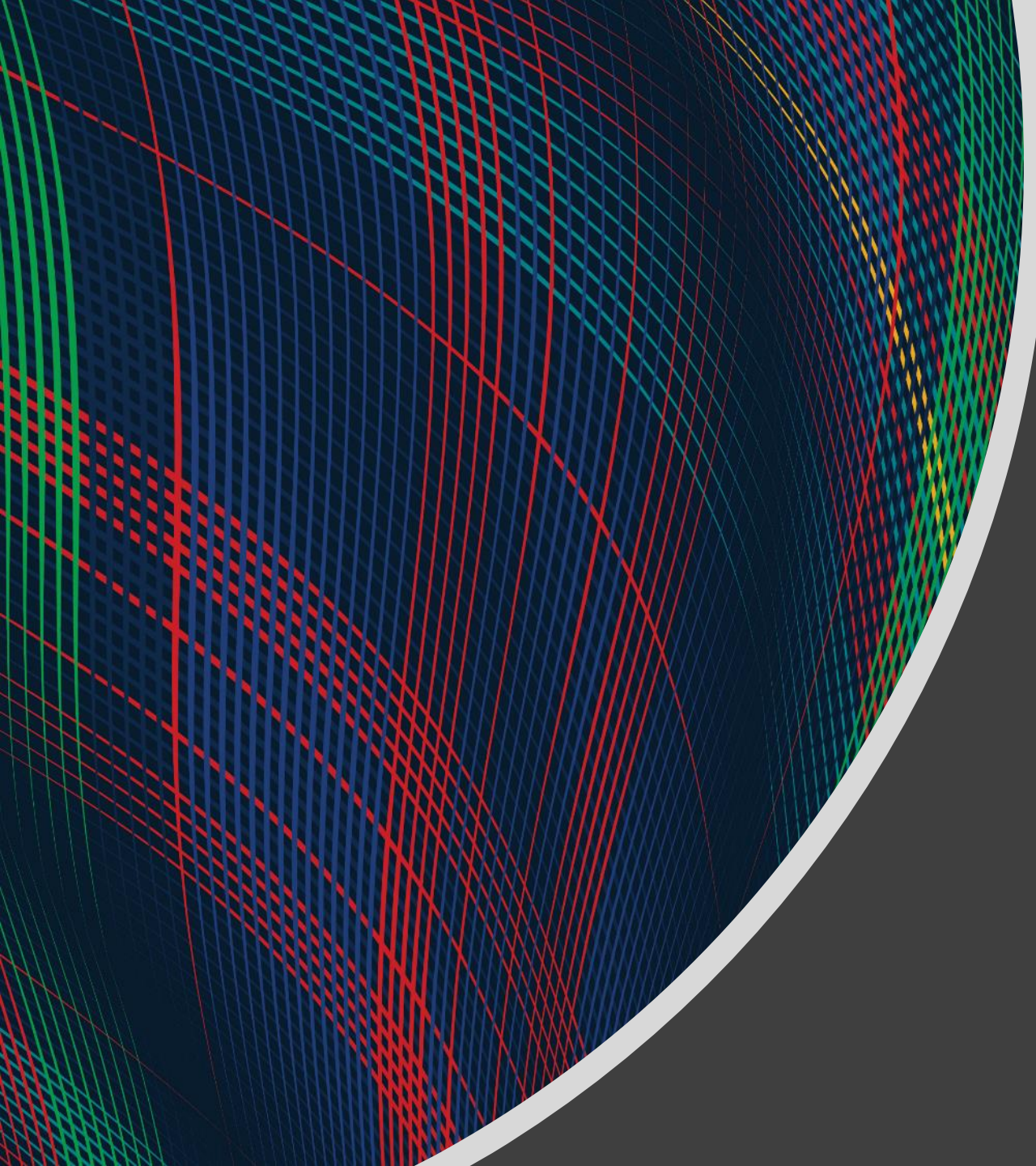
- Due Sat 10/2

Quizzes

- Mon 10/4, last 15 min. of class (calculus, optimization, Lagrange)
- Mon 10/11, last 15 min. of class (probability, statistics) 

Constrained Optimization Clarification

Previous lecture slides



Mathematical Foundations for Machine Learning

Probability

Instructor: Pat Virtue

Plan

Today

Probability

- Vocab
- Notation
- Discrete distributions
- Continuous distributions
- Chain rule
- Independence

Probability

Probability Vocab

Outcomes

Sample space

Events

Probability

Random variable

Discrete random variable

Continuous random variable

Probability mass function

Probability density function

Parameters

Example

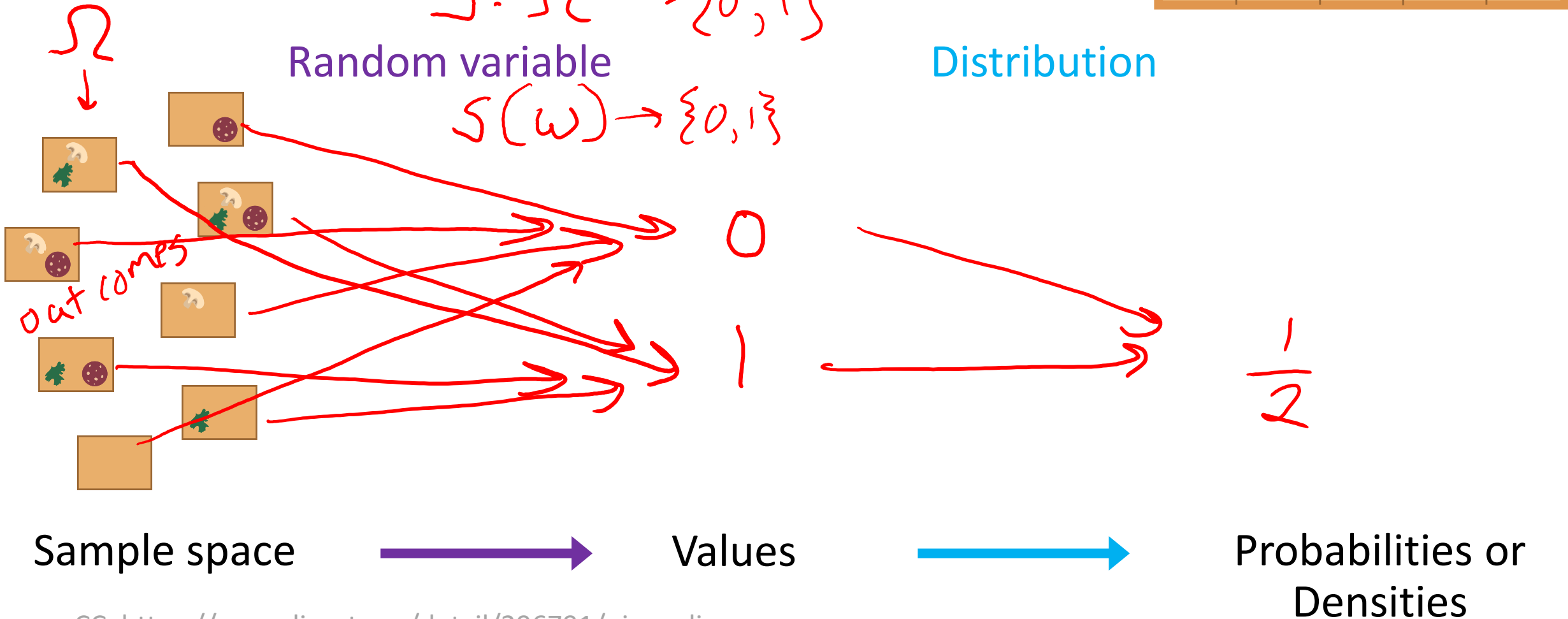
Random variable for spinach or no spinach

$$S: \Omega \rightarrow \{0, 1\}$$

Random variable

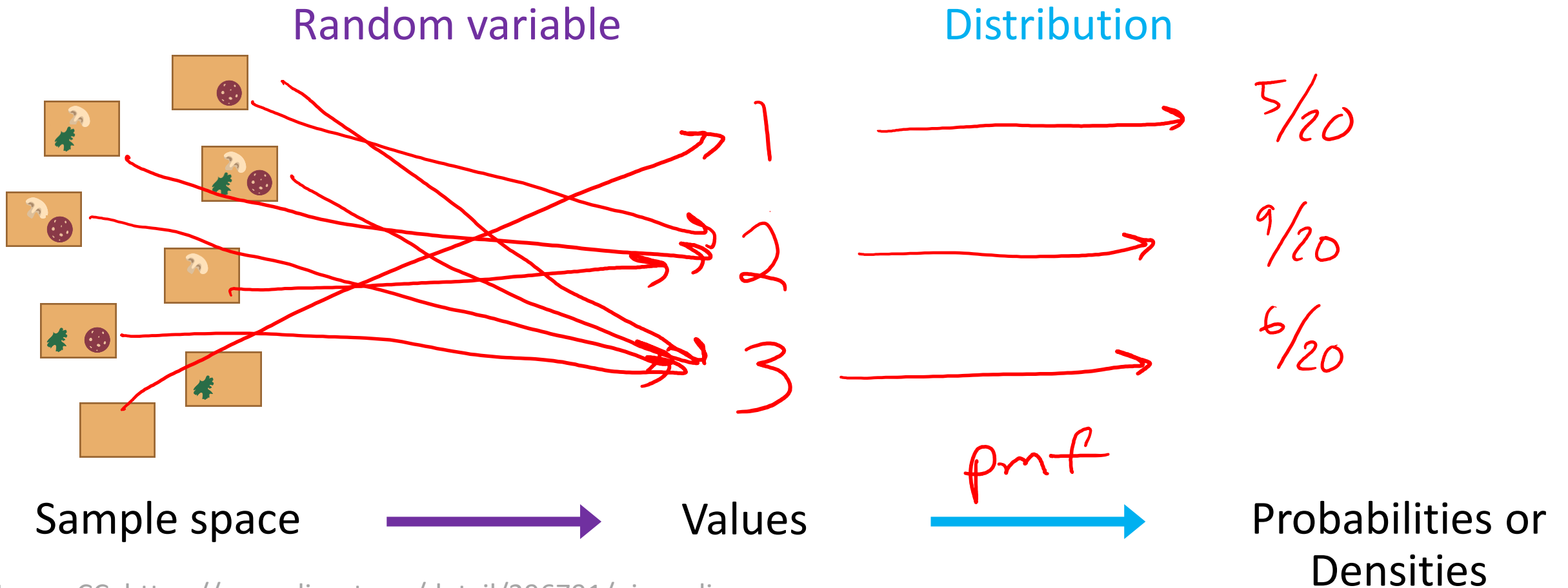
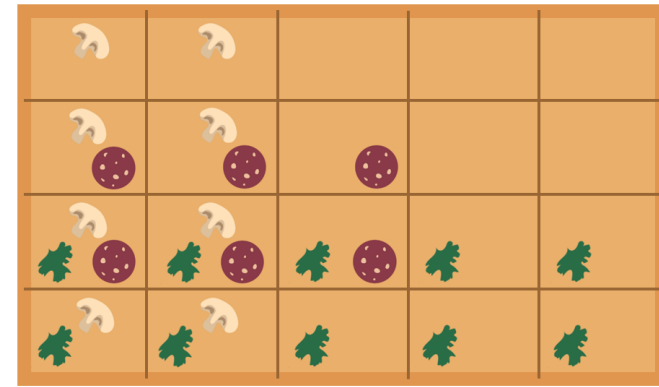
$$S(\omega) \rightarrow \{0, 1\}$$

Distribution



Example

Random variable for topping type with three categories: none, non-meat, meat

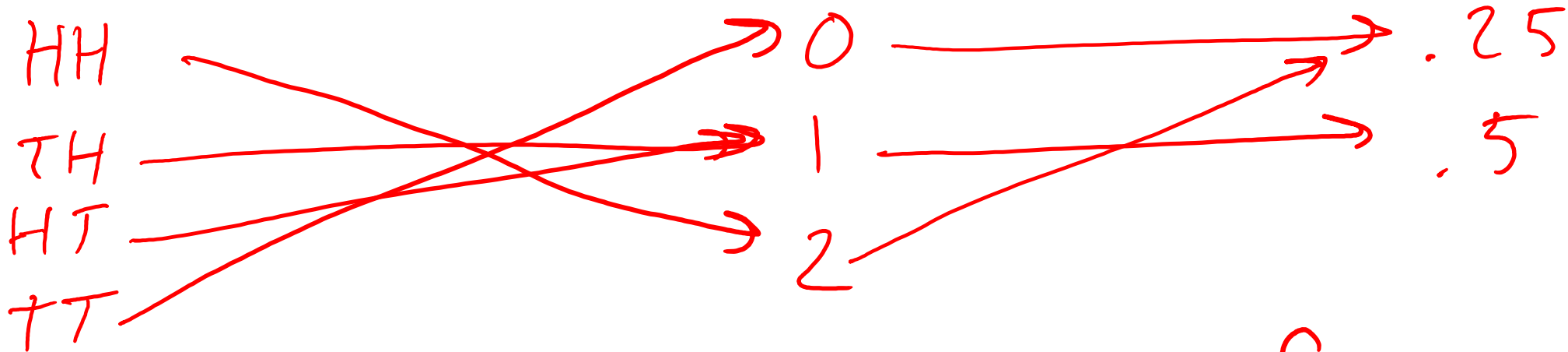


Example

Random variable for number of heads after
two flips of a fair coin

Random variable

Distribution



Sample space



Values



Probabilities or
Densities

Example

Random variable for number of heads after two flips of a *biased* coin that lands heads ~~75%~~

Random variable

Distribution

0

1

2

$$\frac{1}{4} \cdot \frac{1}{4}$$

$$\frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{4}$$

$$\frac{3}{4} \cdot \frac{3}{4}$$

Sample space



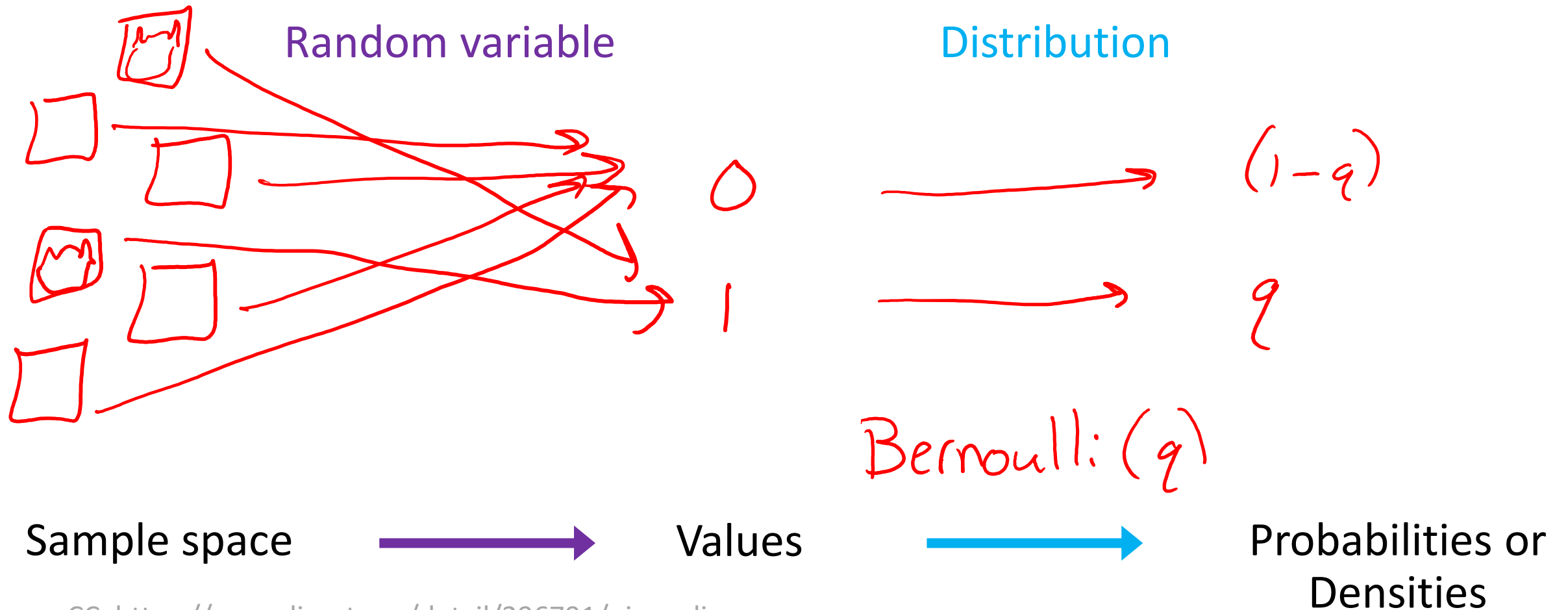
Values



Probabilities or
Densities

Example

Random variable for cat in picture or not



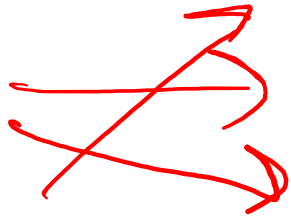
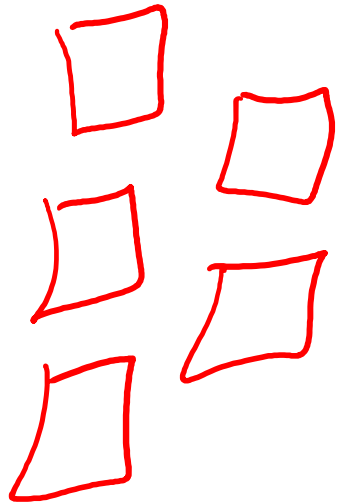
Example

Random variable for animal species in picture assuming one animal picture and available species: dog, cat, pig

Random variable

Distribution

$$\checkmark \sum_{i=1}^3 q_i = 1$$



1
2
3

q_1

q_2

$q_3 = (1 - q_1 - q_2)$

Sample space



Values



Categorical (q_1, q_2, q_3)

Probabilities or
Densities

Example

Random variable for height of student

$$H(\omega) \rightarrow \mathbb{R}$$

Random variable

Continuous

$$f(x) \rightarrow \mathbb{R}^+$$

Distribution

Gaussian
 $N(\mu, \sigma^2)$

$f(x; \mu, \sigma^2)$
pdf
parameters



$\mathbb{R}$

Sample space



Values



Probabilities or
Densities

Probability Vocab

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Random variable

Discrete random variable

Continuous random variable

Probability mass function $p(x) = P(X=x)$

Probability density function $f(x)$

Parameters

$$\begin{array}{c} \text{CDF} \\ \hline F(x) = P(X \leq x) \end{array}$$

Example Discrete Distributions

Bernoulli

← q

Bern	Categorical
Binomial	Multinomial

Categorical

←

K categories

$q_1, \dots, q_K$

$$\sum_{i=1}^K q_i = 1$$

Binomial

Multinomial

Uniform

Example Continuous Distributions

Gaussian

$$x \sim \mathcal{N}(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

pdf

Beta

Laplace

Probability Vocab

Marginal

$$p(x) = \sum_y p(x, y) \quad \text{Marginalizing}$$

Joint

$$p(x, y) \quad p(x, y, z, w)$$

Conditional

$$p(x | y) \quad p(x, y \mid z, w)$$

Notation

Notation doc

$$P(X=x)$$

$$f_x = \frac{\partial f}{\partial x}$$

$$f_x \quad \text{pdf for r.v. } X$$

$$f_\theta \quad \text{pdf with parameters } \theta$$

Probability Toolbox

- Algebra

- Three axioms of probability

- Theorem of total probability

- Definition of conditional probability

- Product rule

- Bayes' theorem

- ■ Chain rule

- ┌ ■ Independence

- ■ Conditional independence

Tools Summary

Adding to our toolbox

1. Definition of conditional probability
2. Product Rule
3. Bayes' theorem
4. Chain Rule...

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

$$P(A, B) = P(A|B)P(B)$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$