Derive the MLE for the Exponential Distribution

Exponential pdf:

$$f(x) = \lambda e^{-\lambda x}$$

Data: $\mathcal{D} = \left\{x^{(i)}\right\}_{i=1}^{N}$

Figure out each derivation step, given the provided justification

	Justification
$\mathcal{L}(\lambda) = p(D \mid \theta)$ $= p\left(x^{(1)}, \dots, x^{(N)} \mid \lambda^{(1)}, \dots, \lambda^{(N)}\right)$	
$= \prod_{i=1}^{N} p\left(x^{(i)} \mid \lambda\right)$	$i.i.d.$ (don't plug in exponential pdf yet)
$\ell(\lambda) = \log \prod_{i=1}^{N} p\left(x^{(i)} \mid \lambda\right)$	Log likelihood
$= \sum_{i=1}^{N} \log\, p\left(x^{(i)} \mid \lambda\right)$	Distribute log over product
$= \sum_{i=1}^{N} \log\left(\lambda e^{-\lambda x^{(i)}}\right)$	Plug in exponential pdf
$= \sum_{i=1}^{N} \log \lambda - \lambda x^{(i)}$	Distribute log over pdf expression
$= N\log(\lambda) - \lambda \sum_{i}^{N} x^{(i)}$	Distribute/simplify summations
$\frac{d\ell}{d\lambda} = \frac{d}{d\lambda}\left[N\log(\lambda) - \lambda \sum_{i}^{N} x^{(i)}\right]$	Derivative with respect to parameter
$0 = \frac{N}{\lambda} - \sum_{i}^{N} x^{(i)}$	Take derivative of each term (and set equal to zero)
$\hat{\lambda}_{MLE} = \frac{N}{\sum_{i}^{N} x^{(i)}}$	Solve for λ