

Derive the MLE for the Exponential Distribution

Exponential pdf:

$$f(x) = \lambda e^{-\lambda x}$$

Data: $\mathcal{D} = \{x^{(i)}\}_{i=1}^N$

Figure out each derivation step, given the provided justification

Justification

$$\mathcal{L}(\lambda) = p(\mathcal{D} \mid \theta)$$

$$= p(x^{(1)}, \dots, x^{(N)} \mid \lambda^{(1)}, \dots, \lambda^{(N)})$$

$$= \prod_{i=1}^N$$

i. i. d. (don't plug in exponential pdf yet)

$$\ell(\lambda) =$$

Log likelihood

$$=$$

Distribute log over product

$$=$$

Plug in exponential pdf

$$=$$

Distribute log over pdf expression

$$= N \log(\lambda) - \lambda \sum_i^N x^{(i)}$$

Distribute/simplify summations

$$\frac{d\ell}{d\lambda} = \frac{d}{d\lambda} [N \log(\lambda) - \lambda \sum_i^N x^{(i)}]$$

Derivative with respect to parameter

$$0 =$$

Take derivative of each term
(and set equal to zero)

$$\hat{\lambda}_{MLE} =$$

Solve for λ