

Warm-up as You Log In



Suppose we have a function that takes in a vector and squares each element individually, returning another vector, $\mathbf{y} = f(\mathbf{x})$.

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) \rightarrow \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{bmatrix}$$

$$f\left(\begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}\right) \rightarrow \begin{bmatrix} 49 \\ 9 \\ 25 \end{bmatrix}$$

What is $\partial \mathbf{y} / \partial \mathbf{x}$?

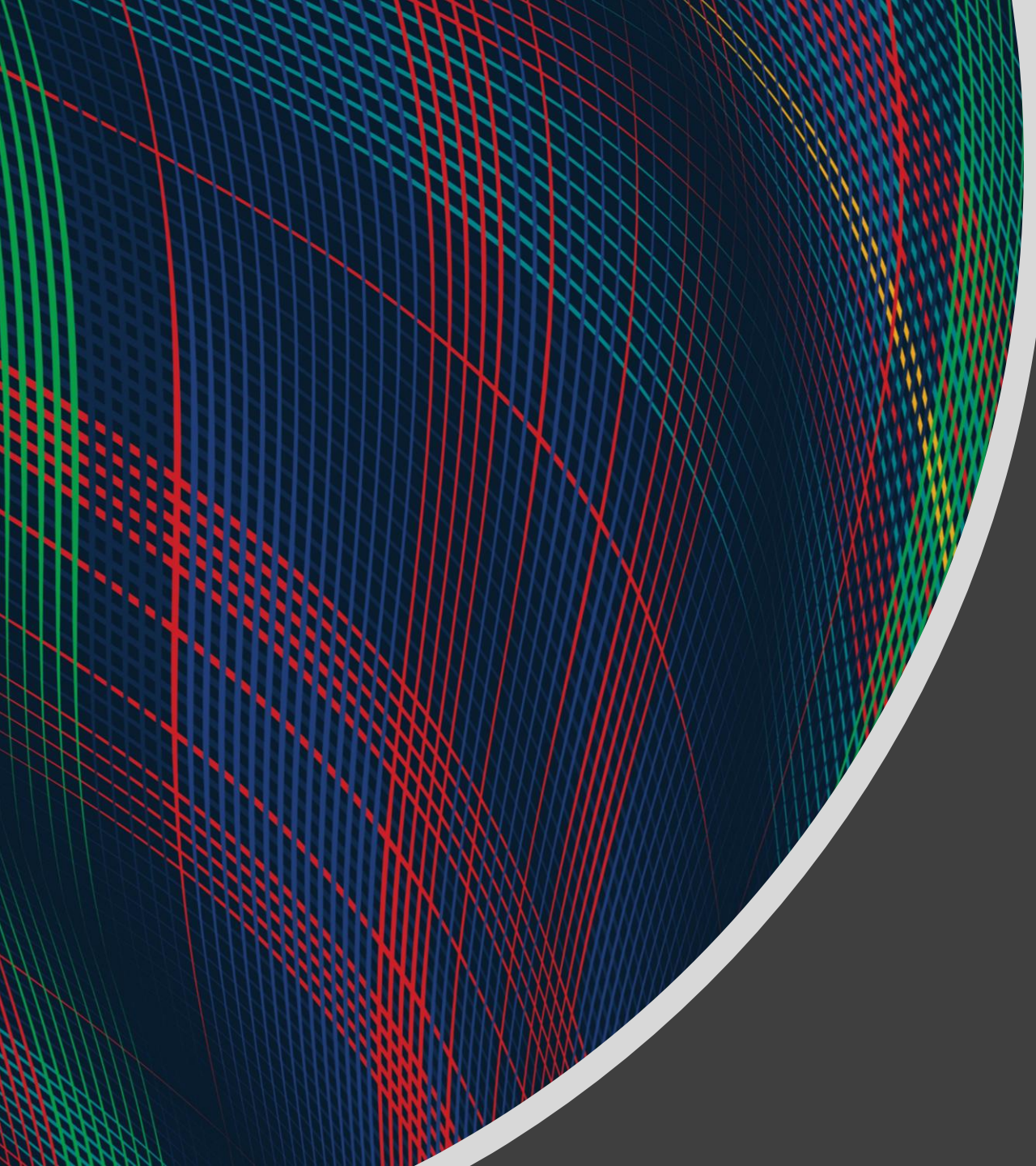
Announcements

Assignments

- HW5
 - Due Mon, 10/26, 11:59 pm
 - Start early

Recitation

- No recitation this Friday

An abstract graphic on the left side of the slide, featuring a sphere-like shape composed of a dense grid of intersecting red, green, and blue lines. The lines are curved and follow the contour of the sphere, creating a complex, woven pattern. The sphere is set against a dark gray background.

Introduction to Machine Learning

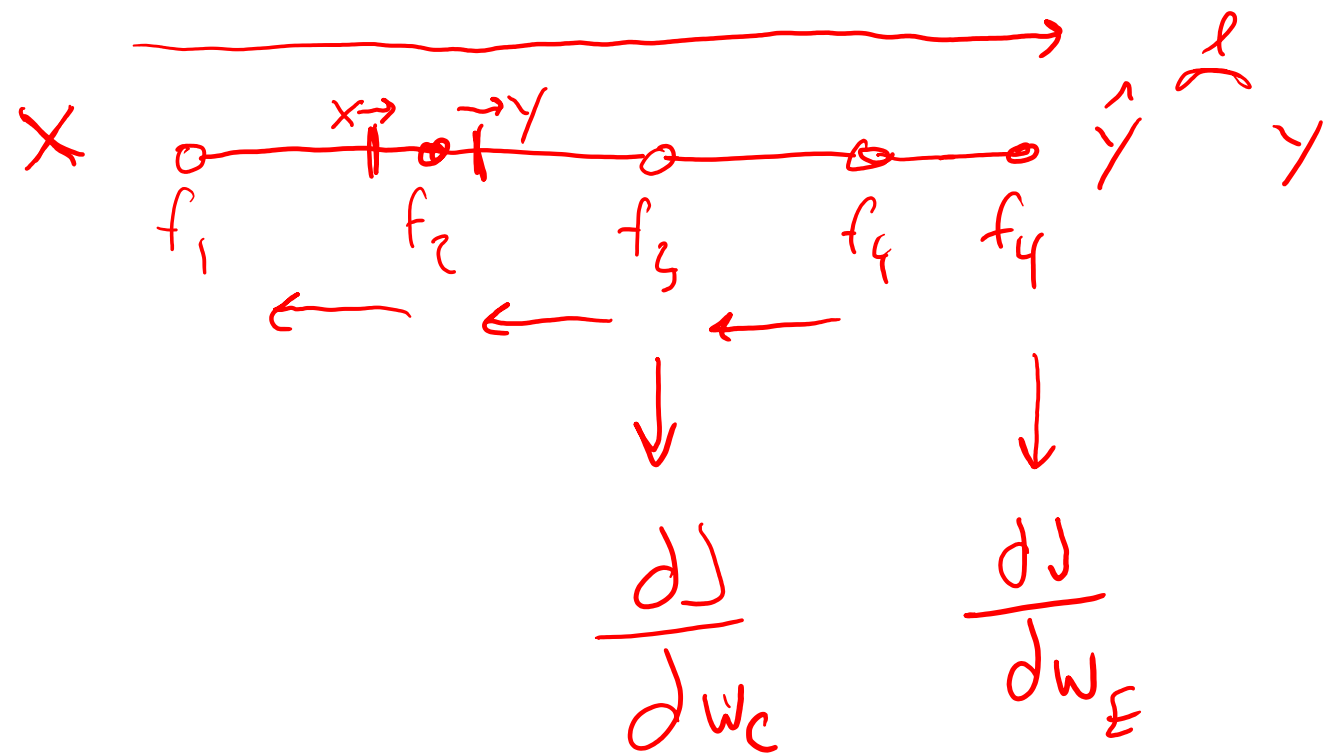
Neural Networks

Instructor: Pat Virtue

Plan

Last Time

- Neural Networks
 - Building blocks
 - Optimization



Today

- Neural Networks
 - Wrap-up calculus 
 - Universal Approximation Theorem
 - Convolutional neural networks

Backpropagation (so-far)

Compute derivatives per layer, utilizing previous derivatives

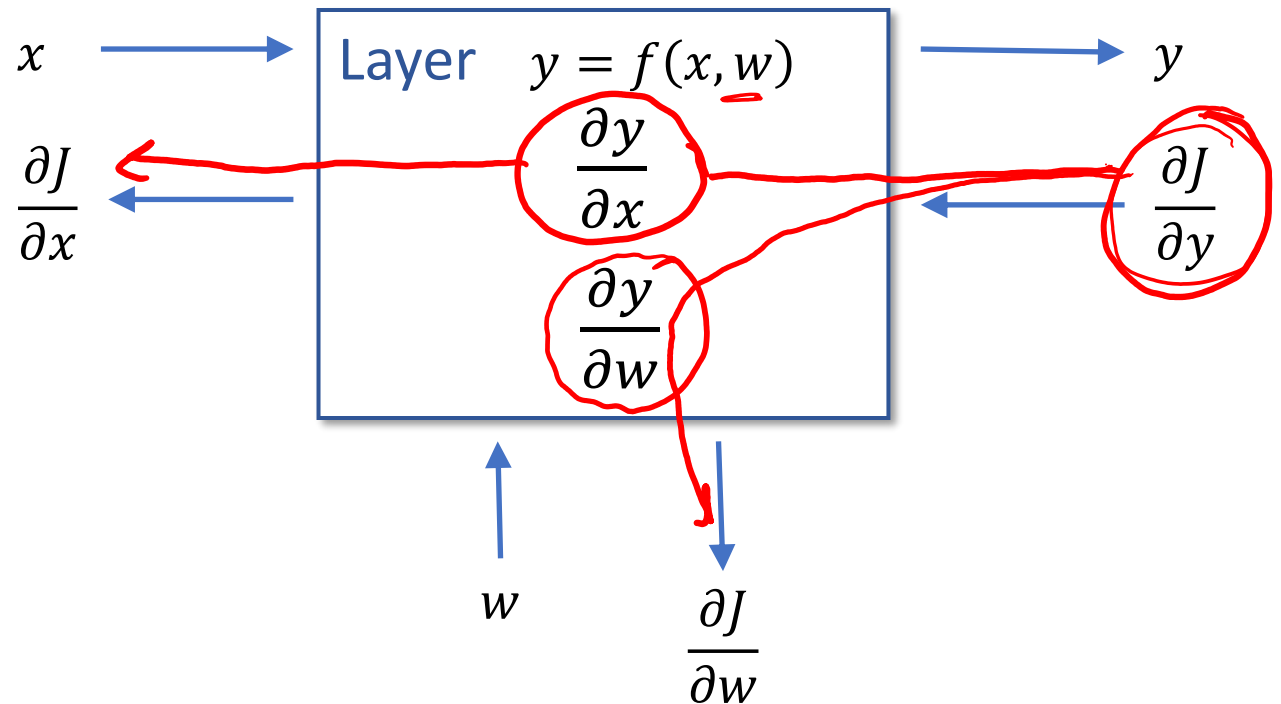
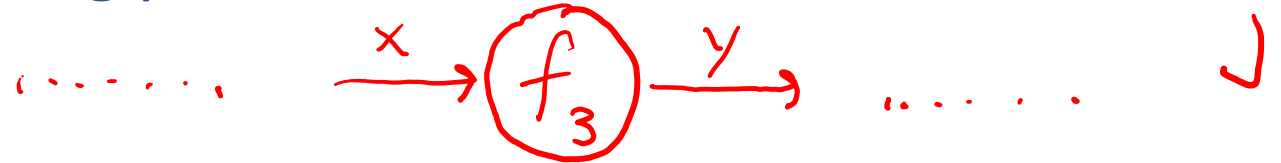
Objective: $J(\mathbf{w})$

Arbitrary layer: $y = f(x, w)$

Need:

$$\blacksquare \frac{\partial J}{\partial x} = \frac{\partial J}{\partial y} \frac{\partial y}{\partial x}$$

$$\blacksquare \frac{\partial J}{\partial w} = \frac{\partial J}{\partial y} \frac{\partial y}{\partial w}$$



Matrix Calculus

→ Jacobian: Vector in, vector out

→ Numerator-layout *num out x num in*

$$\mathbf{y} = f(\mathbf{x}) \quad \mathbf{y} \in \mathbb{R}^N, \quad \mathbf{x} \in \mathbb{R}^M, \quad \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \in \mathbb{R}^{N \times M}$$

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_N}{\partial x_1} & \dots & \frac{\partial y_N}{\partial x_M} \end{bmatrix}$$

Matrix Calculus

Vector in, scalar out

Numerator-layout

$$y = f(\boldsymbol{x}) \quad y \in \mathbb{R}, \quad \boldsymbol{x} \in \mathbb{R}^M, \quad \frac{\partial y}{\partial \boldsymbol{x}} \in \mathbb{R}^{1 \times M}$$

$$\frac{\partial y}{\partial \boldsymbol{x}} = \left[\frac{\partial y}{\partial x_1} \quad \cdots \quad \frac{\partial y}{\partial x_M} \right]$$

Matrix Calculus

Scalar in, vector out

Numerator-layout

$$\mathbf{y} = f(x) \quad \mathbf{y} \in \mathbb{R}^N, \quad x \in \mathbb{R}, \quad \frac{\partial \mathbf{y}}{\partial x} \in \mathbb{R}^{N \times 1}$$

$$\frac{\partial \mathbf{y}}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x} \\ \vdots \\ \frac{\partial y_N}{\partial x} \end{bmatrix}$$

Matrix Calculus

Gradient: Vector in, scalar out

Transpose of numerator-layout

$$y = f(\mathbf{x}) \quad y \in \mathbb{R}, \quad \mathbf{x} \in \mathbb{R}^M, \quad \frac{\partial y}{\partial \mathbf{x}} \in \mathbb{R}^{1 \times M}, \quad \nabla_{\mathbf{x}} f \in \mathbb{R}^{M \times 1}$$

$$\frac{\partial y}{\partial \mathbf{x}} = (\nabla_{\mathbf{x}} f)^T$$

$$\boxed{\phantom{\frac{\partial y}{\partial \mathbf{x}}}} = \boxed{\phantom{\nabla_{\mathbf{x}} f}}^T$$

Matrix Calculus

Matrix in, scalar out

Keep same dimensions as matrix

$$y = f(\mathbf{X}) \quad y \in \mathbb{R}, \quad \mathbf{X} \in \mathbb{R}^{N \times M}, \quad \frac{\partial y}{\partial \mathbf{X}} \in \mathbb{R}^{N \times M}$$

$$\frac{\partial y}{\partial \mathbf{X}} = \begin{bmatrix} \frac{\partial y}{\partial X_{1,1}} & \cdots & \frac{\partial y}{\partial X_{1,M}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y}{\partial X_{N,1}} & \cdots & \frac{\partial y}{\partial X_{N,M}} \end{bmatrix}$$

$$W \vec{x} \rightarrow \vec{a}$$

Warm-up as You Log In

$2\vec{x}$ $\leftarrow$

Suppose we have a function that takes in a vector and squares each element individually, returning another vector, $\mathbf{y} = f(\mathbf{x})$.

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) \rightarrow \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{bmatrix}$$

$$f\left(\begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}\right) \rightarrow \begin{bmatrix} 49 \\ 9 \\ 25 \end{bmatrix}$$

What is $\partial \mathbf{y} / \partial \mathbf{x}$?

$$= \begin{bmatrix} 2x_1 & 0 & 0 \\ 0 & 2x_2 & 0 \\ 0 & 0 & 2x_3 \end{bmatrix}$$

$$\frac{dy_i}{dx_j} = 2x_j \mathbb{1}(i=j)$$

Calculus Chain Rule

Scalar:

$$y = f(z)$$

$$z = g(x)$$

$$\frac{dy}{dx} = \frac{dy}{dz} \frac{dz}{dx}$$

Multivariate:

$$y = f(\mathbf{z})$$

$$\mathbf{z} = g(x)$$

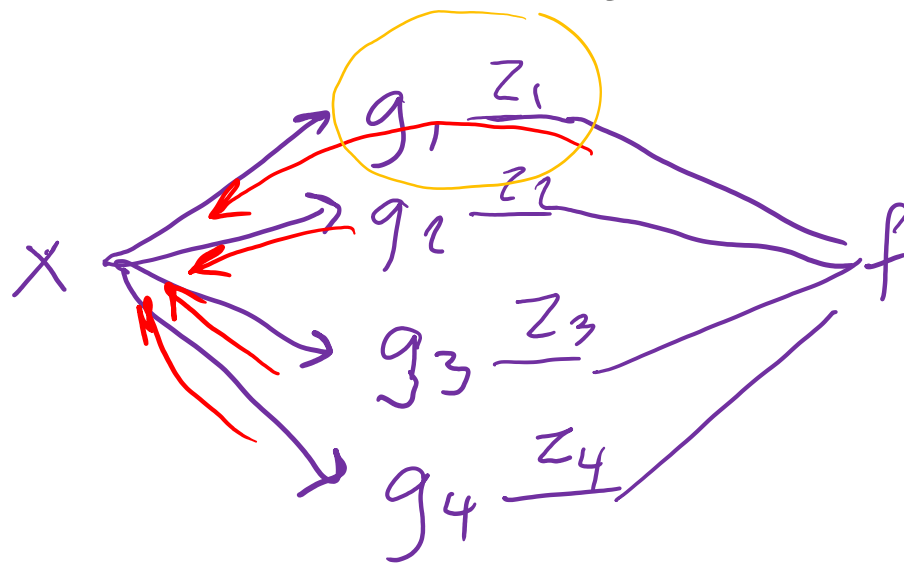
$$\frac{dy}{dx} = \sum_j \frac{\partial y}{\partial z_j} \frac{\partial z_j}{\partial x}$$

Multivariate:

$$\mathbf{y} = f(\mathbf{z})$$

$$\mathbf{z} = g(\mathbf{x})$$

$$\frac{\partial y_i}{\partial x_k} = \sum_j \frac{\partial y_i}{\partial z_j} \frac{\partial z_j}{\partial x_k}$$



Piazza Poll 1

out-by-in

$$y = f(\mathbf{z}) \quad y \in \mathbb{R}, \mathbf{z} \in \mathbb{R}^N, x \in \mathbb{R}$$

$$\mathbf{z} = g(x)$$

Select all that apply

$$\frac{\partial y}{\partial x} = \dots$$

75% A. $\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial x}$

B. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \frac{\partial \mathbf{z}}{\partial x}$

C. $\frac{\partial y}{\partial \mathbf{z}} \left(\frac{\partial \mathbf{z}}{\partial x}\right)^T$

D. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \left(\frac{\partial \mathbf{z}}{\partial x}\right)^T$

50% E. $\left(\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial x}\right)^T$

F. None of the above

$\frac{\partial y}{\partial x}$ $\frac{\partial y}{\partial \mathbf{z}}$ $\frac{\partial \mathbf{z}}{\partial x}$

$\square = \begin{matrix} \text{---} \\ N \end{matrix}$ $\begin{matrix} N \\ \text{---} \end{matrix}$

$[1 \times N] [N \times 1]$

Piazza Poll 1

$$y = f(\mathbf{z}) \quad y \in \mathbb{R}, \mathbf{z} \in \mathbb{R}^N, x \in \mathbb{R}$$

$$\mathbf{z} = g(x)$$

Select all that apply

$$\frac{\partial y}{\partial x} = \dots$$

A. $\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial x}$

B. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \frac{\partial \mathbf{z}}{\partial x}$

C. $\frac{\partial y}{\partial \mathbf{z}} \left(\frac{\partial \mathbf{z}}{\partial x}\right)^T$

D. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \left(\frac{\partial \mathbf{z}}{\partial x}\right)^T$

E. $\left(\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial x}\right)^T$

F. None of the above

Piazza Poll 2

out x in

$$y = f(\mathbf{z}) \quad y \in \mathbb{R}, \mathbf{z} \in \mathbb{R}^N, \underline{\mathbf{x} \in \mathbb{R}^M}$$

$$\mathbf{z} = g(\mathbf{x})$$

Select all that apply

$$\frac{\partial y}{\partial \mathbf{x}} = \dots$$

- A. $\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{x}}$
 B. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \frac{\partial \mathbf{z}}{\partial \mathbf{x}}$
 C. $\frac{\partial y}{\partial \mathbf{z}} \left(\frac{\partial \mathbf{z}}{\partial \mathbf{x}}\right)^T$
 D. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \left(\frac{\partial \mathbf{z}}{\partial \mathbf{x}}\right)^T$
 E. $\left(\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{x}}\right)^T$
 F. None of the above

$$\frac{\partial y}{\partial \vec{x}} \quad \begin{matrix} M \\ 1 \times M \end{matrix} = \frac{\partial y}{\partial \vec{z}} \quad \begin{matrix} \frac{\partial \vec{z}}{\partial \vec{x}} \\ N \quad M \end{matrix}$$

$[1 \times N][N \times M]$

Piazza Poll 2

$$y = f(\mathbf{z})$$

$$\mathbf{z} = g(\mathbf{x})$$

Select all that apply

$$\frac{\partial y}{\partial \mathbf{x}} = \dots$$

A. $\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{x}}$

B. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \frac{\partial \mathbf{z}}{\partial \mathbf{x}}$

C. $\frac{\partial y}{\partial \mathbf{z}} \left(\frac{\partial \mathbf{z}}{\partial \mathbf{x}}\right)^T$

D. $\left(\frac{\partial y}{\partial \mathbf{z}}\right)^T \left(\frac{\partial \mathbf{z}}{\partial \mathbf{x}}\right)^T$

E. $\left(\frac{\partial y}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{x}}\right)^T$

F. None of the above

Network Optimization

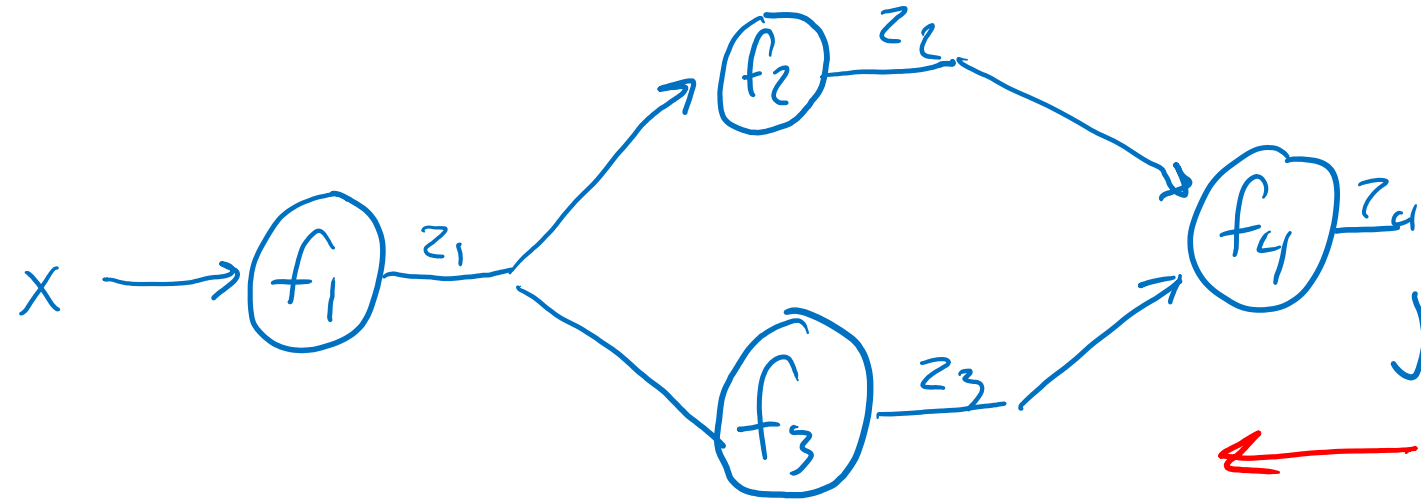
$$J(\mathbf{w}) = z_4$$

$$z_4 = f_4(w_D, w_E, z_2, z_3)$$

$$z_3 = f_3(w_C, z_1)$$

$$z_2 = f_2(w_B, z_1)$$

$$z_1 = f_1(w_A, x)$$



Need multivariate chain rule!

$$\frac{\partial J}{\partial w_E}, \frac{\partial J}{\partial w_D}, \frac{\partial J}{\partial w_C}, \frac{\partial J}{\partial w_B}, \frac{\partial J}{\partial w_A}$$

Network Optimization

$$J(\mathbf{w}) = z_4$$

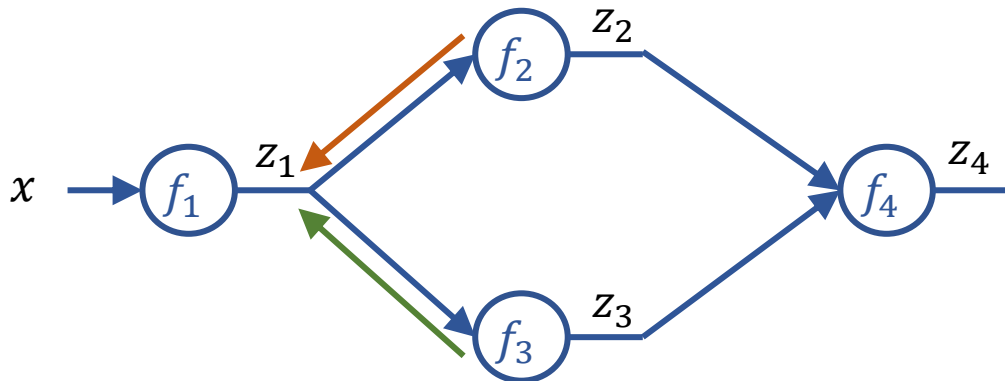
$$z_4 = f_4(w_D, w_E, z_2, z_3)$$

$$z_3 = f_3(w_C, z_1)$$

$$z_2 = f_2(w_B, z_1)$$

$$z_1 = f_1(w_A, x)$$

Need multivariate chain rule!



$$\frac{\partial J}{\partial w_E} = \frac{\partial J}{\partial z_4} \frac{\partial z_4}{\partial w_E}$$

$$\frac{\partial J}{\partial w_D} = \frac{\partial J}{\partial z_4} \frac{\partial z_4}{\partial w_D}$$

$$\frac{\partial J}{\partial z_3} = \frac{\partial J}{\partial z_4} \frac{\partial z_4}{\partial z_3}$$

$$\frac{\partial J}{\partial z_2} = \frac{\partial J}{\partial z_4} \frac{\partial z_4}{\partial z_2}$$

$$\frac{\partial J}{\partial w_C} = \frac{\partial J}{\partial z_3} \frac{\partial z_3}{\partial w_C}$$

$$\frac{\partial J}{\partial w_B} = \frac{\partial J}{\partial z_2} \frac{\partial z_2}{\partial w_B}$$

$$\frac{\partial J}{\partial z_1} = \frac{\partial J}{\partial z_2} \frac{\partial z_2}{\partial z_1} + \frac{\partial J}{\partial z_3} \frac{\partial z_3}{\partial z_1}$$

$$\frac{\partial J}{\partial w_A} = \frac{\partial J}{\partial z_1} \frac{\partial z_1}{\partial w_A}$$

Backpropagation (updated)

Compute derivatives per layer, utilizing previous derivatives

Objective: $J(\mathbf{w})$

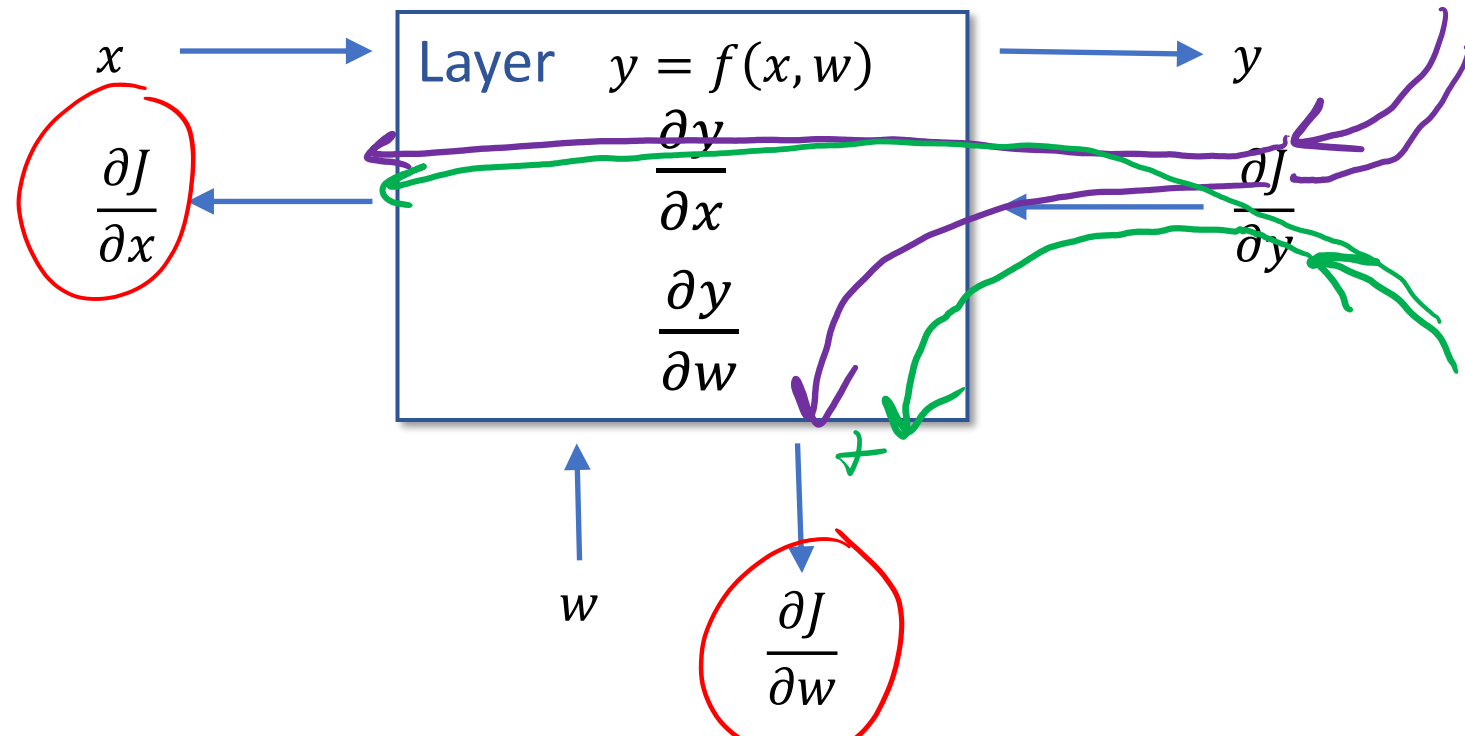
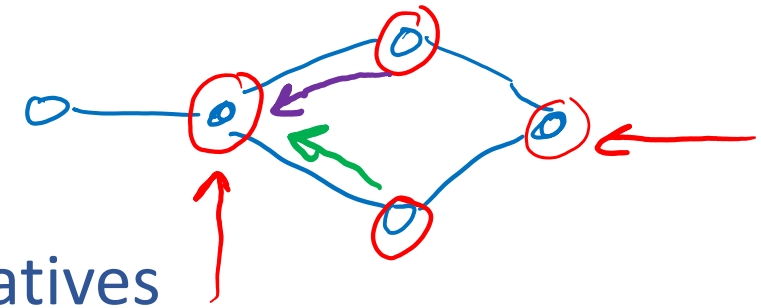
Arbitrary layer: $y = f(x, w)$

Init:

- $\frac{\partial J}{\partial x} = 0$
- $\frac{\partial J}{\partial w} = 0$

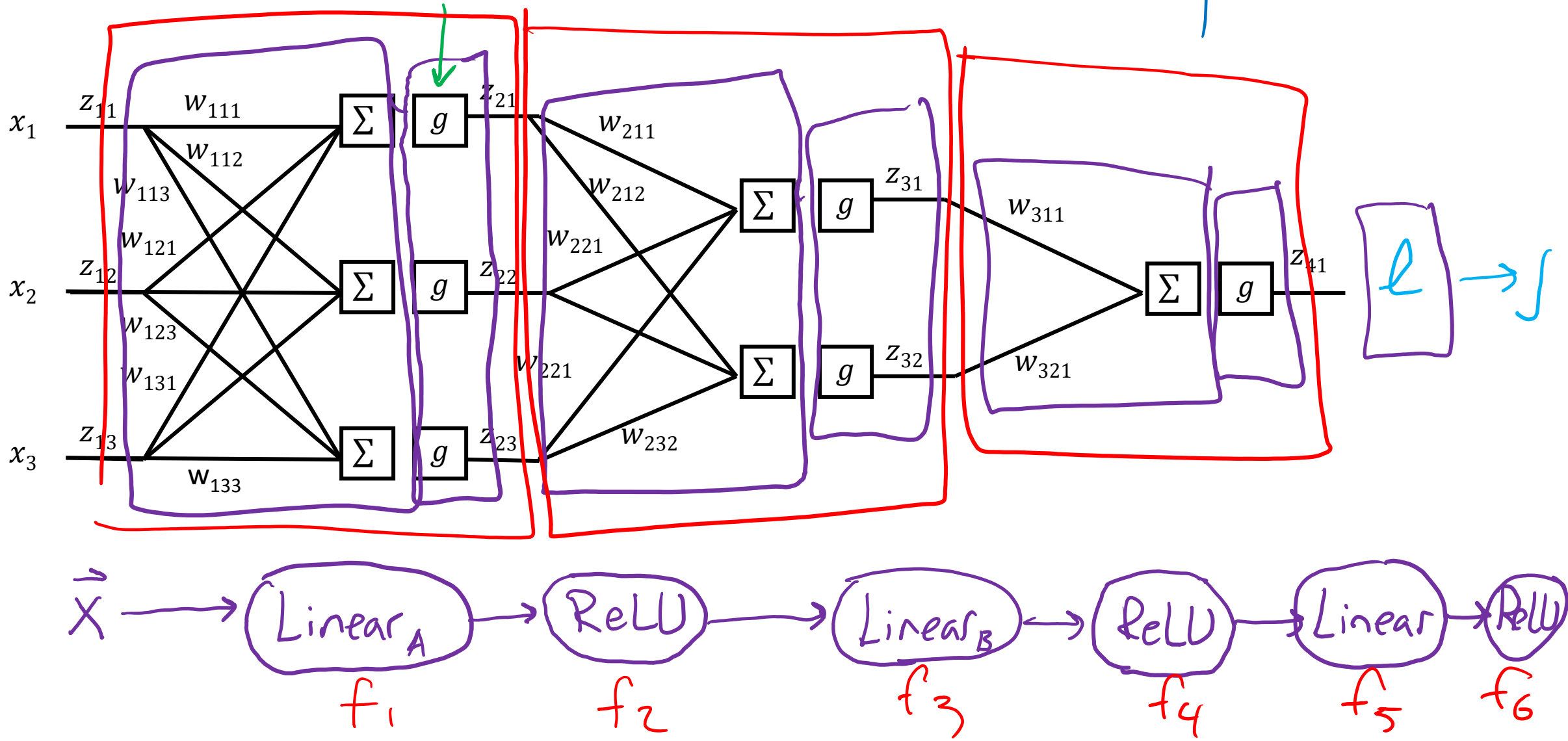
Compute:

- $\frac{\partial J}{\partial x} \oplus = \frac{\partial J}{\partial y} \frac{\partial y}{\partial x}$
- $\frac{\partial J}{\partial w} \oplus = \frac{\partial J}{\partial y} \frac{\partial y}{\partial w}$

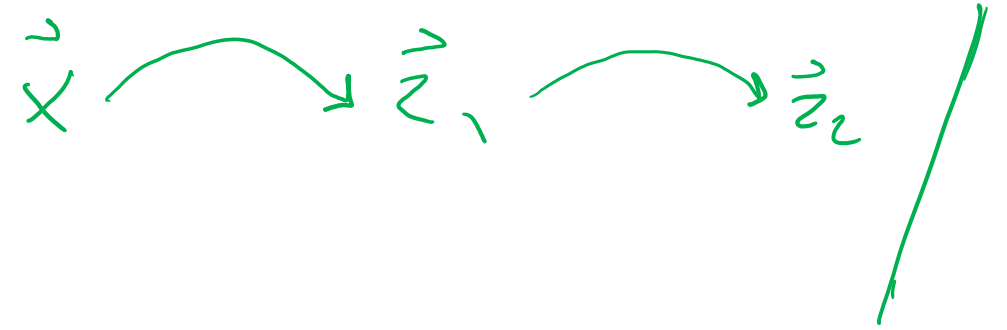


Neural Network Implementation

Which pieces do we treat as functions?



Neural Networks Properties



Practical considerations

- Large number of neurons
 - Danger for overfitting
- Modelling assumptions vs data assumptions trade-off
- Gradient descent can easily get stuck local optima

What if there are no non-linear activations?

- A deep neural network with only linear layers can be reduced to an exactly equivalent single linear layer

Universal Approximation Theorem:

- A two-layer neural network with a sufficient number of neurons can approximate any continuous function to any desired accuracy.

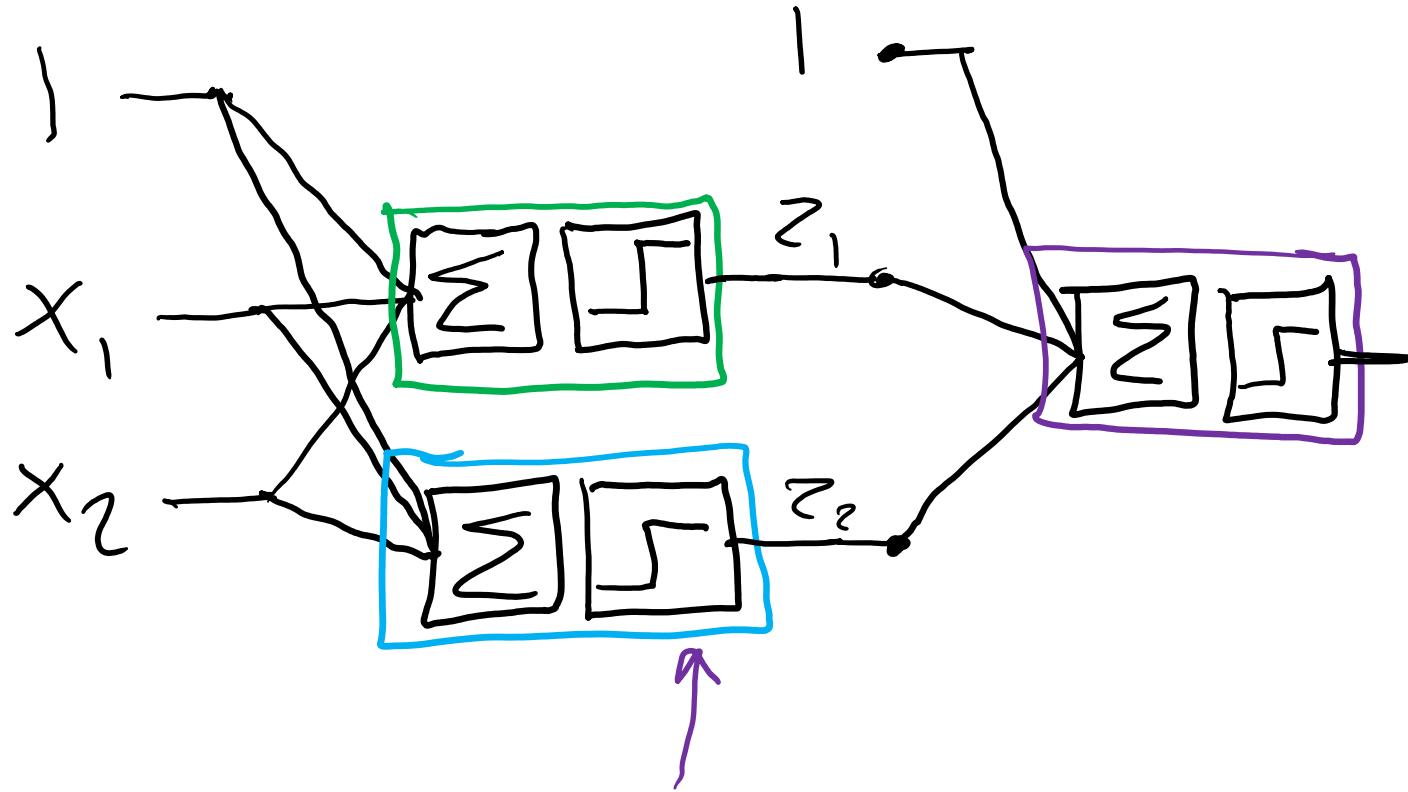
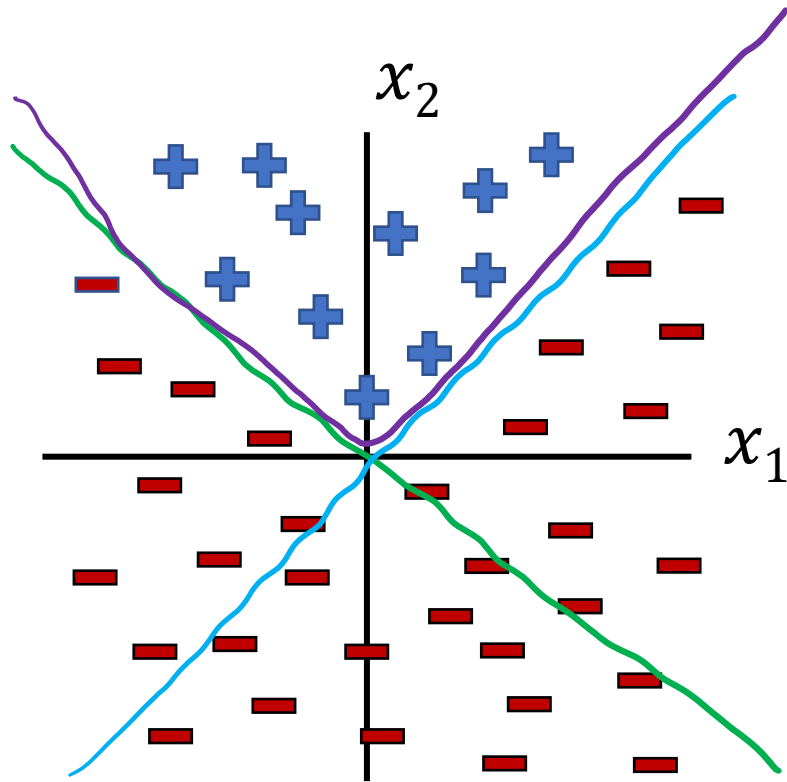
Classification Design Challenge

How could you configure three specific perceptrons to classify this data?

$$h_A(\mathbf{x}) = \text{sign}(\mathbf{w}_A^T \mathbf{x} + b_A)$$

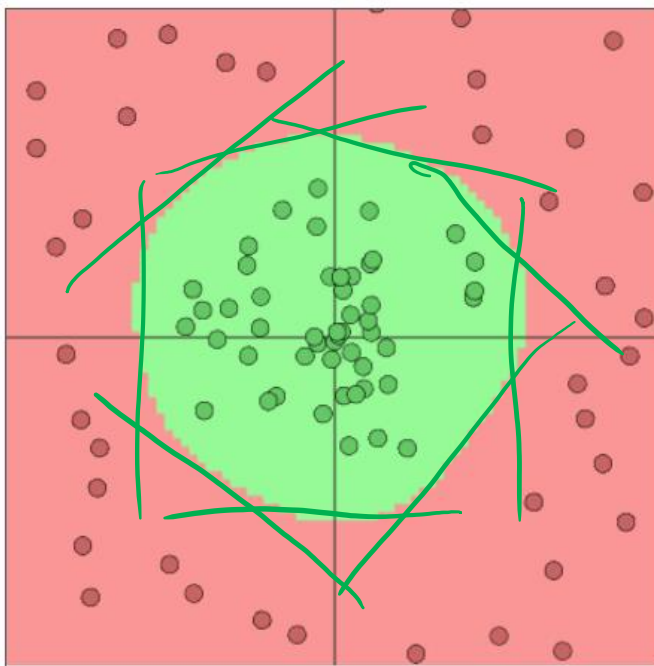
$$h_B(\mathbf{x}) = \text{sign}(\mathbf{w}_B^T \mathbf{x} + b_B)$$

$$h_C(\mathbf{z}) = \text{sign}(\mathbf{w}_C^T \mathbf{z} + b_C)$$



Network to Approximate Binary Classification

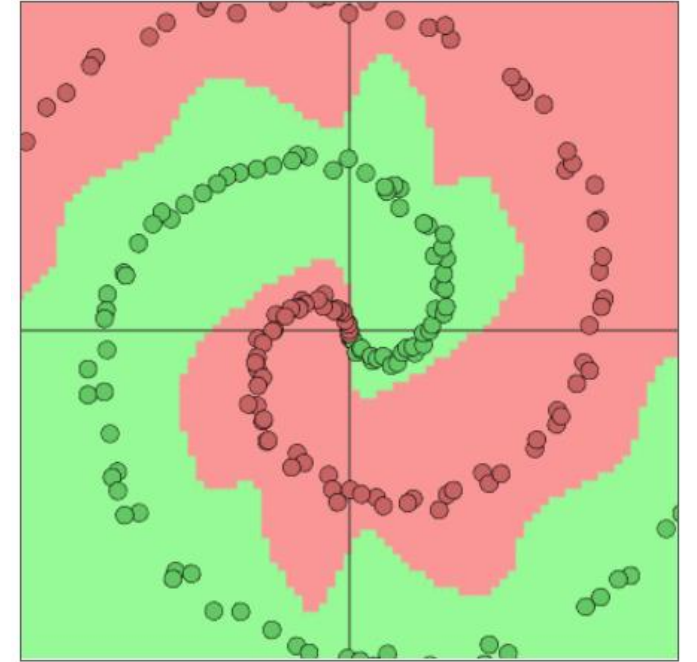
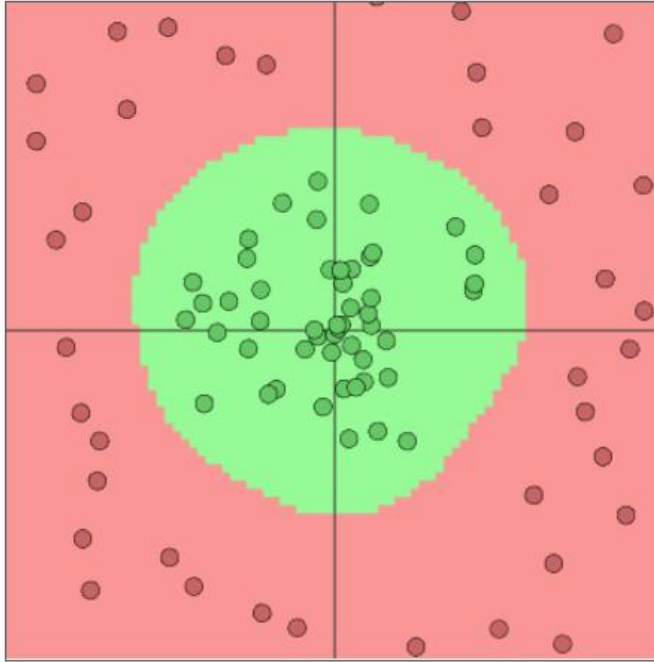
Approximate arbitrary decision boundary



<https://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html>

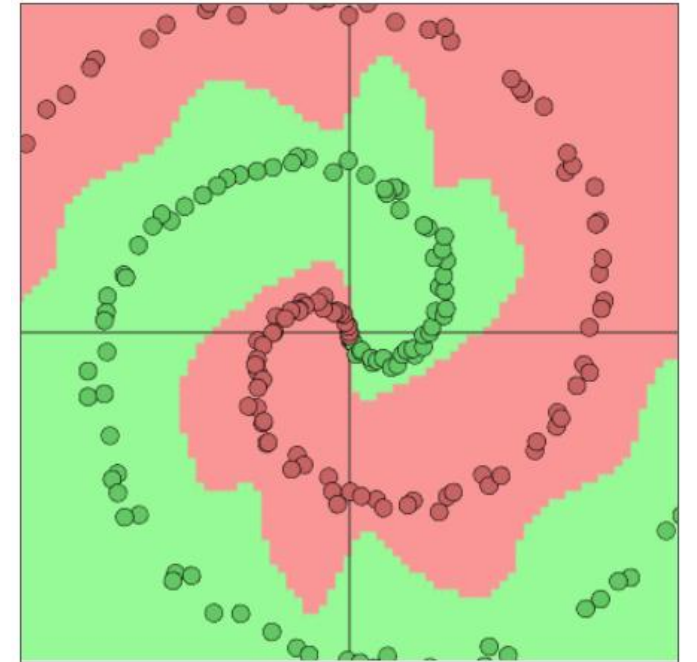
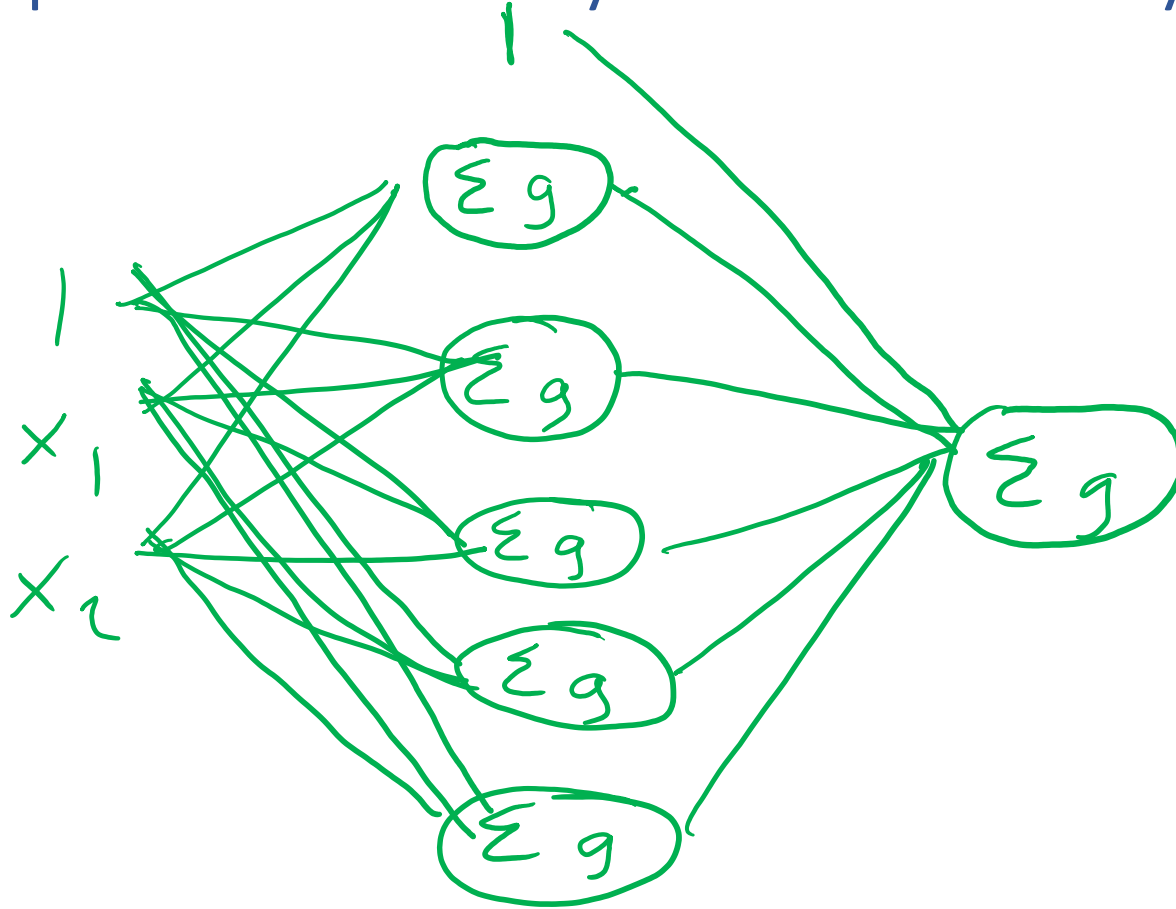
Network to Approximate Binary Classification

Approximate arbitrary decision boundary



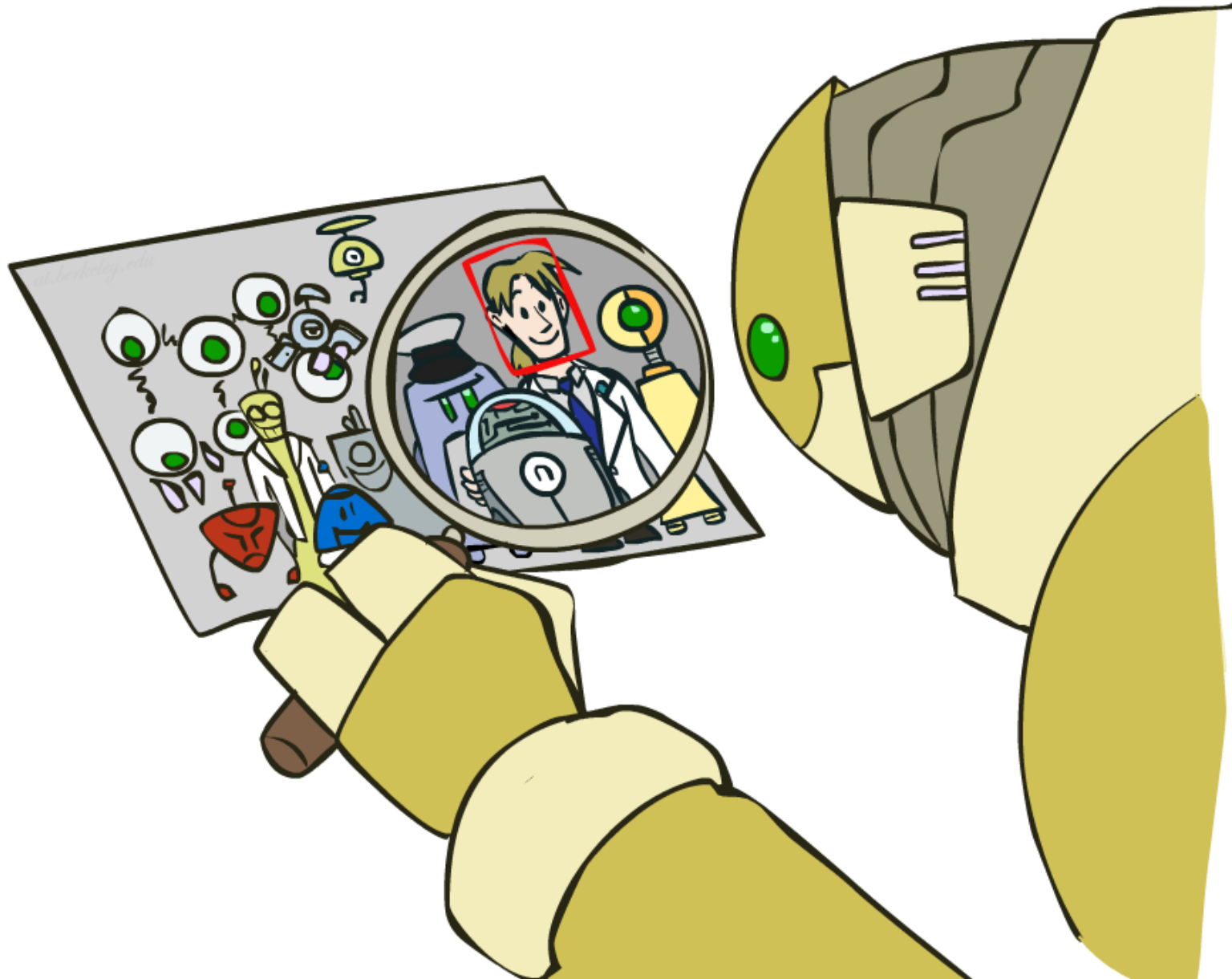
Network to Approximate Binary Classification

Approximate arbitrary decision boundary



Convolutional Neural Nets

Computer Vision: How far along are we?

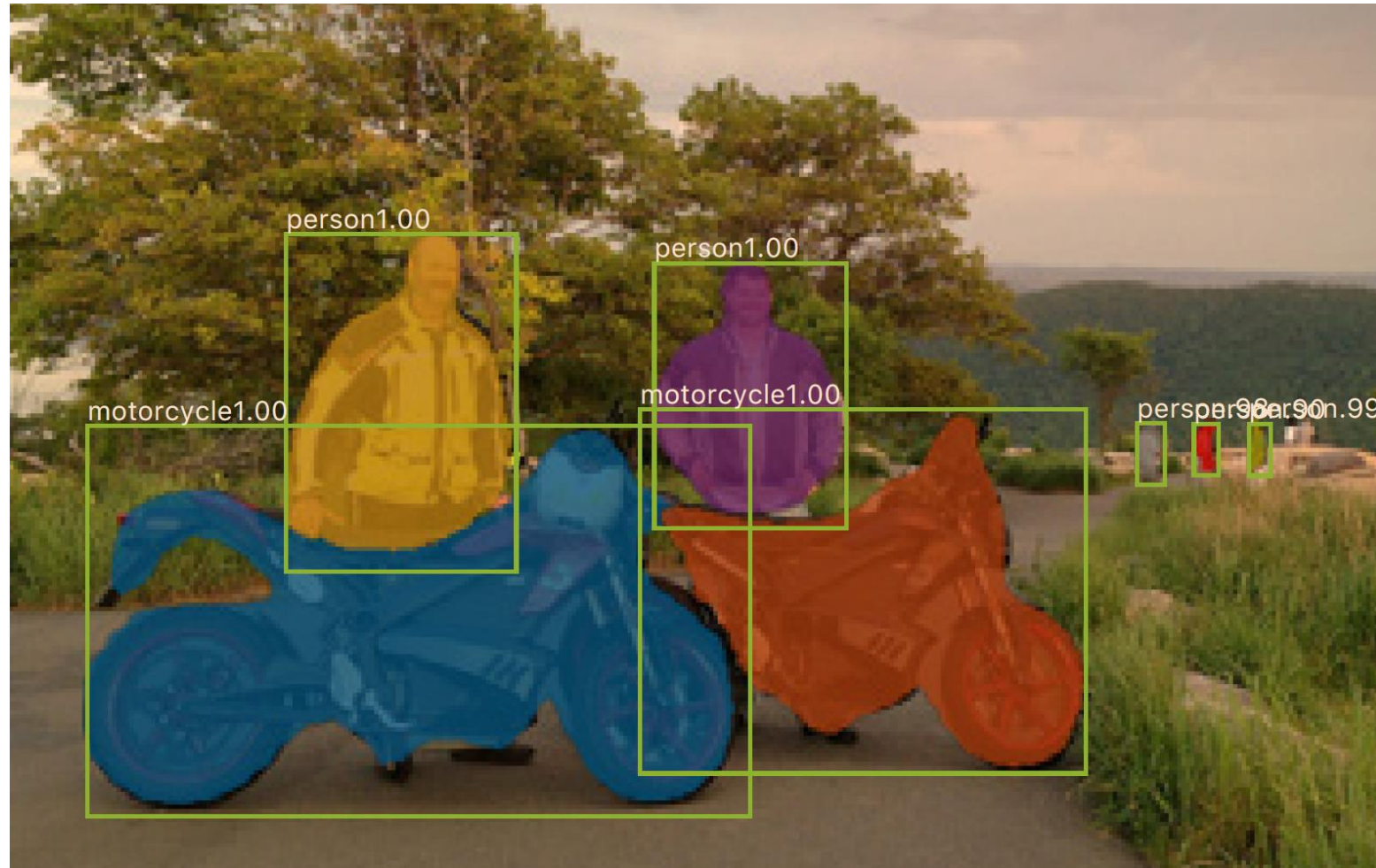


Computer Vision: How far along are we?



Terminator 2, 1991 <https://www.youtube.com/watch?v=9MeaaCwBW28>

Computer Vision: How far along are we?



0.2 seconds
per image

Mask R-CNN

He, Kaiming, et al. "Mask R-CNN." *Computer Vision (ICCV), 2017 IEEE International Conference on*. IEEE, 2017.

Computer Vision: How far along are we?



“My CPU is a neural net processor, a learning computer”

Terminator 2, 1991

Computer Vision: Autonomous Driving



Tesla, Inc: <https://vimeo.com/192179726>

Computer Vision: Domain Transfer

CycleGAN



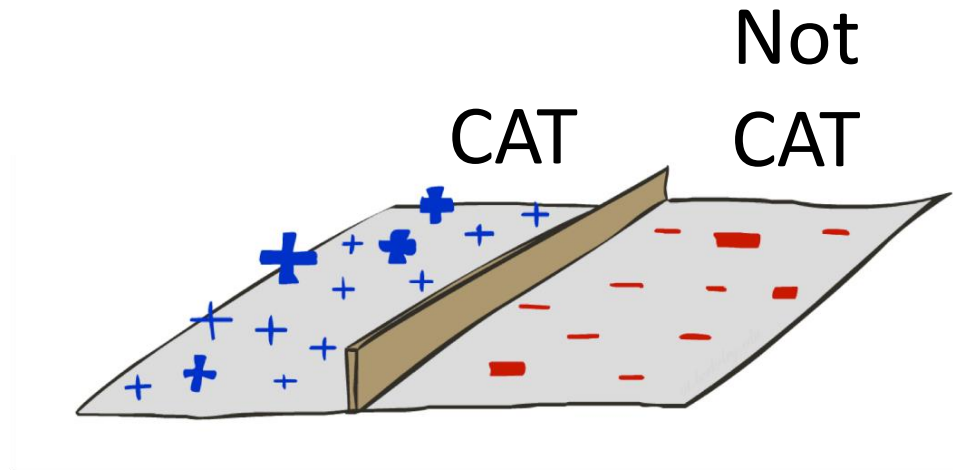
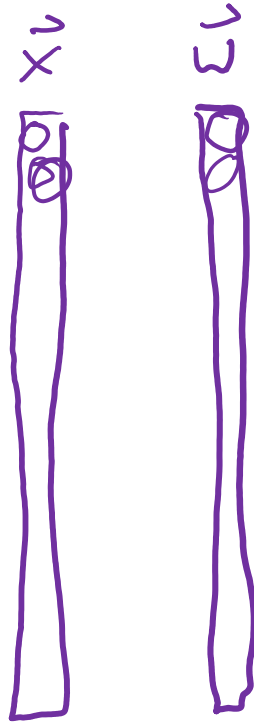
Jun-Yan Zhu*, Taesung Park*, Phillip Isola, and Alexei A. Efros. "Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks", ICCV 2017.

Outline

1. Measuring the current state of computer vision
2. Why convolutional neural networks
 - Old school computer vision
 - Image features and classification
3. Convolution “nuts and bolts”

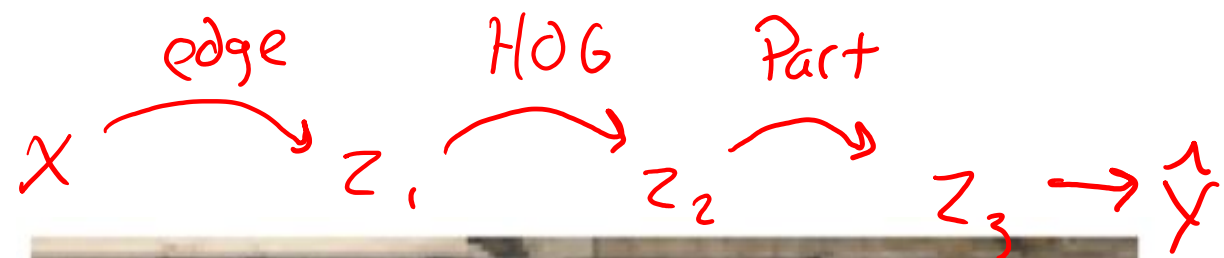
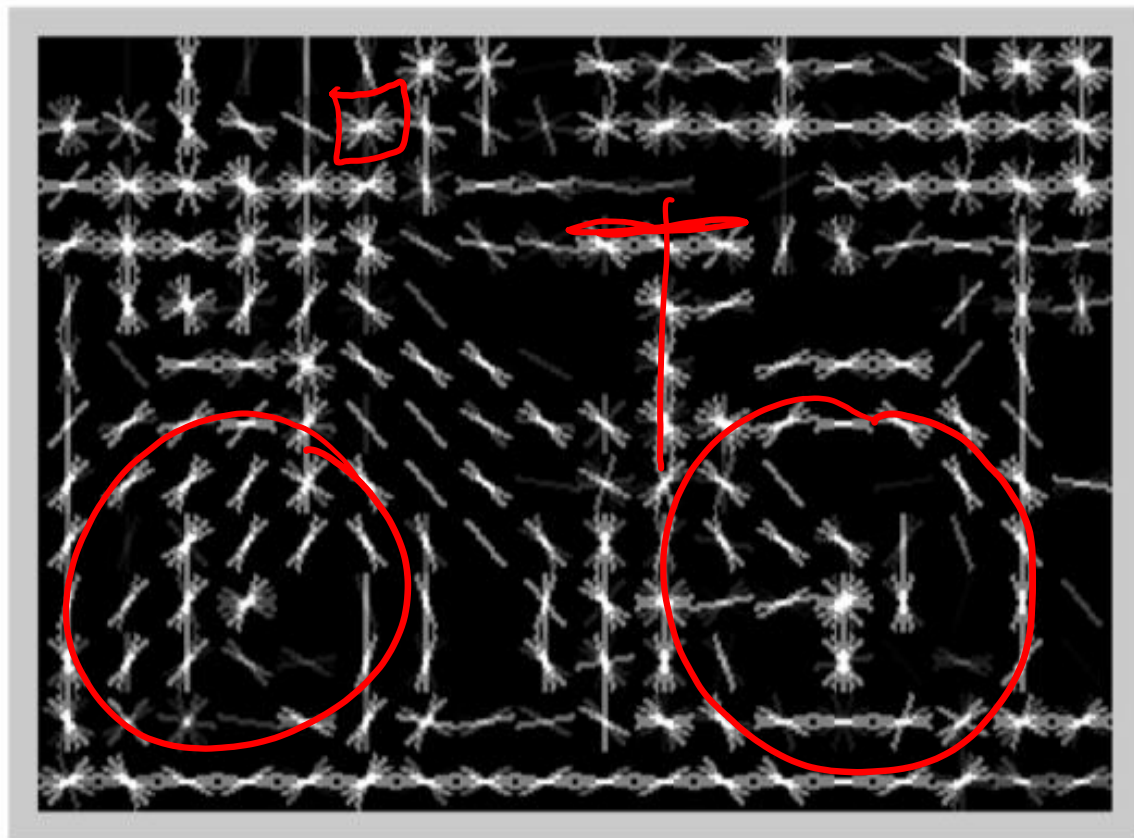
Image Classification

What's the problem with just directly classifying raw pixels in high dimensional space?



CAT

Image Classification



HoG Filter

HoG: Histogram of oriented gradients

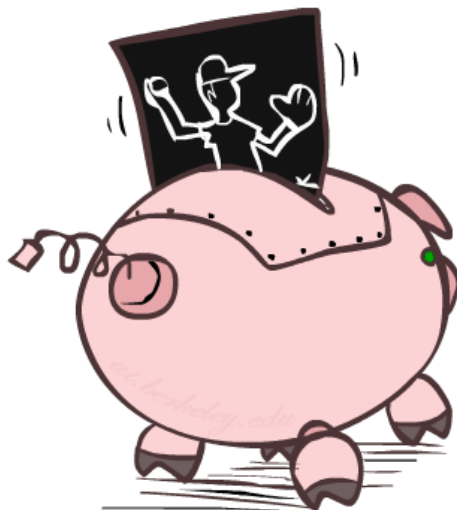
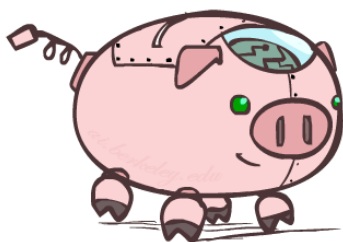
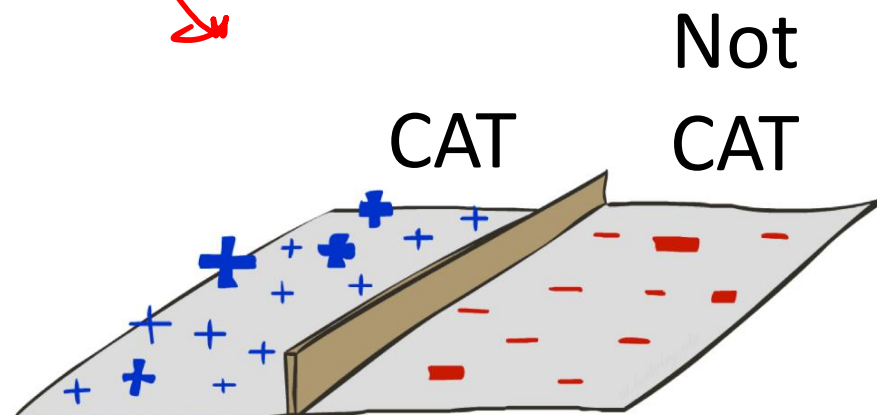
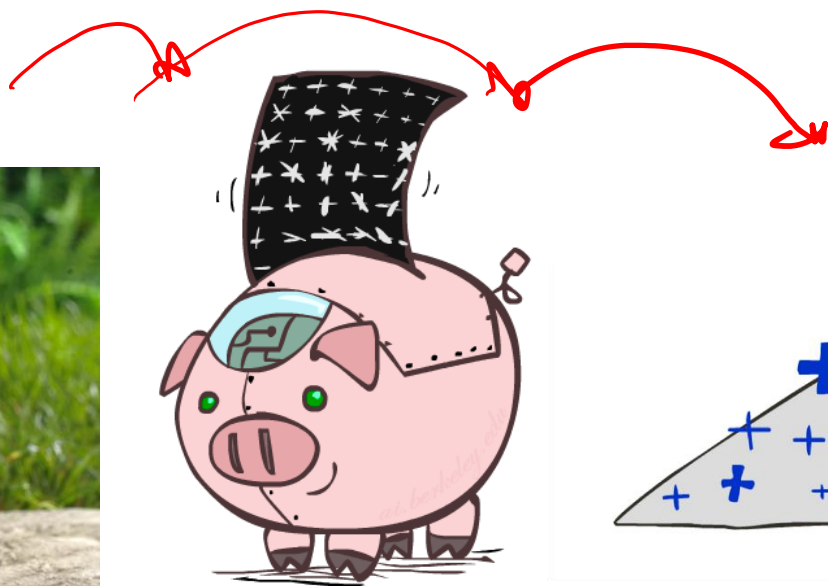


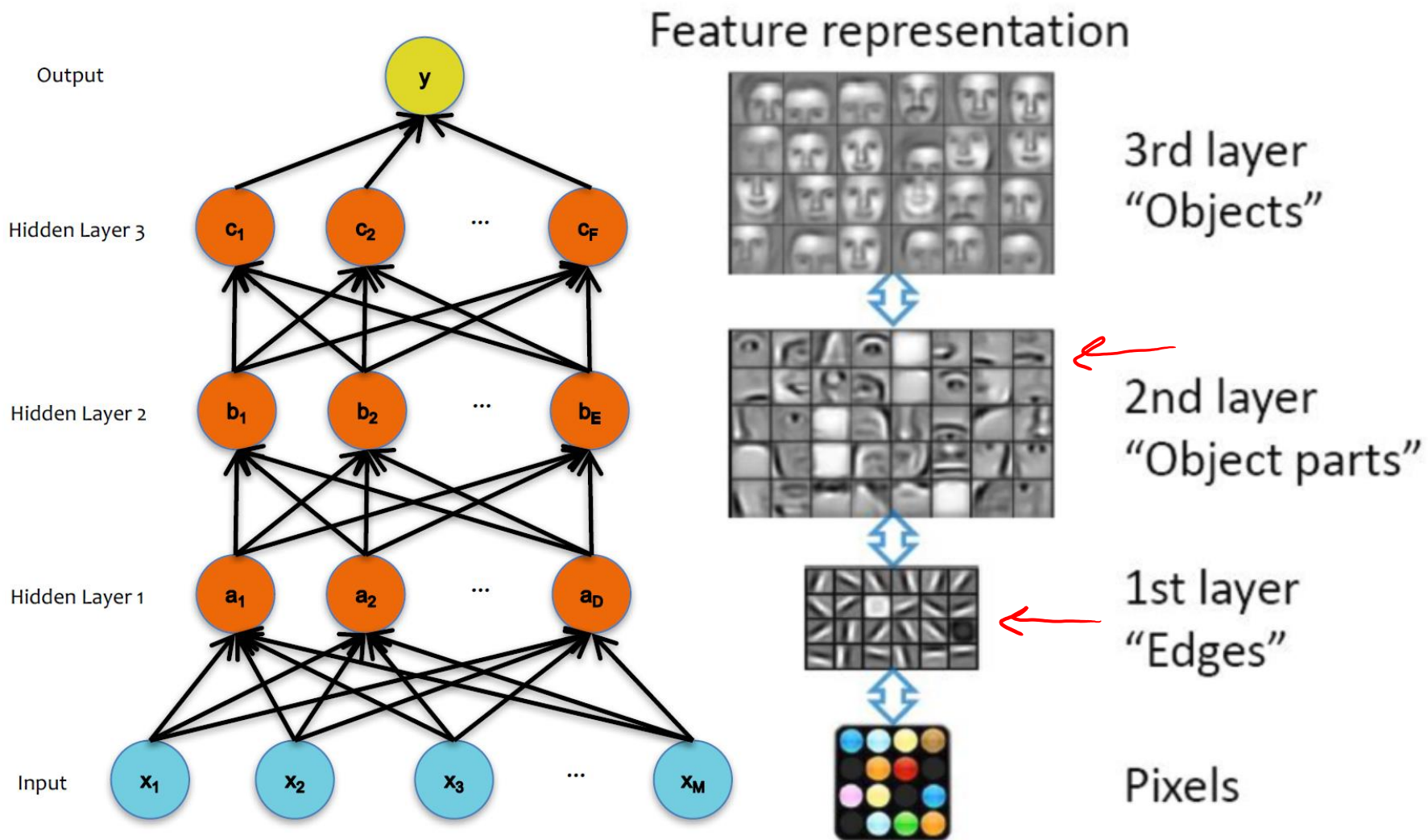
Image Classification

HOG features passed to a linear classifier (logistic regression / SVM)



CAT

Classification: Learning Features

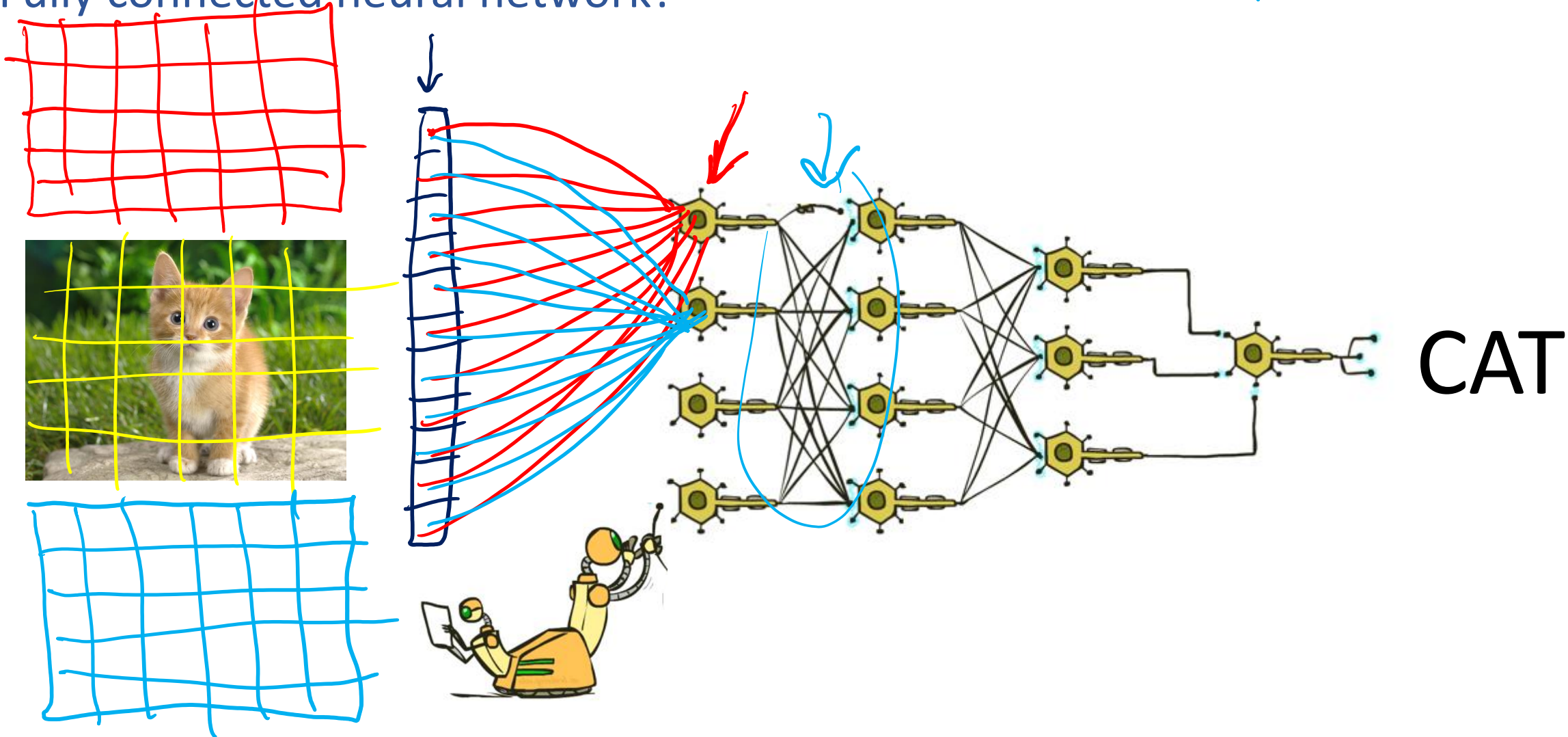


Example from Honglak Lee (NIPS 2010)

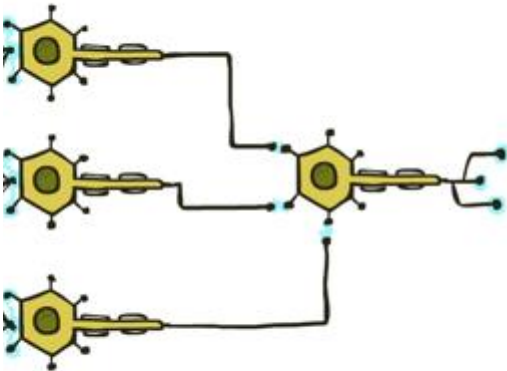
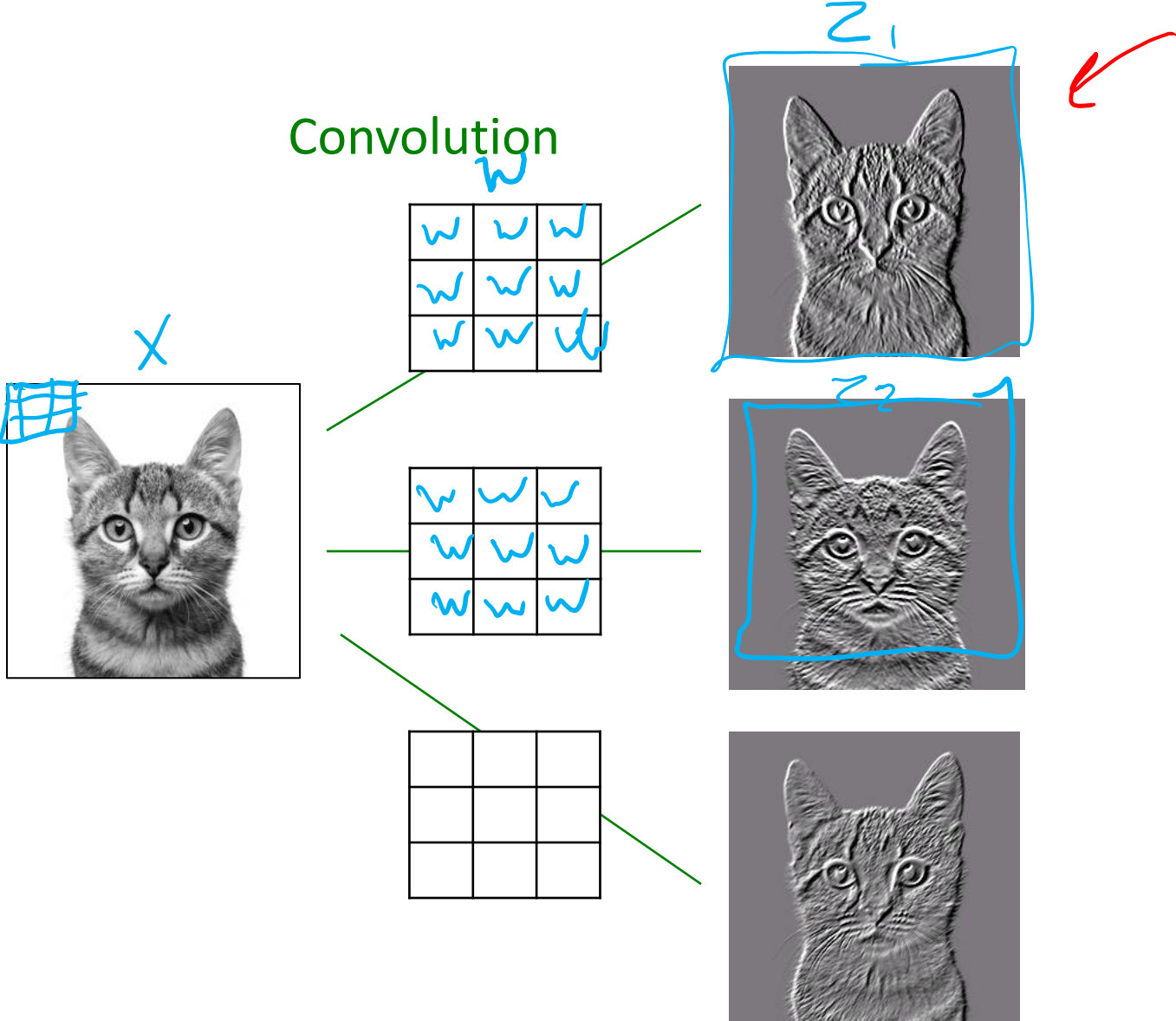
Classification: Deep Learning

w	w	w
w	w	w
w	w	w

Fully connected neural network?



Convolutional Neural Networks



CAT

Convolution

x

w



-1	0	1
-1	0	1
-1	0	1



Convolution

x

0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0

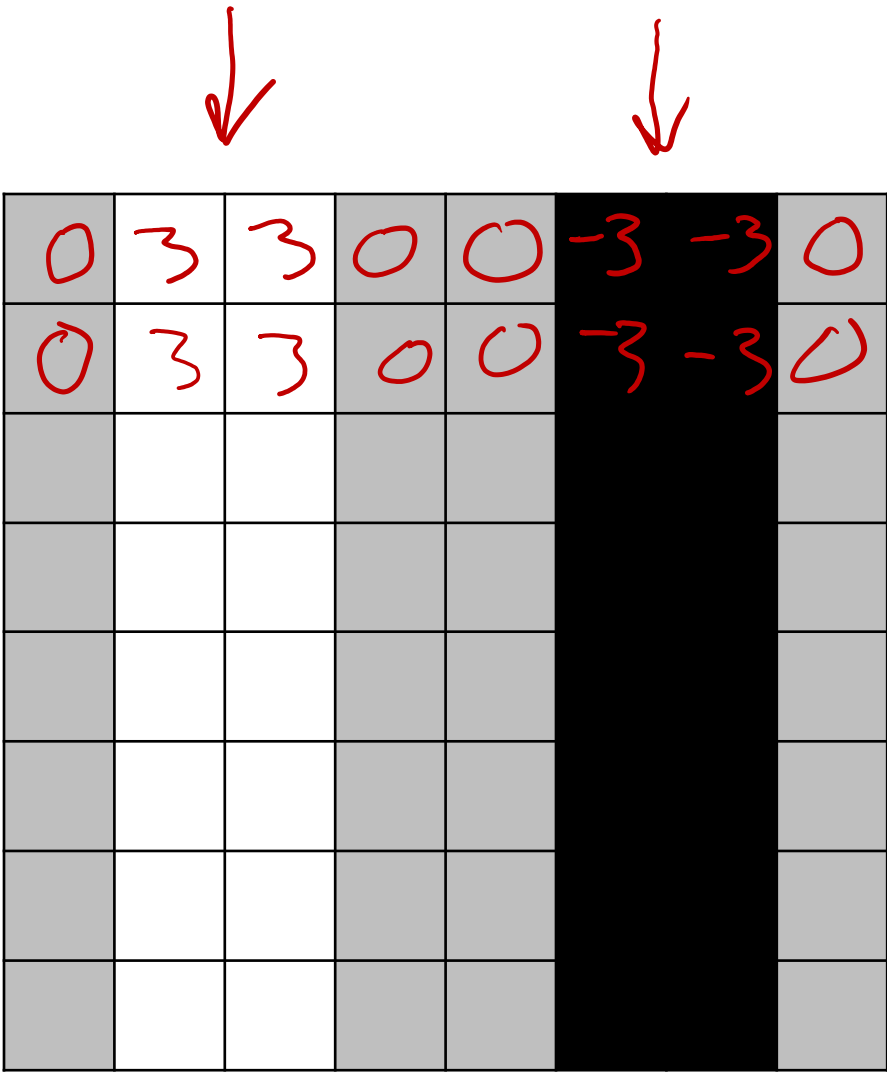
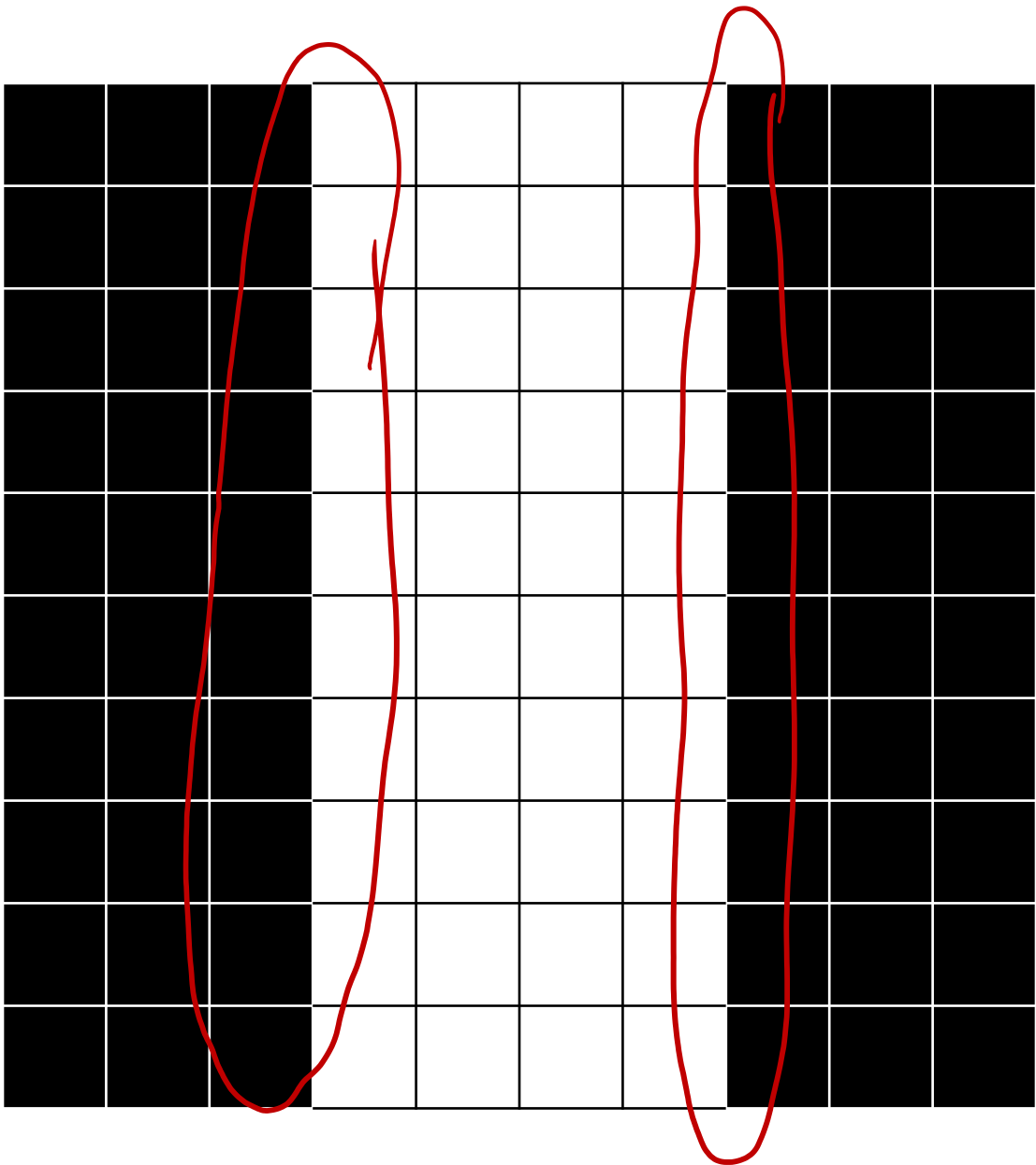
0 3 3 0

$$\sum w_{ij} x_{uv}$$

w

-1	0	1
-1	0	1
-1	0	1

Convolution



-1	0	1
-1	0	1
-1	0	1

$$\sum x_j w_j$$


- Drop infinity; don't flip kernel

$$z[i,j] = \sum_{u=0}^{K-1} \sum_{v=0}^{K-1} x[i+u,j+v] \cdot w[u,v]$$

-1	0	1
-2	0	2
-1	0	1

A 6x6 grid of squares. The top-left 3x3 subgrid is outlined with a thick blue border. The remaining cells in the grid are outlined with thin black borders.

Convolution

Relaxed definition

$$z[i, j] = \sum_{u=0}^{K-1} \sum_{v=0}^{K-1} x[i+u, j+v] \cdot w[u, v]$$

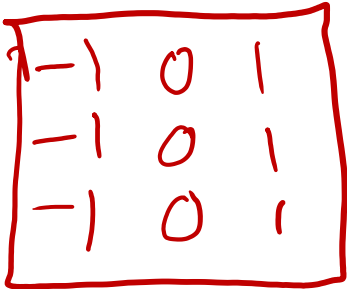
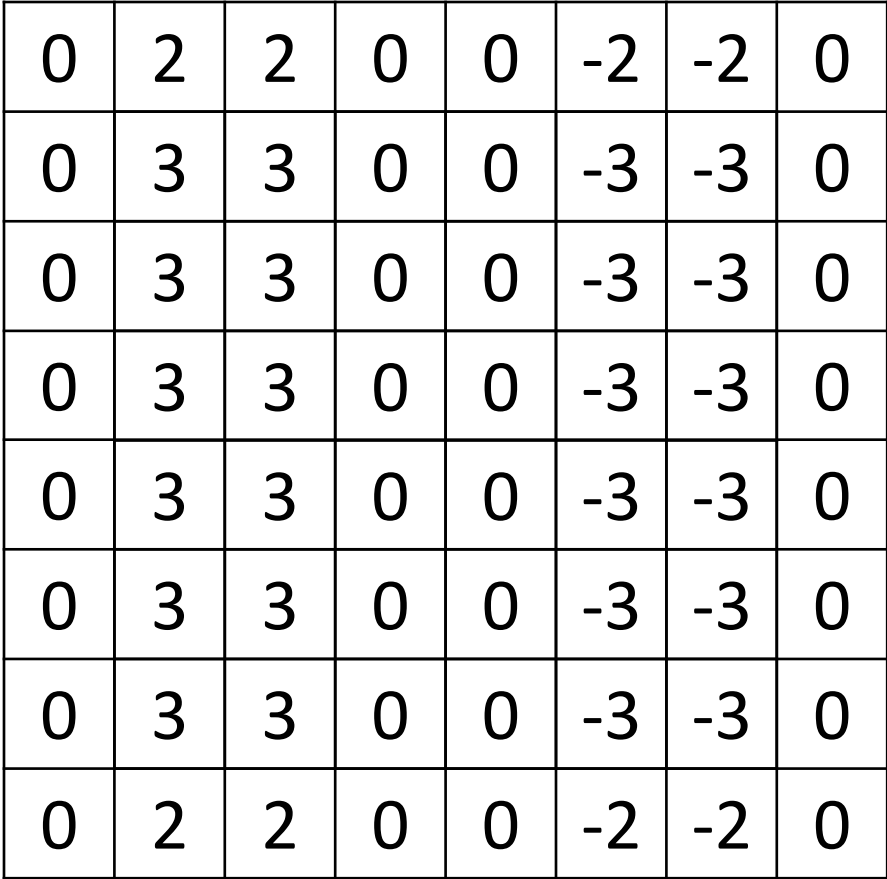
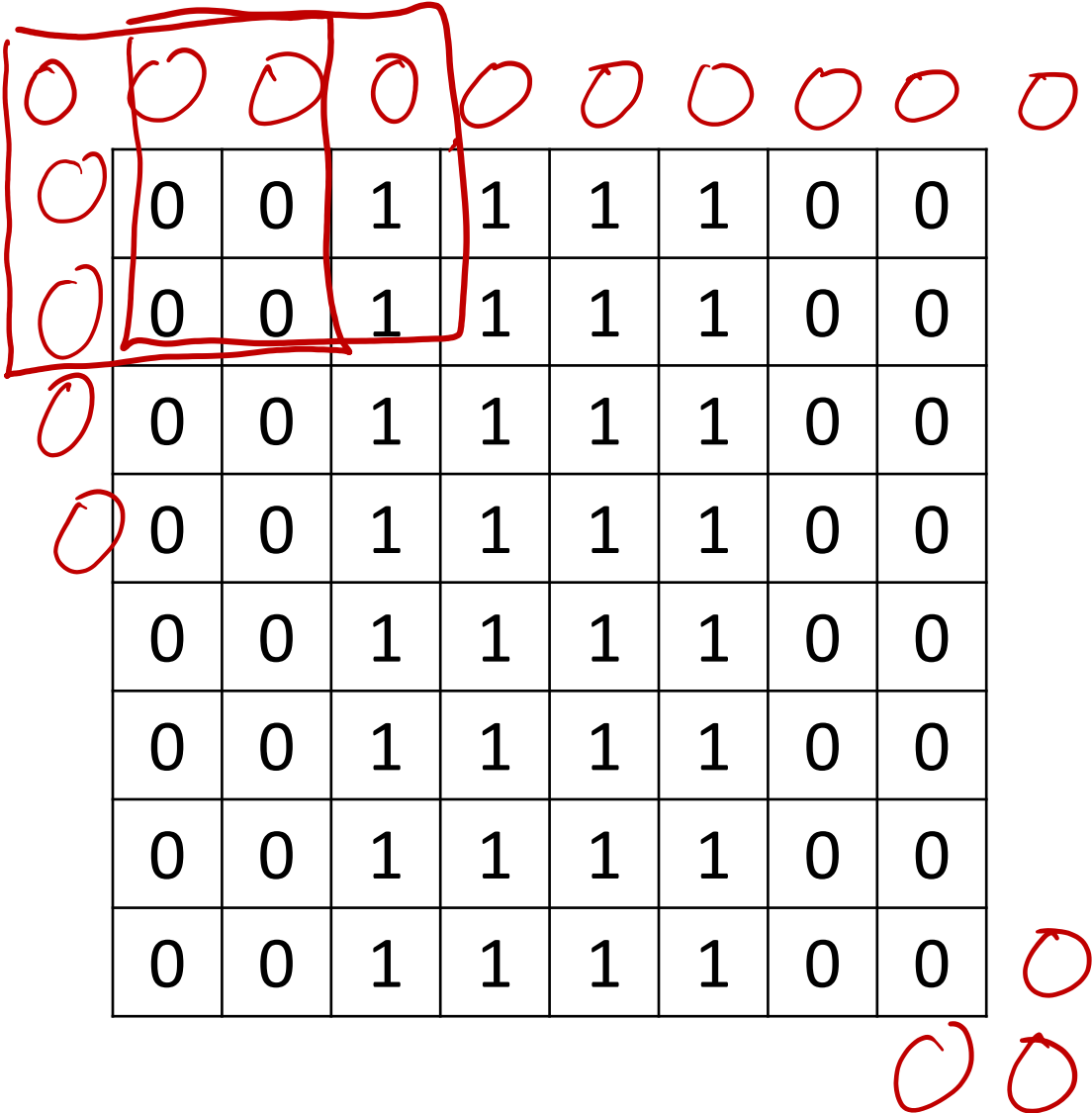
-1	0	1
-2	0	2
-1	0	1

```
for i in range(0, im_width - K + 1):  
    for j in range(0, im_height - K + 1):  
        im_out[i, j] = 0  
        for u in range(0, K):  
            for v in range(0, K):  
                im_out[i, j] += im[i+u, j+v] * kernel[u, v]
```

GPU!!

0	0	0			
0	0	0			
0	0	0			

Convolution: Padding



Piazza Poll 3 : Which kernel goes with which output image?

Input



K1

-1	0	1
-2	0	2
-1	0	1

K2

-1	-2	-1
0	0	0
1	2	1

K3

0	0	-1	0
0	-2	0	1
-1	0	2	0
0	1	0	0

Im1



Im2



Im3



Piazza Poll 3: Which kernel goes with which output image?

Input



K1

-1	0	1
-2	0	2
-1	0	1

K2

-1	-2	-1
0	0	0
1	2	1

K3

0	0	-1	0
0	-2	0	1
-1	0	2	0
0	1	0	0

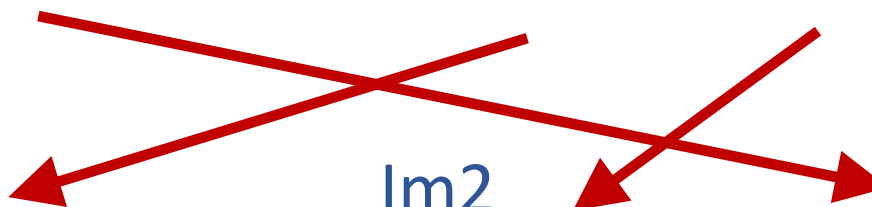
Im1



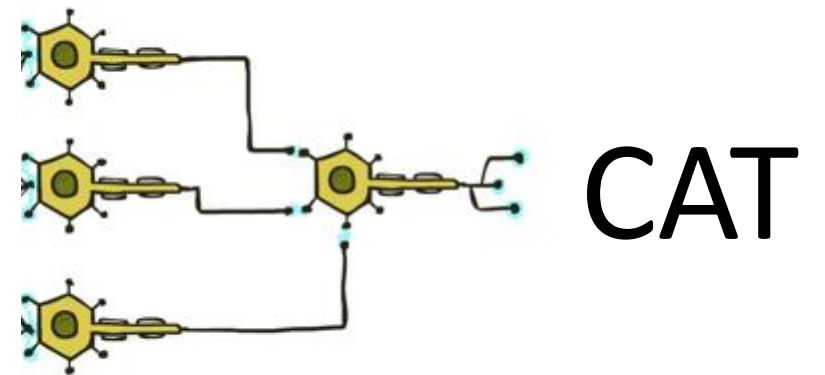
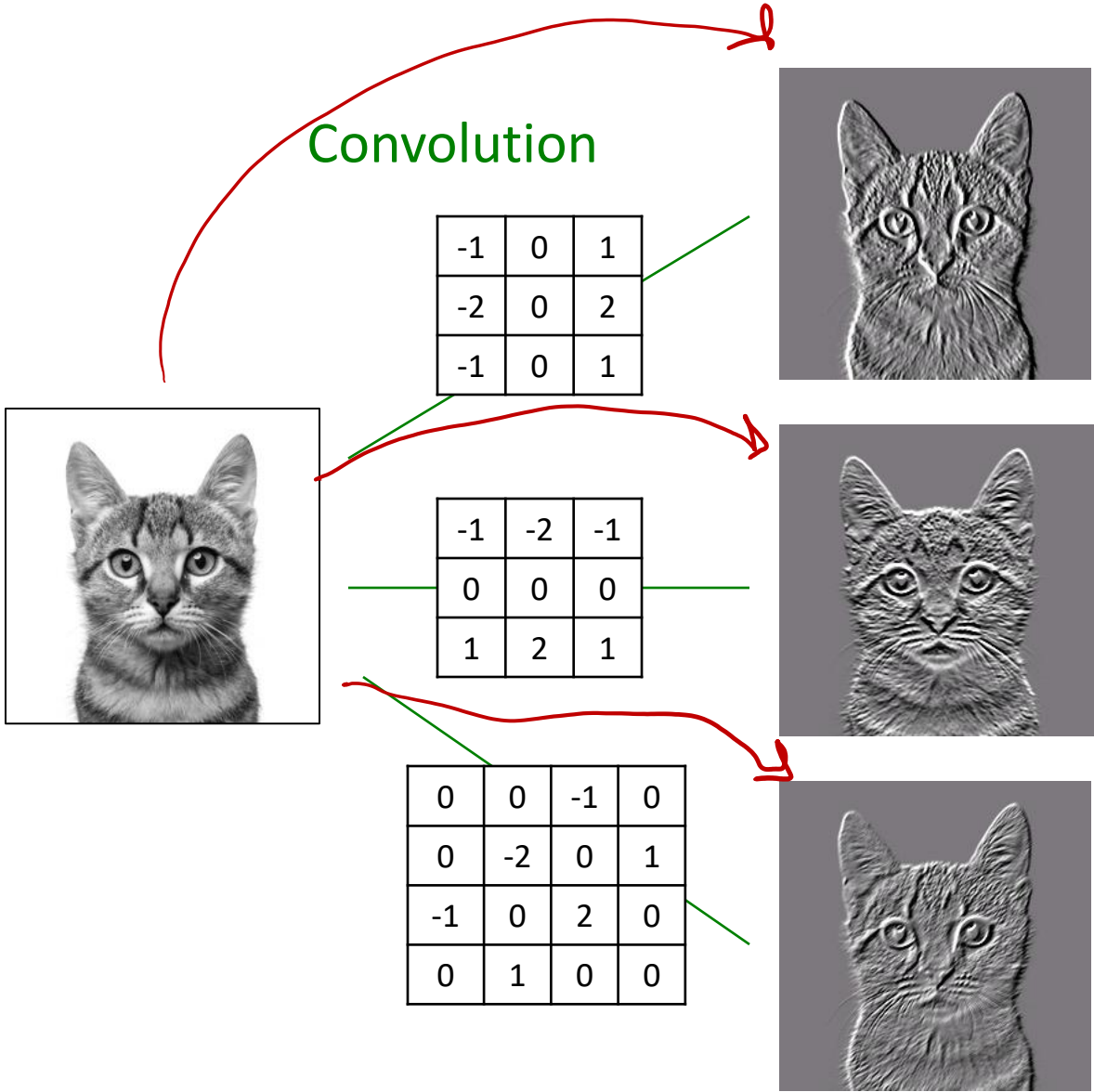
Im2



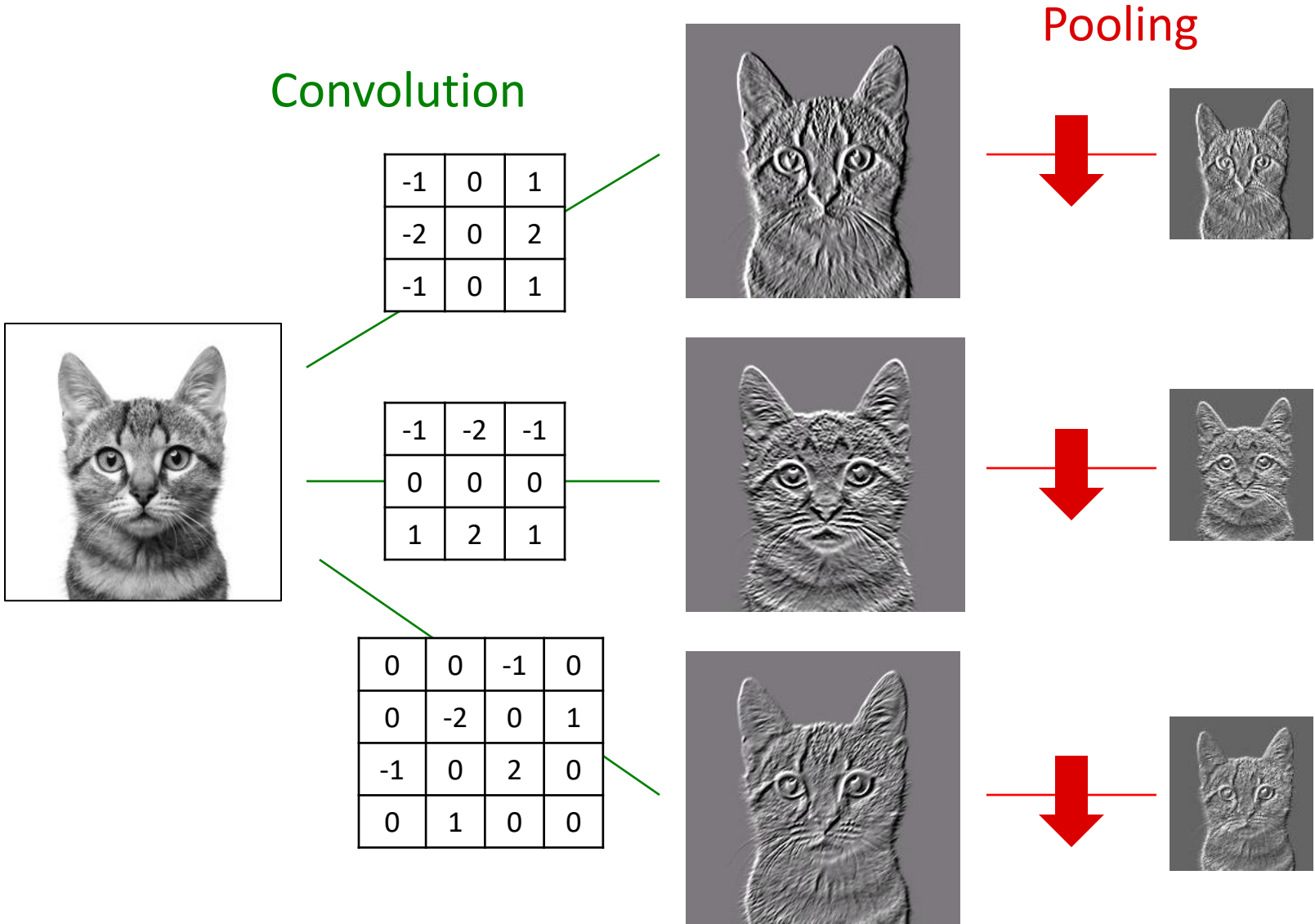
Im3



Convolutional Neural Networks



Convolutional Neural Networks



Convolution: Stride=2

0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0

0 2 4 2 0
0 2 4

average
kernel

.25	.25
.25	.25

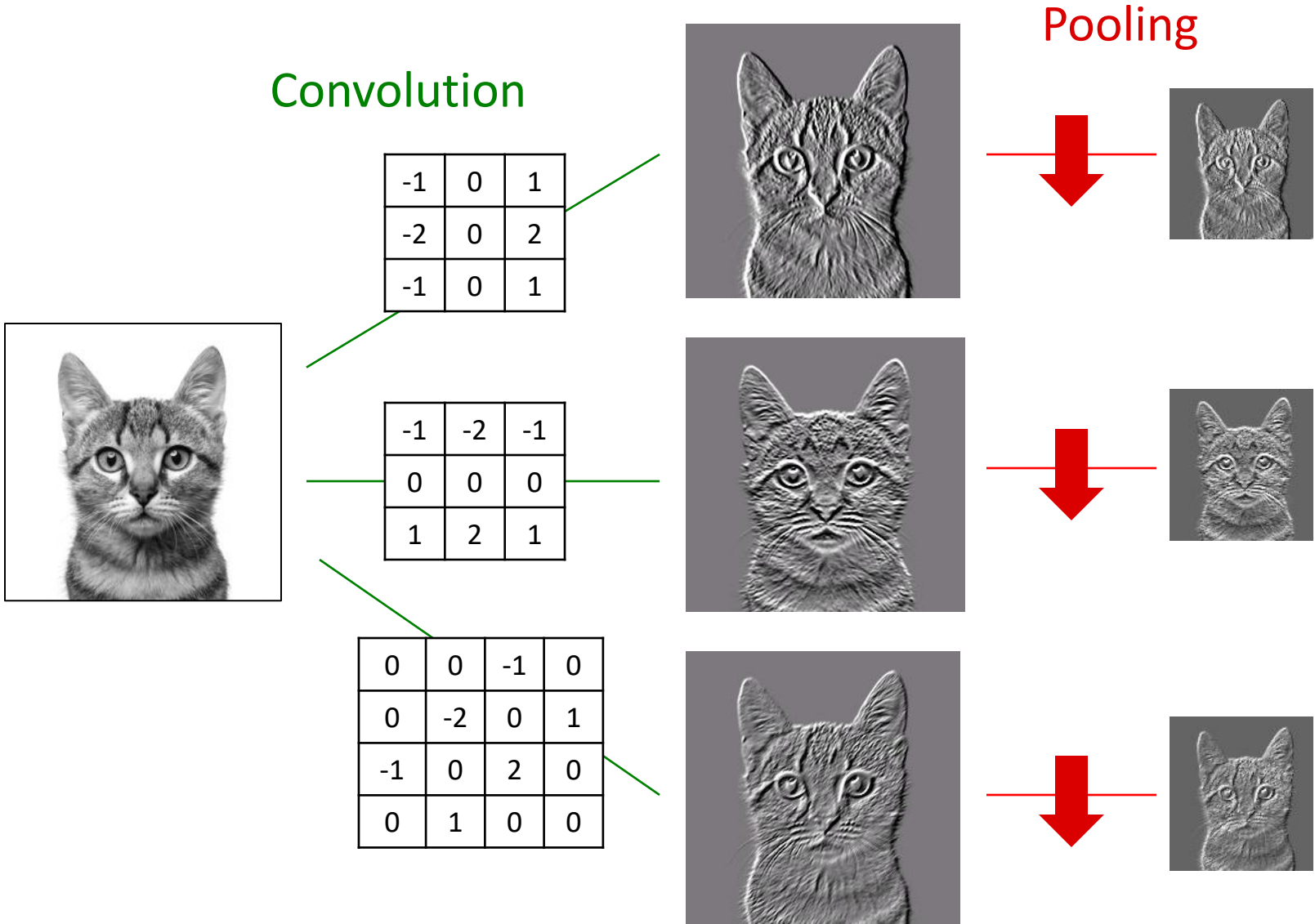
Stride: Max Pooling

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

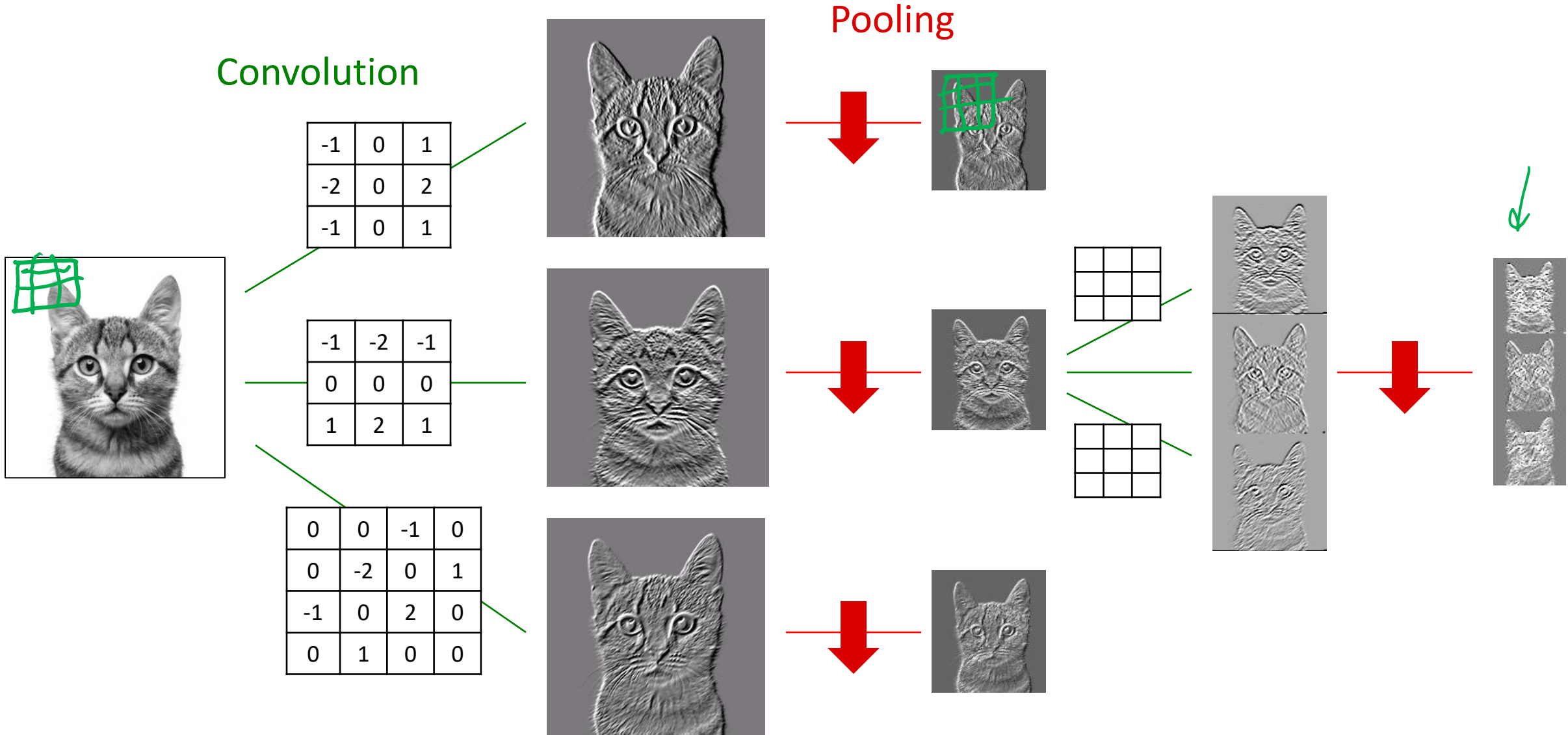
max pool with 2x2 filters
and stride 2

6	8
3	4

Convolutional Neural Networks



Convolutional Neural Networks



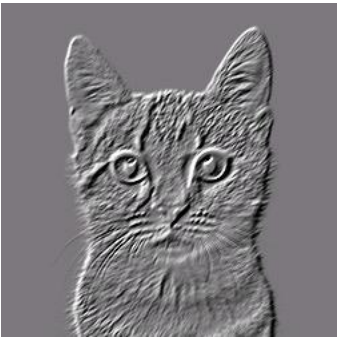
Convolutional Neural Networks

2 @ 3x3
1 @ 4x4
Convolution

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

0	0	-1	0
0	-2	0	1
-1	0	2	0
0	1	0	0



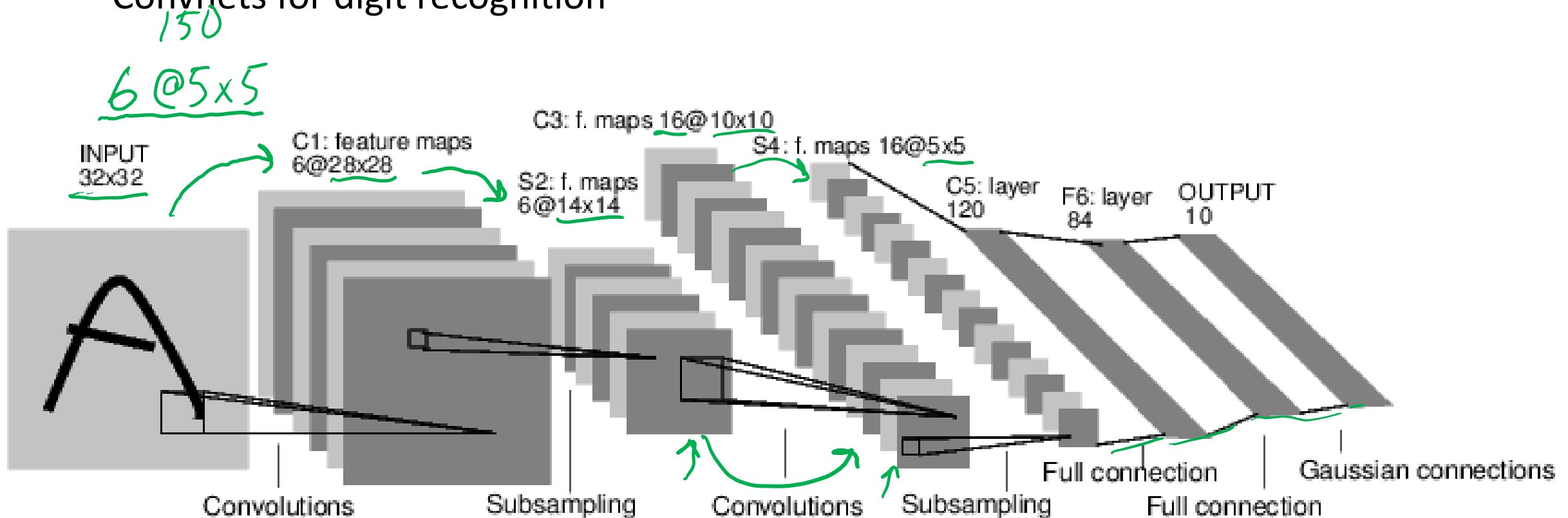
Pooling



Convolutional Neural Networks

Lenet5 – Lecun, et al, 1998

- Convnets for digit recognition

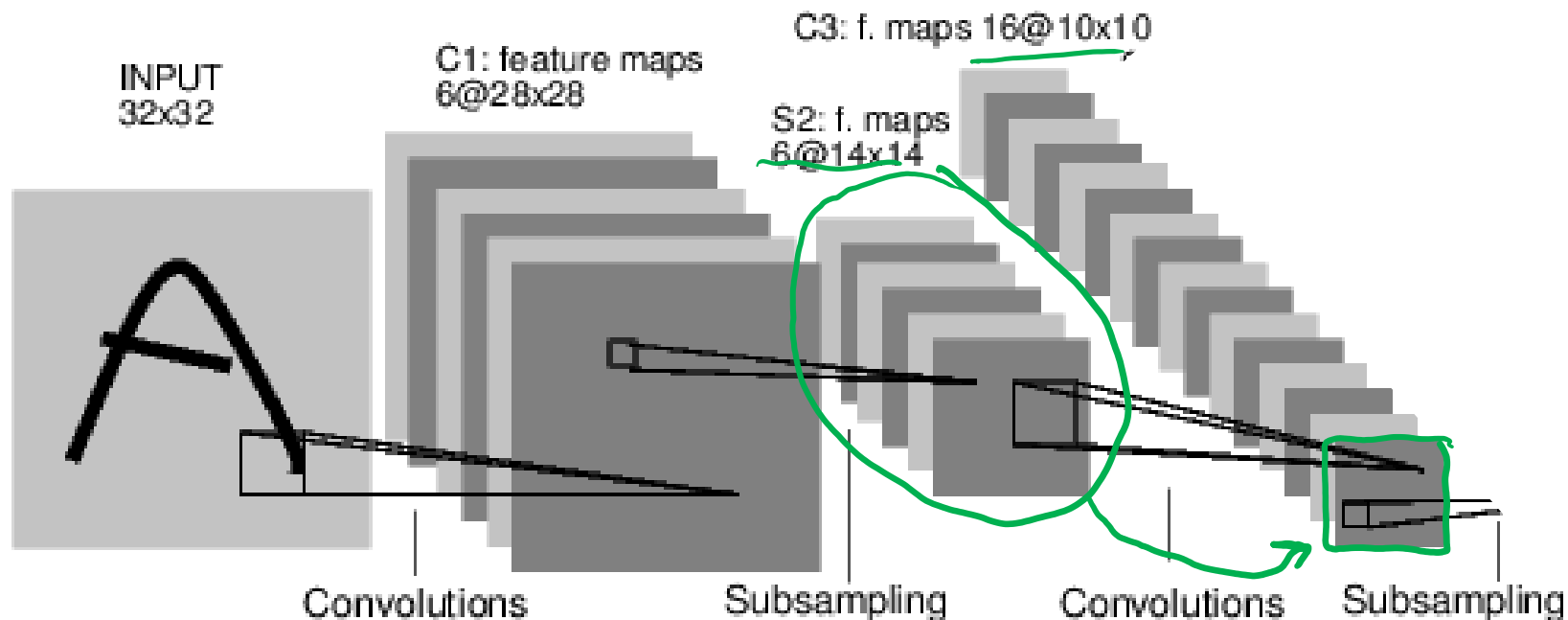


LeCun, Yann, et al. "Gradient-based learning applied to document recognition." Proceedings of the IEEE 86.11 (1998): 2278-2324.

Question:

How big many convolutional weights between S2 and C3?

- S2: 6 channels @14x14
- Conv: 5x5, pad=0, stride=1
- C3: 16 channels @ 10x10



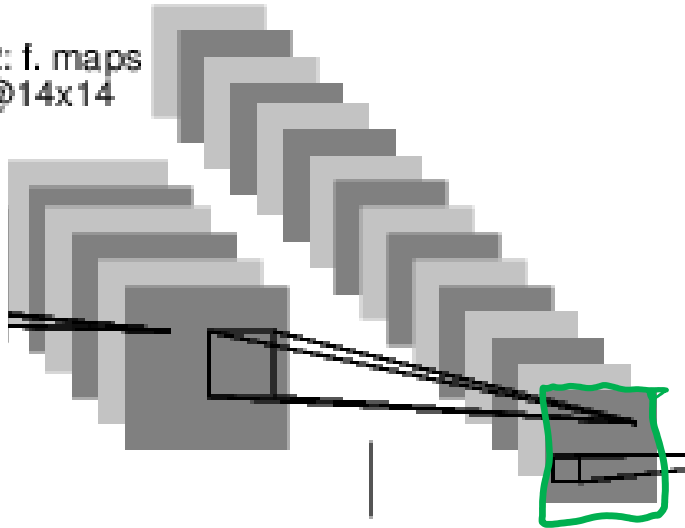
Question:

How big many convolutional weights between S2 and C3?

- S2: 6 channels @14x14
- Conv: 5x5, pad=0, stride=1
- C3: 16 channels @ 10x10

C3: f. maps 16@10x10

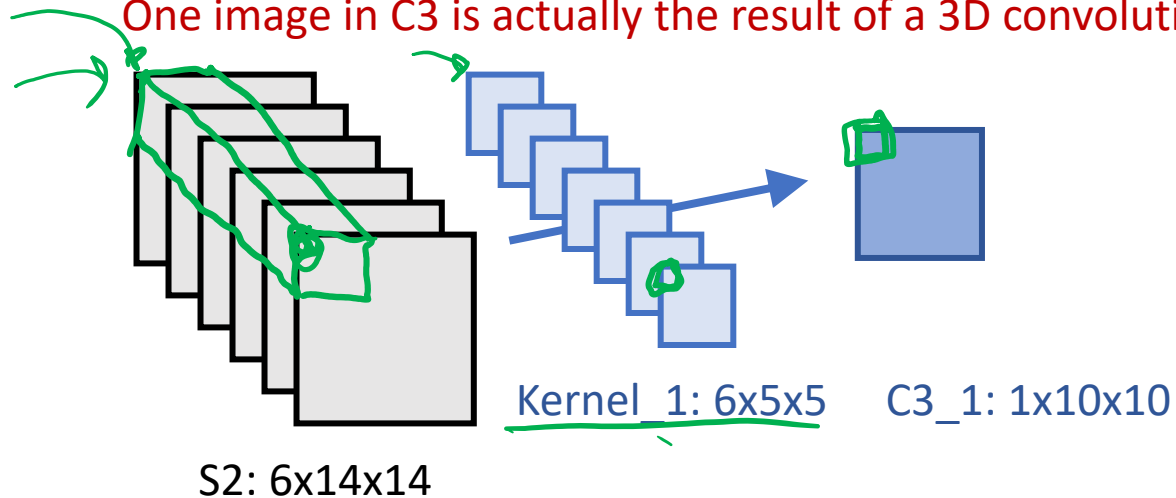
S2: f. maps 6@14x14



Convolutions

$$\sum w_{ijk} x_{uvz}$$

One image in C3 is actually the result of a 3D convolution



S2: 6x14x14

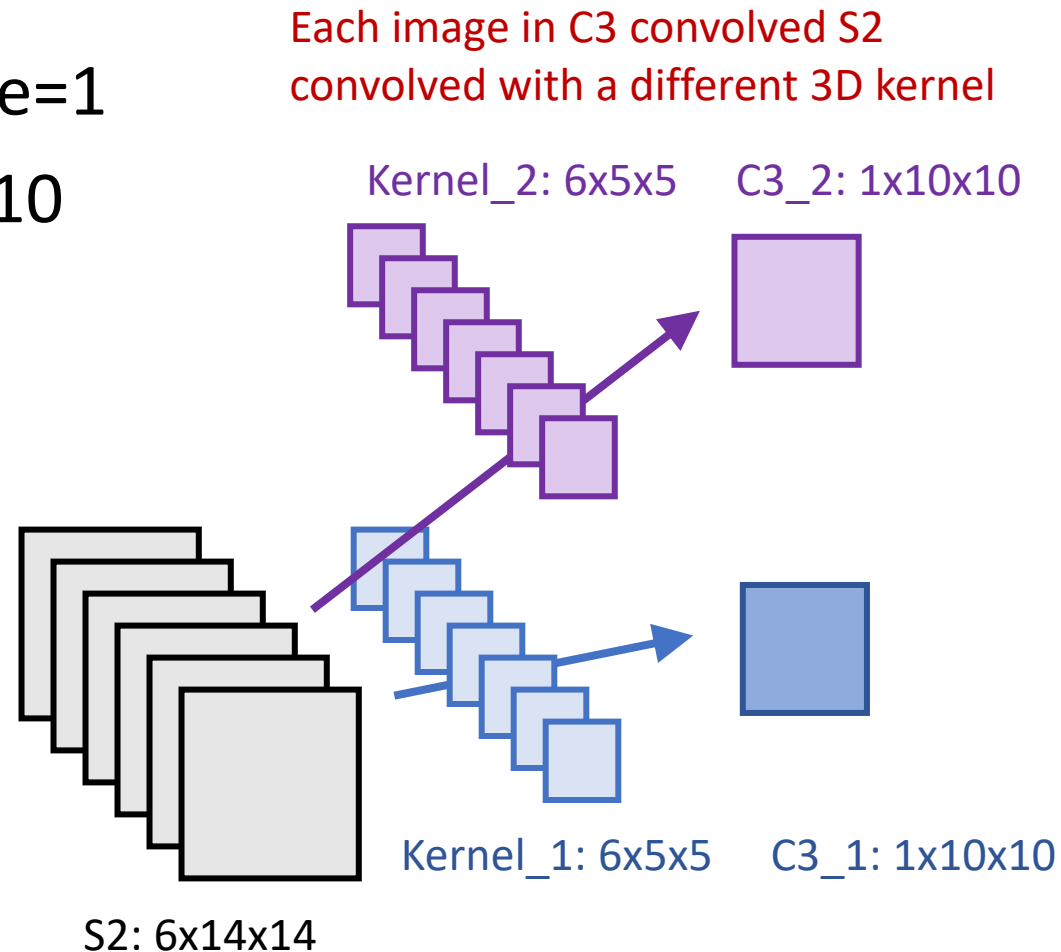
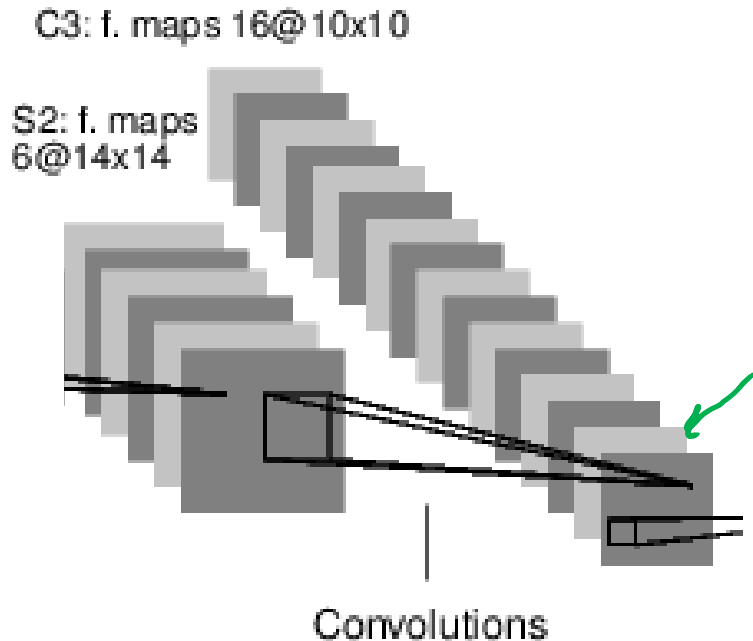
Kernel 1: 6x5x5

C3_1: 1x10x10

Question:

How big many convolutional weights between S2 and C3?

- S2: 6 channels @14x14
- Conv: 5x5, pad=0, stride=1
- C3: 16 channels @ 10x10



Question:

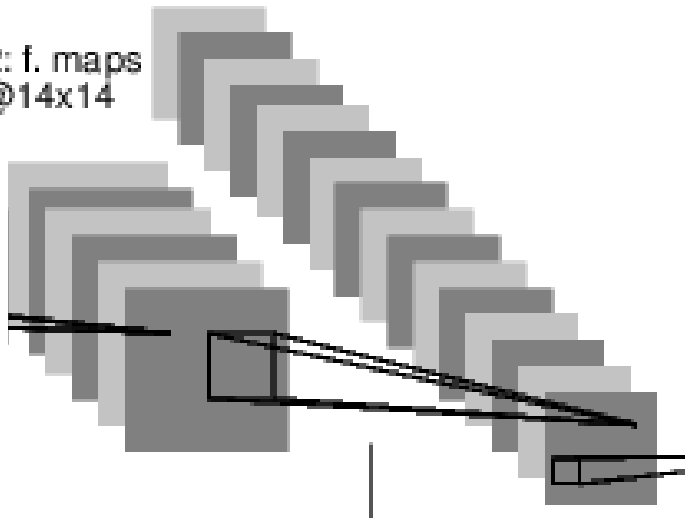
How big many convolutional weights between S2 and C3?

- S2: 6 channels @14x14
- Conv: 5x5, pad=0, stride=1
- C3: 16 channels @ 10x10

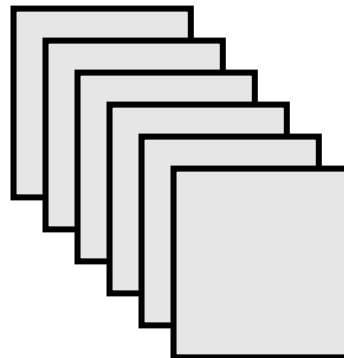
The 16 images in C3 are the result of doing 16 3D convolutions of S2 with 16 different 6x5x5 kernels. Assuming no bias term, this is 16x6x5x5 weights!

C3: f. maps 16@10x10

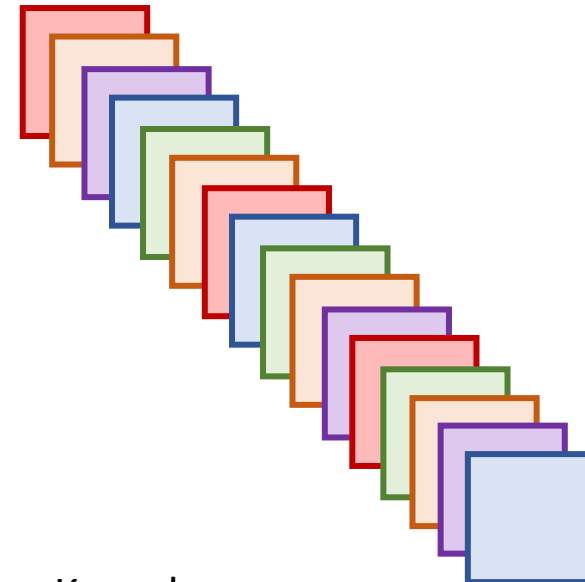
S2: f. maps 6@14x14



Convolutions



S2: 6x14x14



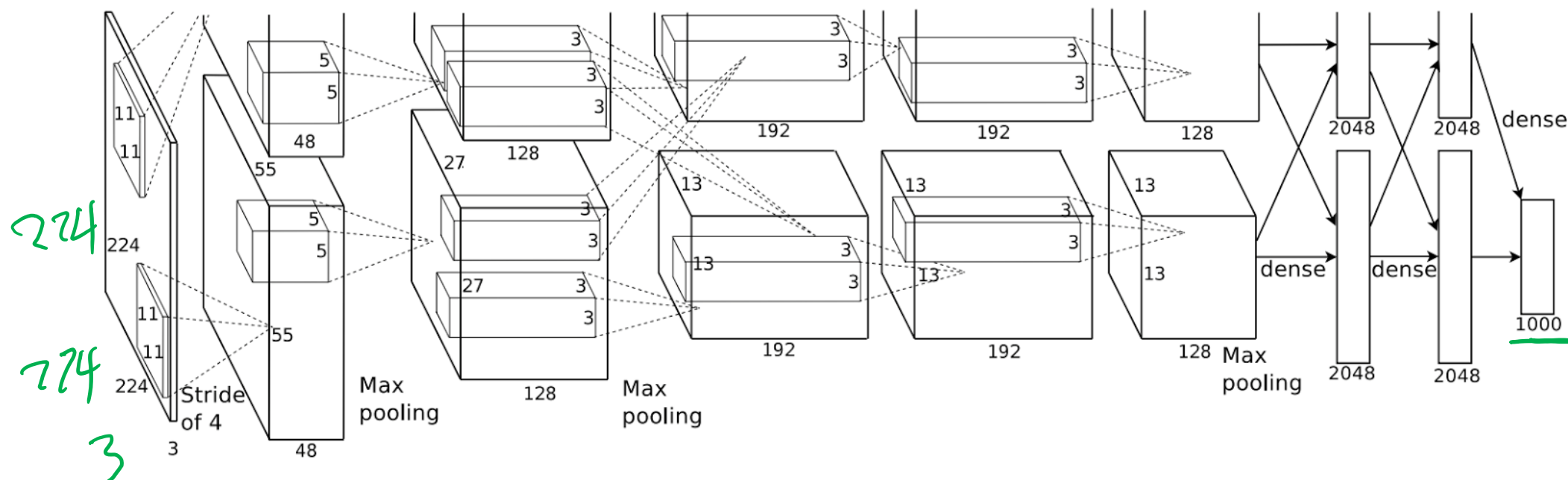
Kernels:
16@6x5x5

C3: 16@10x10

Convolutional Neural Networks

Alexnet – Lecun, et al, 2012

- Convnets for image classification
- More data & more compute power



Krizhevsky, Alex, Ilya Sutskever, and Geoffrey E. Hinton. "ImageNet classification with deep convolutional neural networks." NIPS, 2012.

CNNs for Image Recognition

