

Announcements



Canvas

- Up-to-date with scores and slip days

Assignments

- HW7: Thu, 11/19, 11:59 pm

Schedule change

- Friday: Lecture in all three recitation slots
- Monday: Recitation in both lecture slots

Final exam scheduled

Study groups

Wrap Up HMMs

HMM slides from last time





Introduction to Machine Learning

Markov Decision Processes

Instructor: Pat Virtue

Learning Paradigms

Paradigm	Data
Supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^N \quad \mathbf{x} \sim p^*(\cdot) \text{ and } y = c^*(\cdot)$
\hookrightarrow Regression	$y^{(i)} \in \mathbb{R}$
\hookrightarrow Classification	$y^{(i)} \in \{1, \dots, K\}$
\hookrightarrow Binary classification	$y^{(i)} \in \{+1, -1\}$
\hookrightarrow Structured Prediction	$\mathbf{y}^{(i)}$ is a vector
 Unsupervised	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N \quad \mathbf{x} \sim p^*(\cdot)$
Semi-supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^{N_1} \cup \{\mathbf{x}^{(j)}\}_{j=1}^{N_2}$
Online	$\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}), (\mathbf{x}^{(2)}, y^{(2)}), (\mathbf{x}^{(3)}, y^{(3)}), \dots\}$
Active Learning	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N$ and can query $y^{(i)} = c^*(\cdot)$ at a cost
Imitation Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}), (s^{(2)}, a^{(2)}), \dots\}$
 Reinforcement Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}, r^{(1)}), (s^{(2)}, a^{(2)}, r^{(2)}), \dots\}$

Learning Paradigms

Paradigm	Data
Supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^N \quad \mathbf{x} \sim p^*(\cdot) \text{ and } y = c^*(\cdot)$
\hookrightarrow Regression	$y^{(i)} \in \mathbb{R}$
\hookrightarrow Classification	$y^{(i)} \in \{1, \dots, K\}$
\hookrightarrow Binary classification	$y^{(i)} \in \{+1, -1\}$
\hookrightarrow Structured Prediction	$\mathbf{y}^{(i)}$ is a vector
Unsupervised	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N \quad \mathbf{x} \sim p^*(\cdot)$
Semi-supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^{N_1} \cup \{\mathbf{x}^{(j)}\}_{j=1}^{N_2}$
Online	$\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}), (\mathbf{x}^{(2)}, y^{(2)}), (\mathbf{x}^{(3)}, y^{(3)}), \dots\}$
Active Learning	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N$ and can query $y^{(i)} = c^*(\cdot)$ at a cost
Imitation Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}), (s^{(2)}, a^{(2)}), \dots\}$
Reinforcement Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}, r^{(1)}), (s^{(2)}, a^{(2)}, r^{(2)}), \dots\}$

Interacting with the World

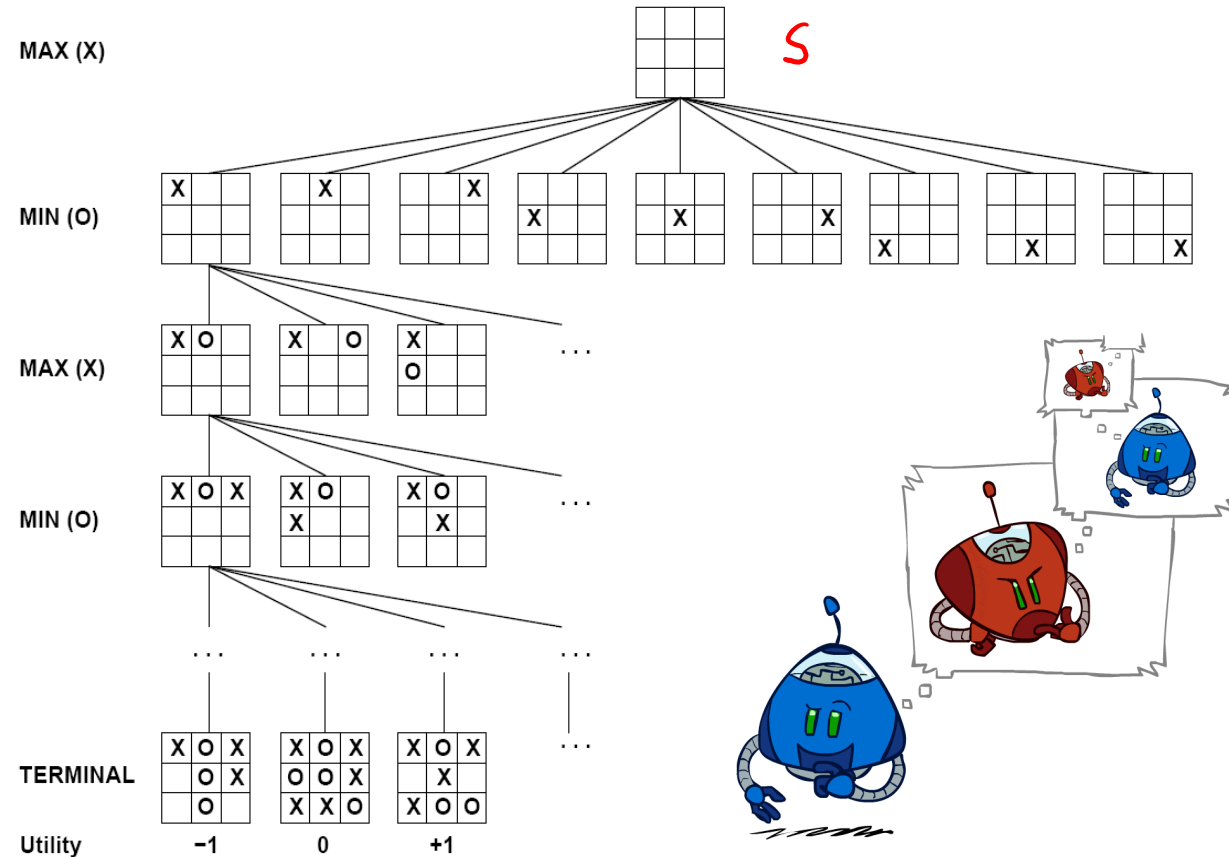
Sequential decision making

Games

- Simple: Tic-tac-toe
- Go
- Arcade games

Worlds

- Simple: Grid World
- Open AI Gym <https://gym.openai.com/>
- Autonomous Driving



Interacting with the World

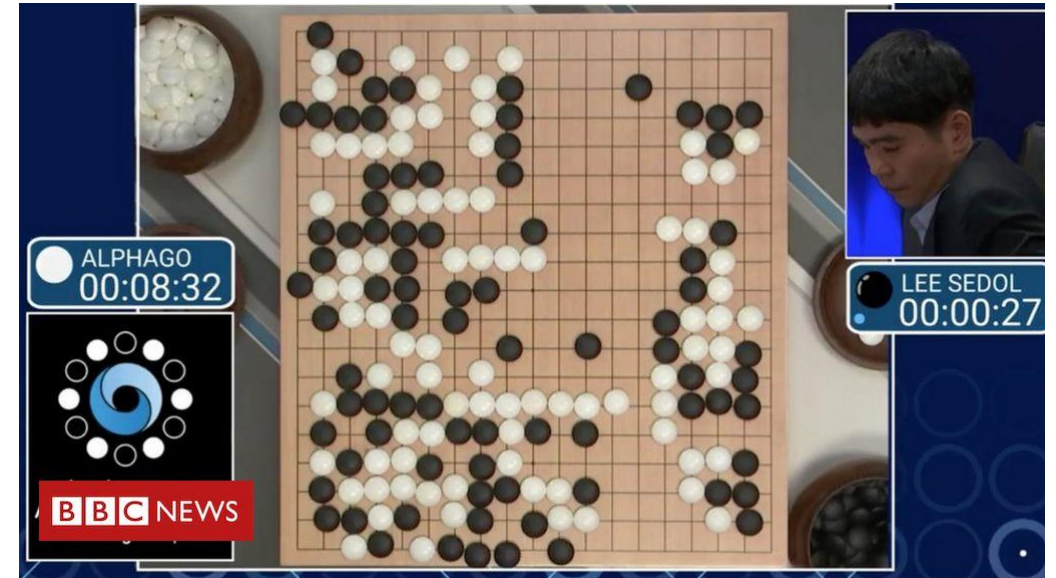
Sequential decision making

Games

- Simple: Tic-tac-toe
- Go
- Arcade games

Worlds

- Simple: Grid World
- Open AI Gym <https://gym.openai.com/>
- Autonomous Driving



Interacting with the World

Sequential decision making

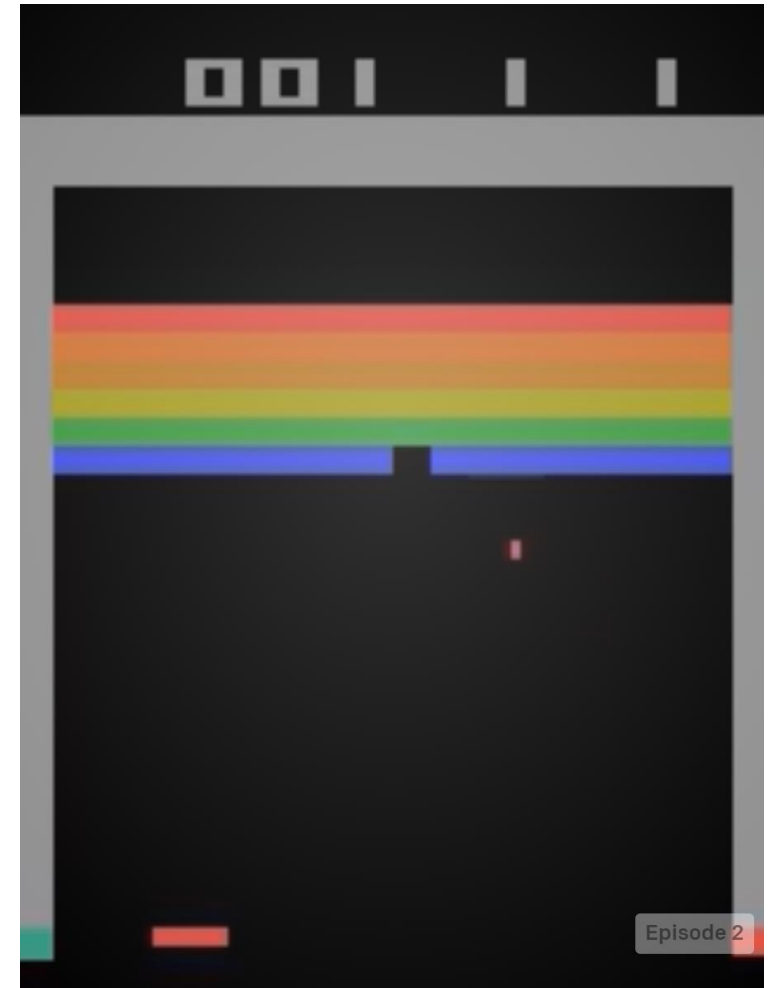
Games

- Simple: Tic-tac-toe
- Go
- Arcade games

Worlds

- Simple: Grid World
- Open AI Gym <https://gym.openai.com/>
- Autonomous Driving

Image: <https://gym.openai.com/>



Interacting with the World

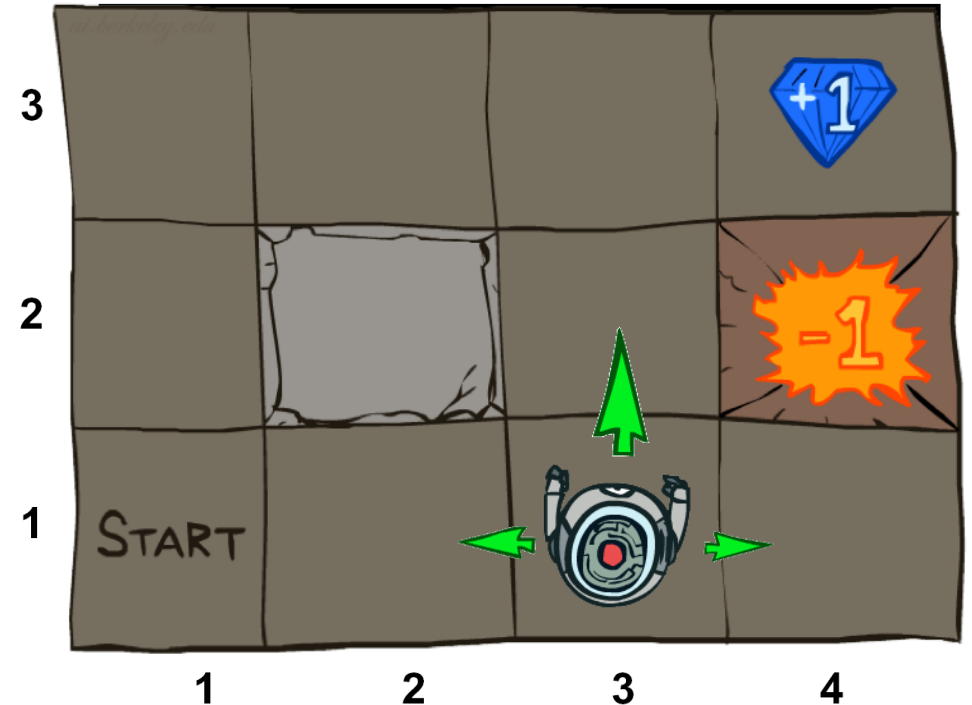
Sequential decision making

Games

- Simple: Tic-tac-toe
- Go
- Arcade games

Worlds

- Simple: Grid World
- Open AI Gym <https://gym.openai.com/>
- Autonomous Driving



Interacting with the World

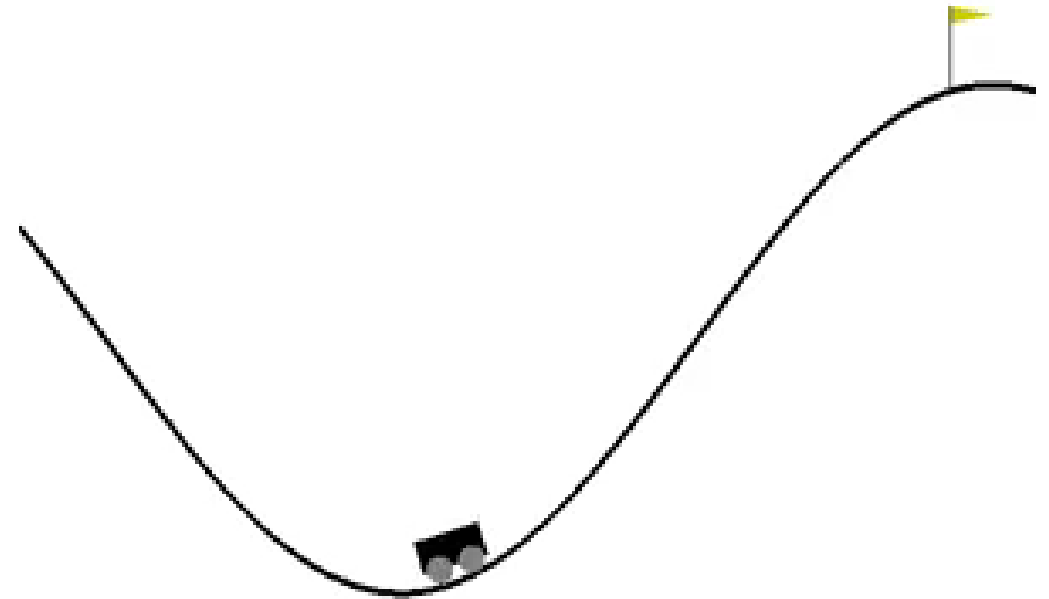
Sequential decision making

Games

- Simple: Tic-tac-toe
- Go
- Arcade games

Worlds

- Simple: Grid World
- Open AI Gym <https://gym.openai.com/>
- Autonomous Driving



Interacting with the World

Sequential decision making

Games

- Simple: Tic-tac-toe
- Go
- Arcade games

Worlds

- Simple: Grid World
- Open AI Gym <https://gym.openai.com/>
- Autonomous Driving

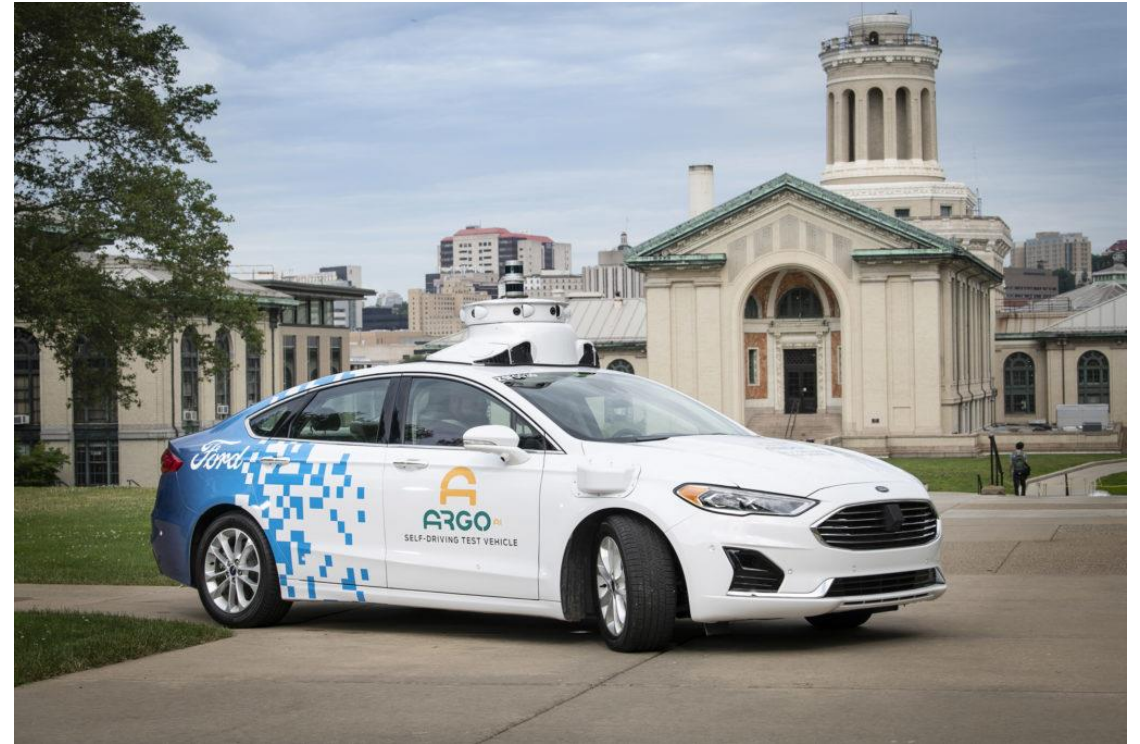
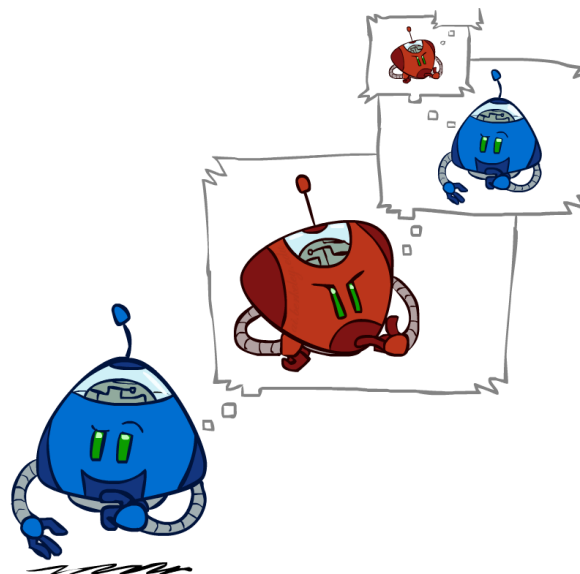


Image: <https://www.argo.ai/2019/06/pushing-the-self-driving-frontier-argo-ai-partners-with-carnegie-mellon-to-form-autonomous-vehicle-research-center/>

Minimax Search

$$s' = \text{result}(s, a)$$


Piazza Poll 1

What is the minimax value at the root?

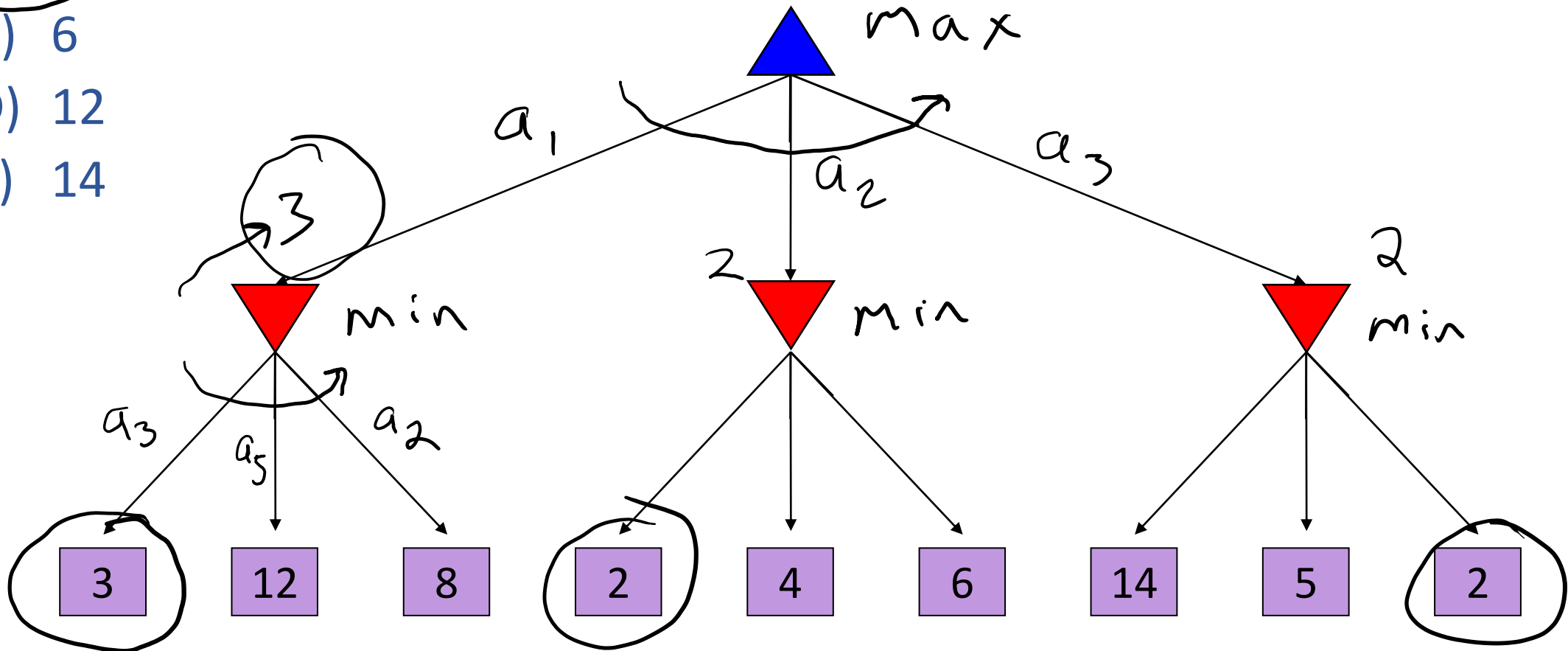
A) 2

B) 3

C) 6

D) 12

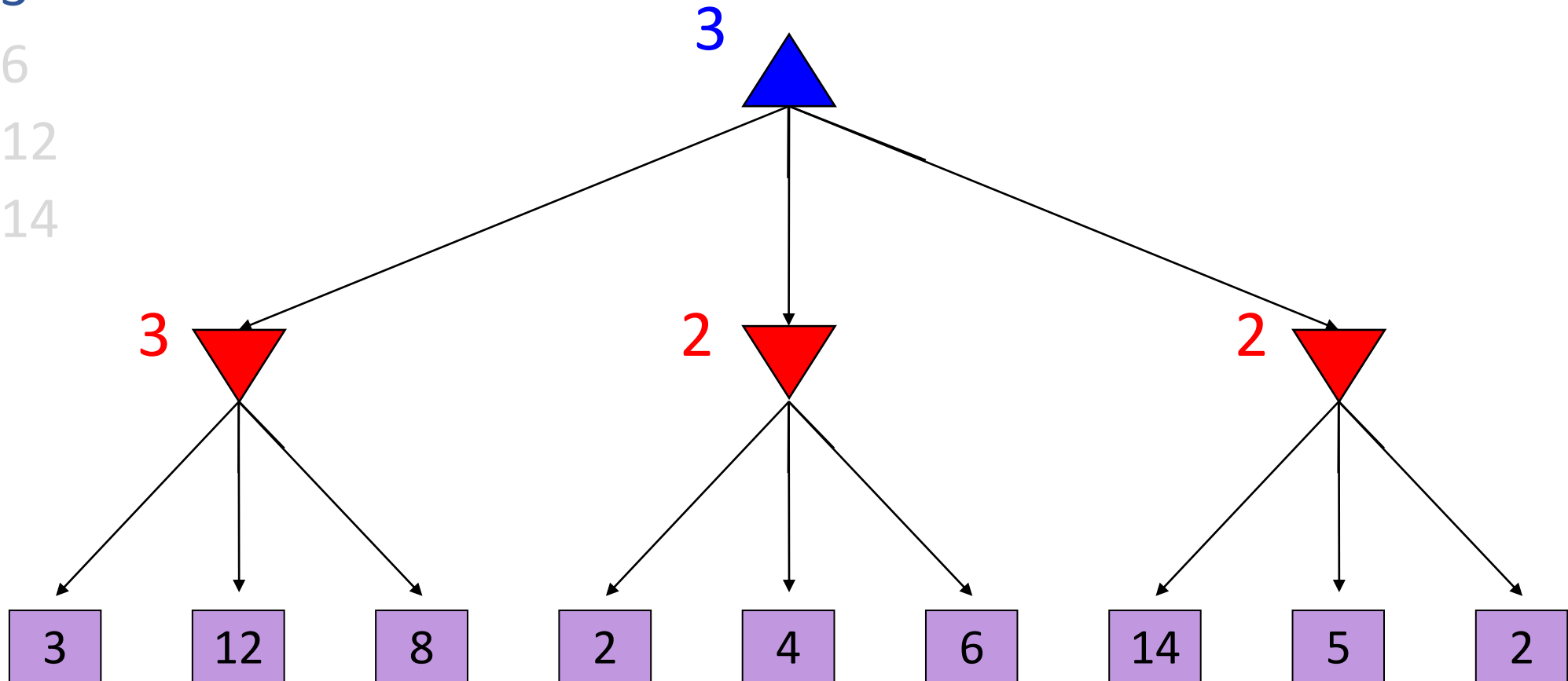
E) 14



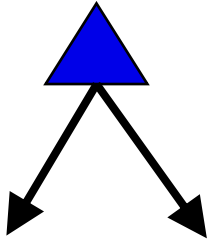
Piazza Poll 1

What is the minimax value at the root?

- A) 2
- B) 3
- C) 6
- D) 12
- E) 14

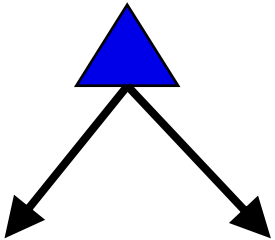


Minimax Notation



$$V(s) = \max_a V(s'),$$

where $s' = \text{result}(s, a)$

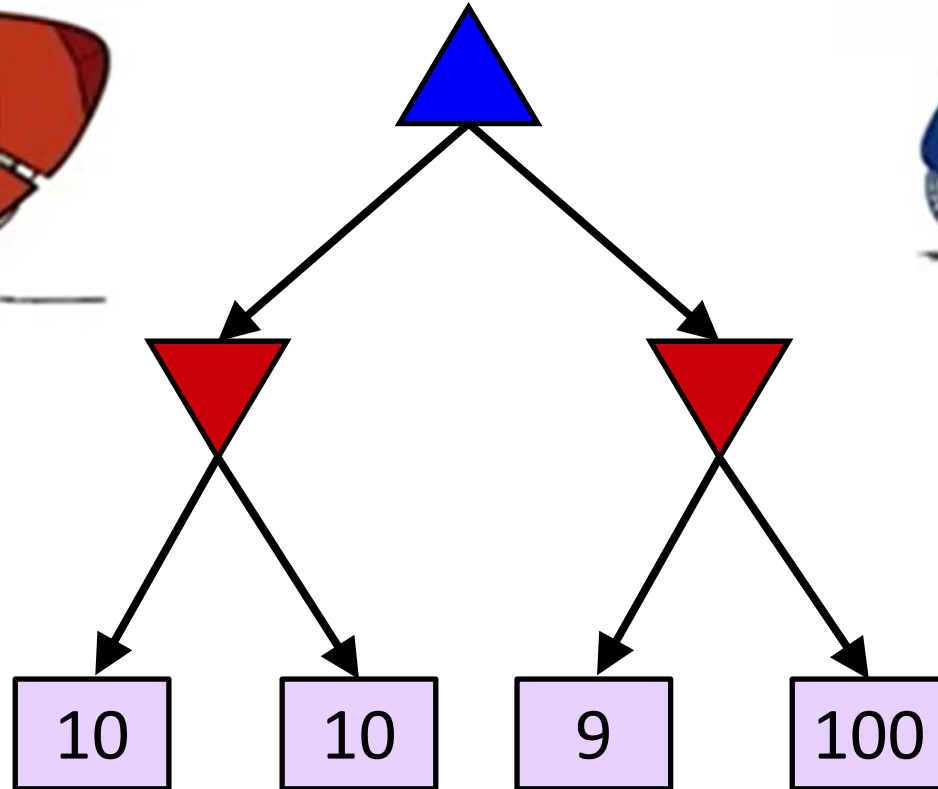
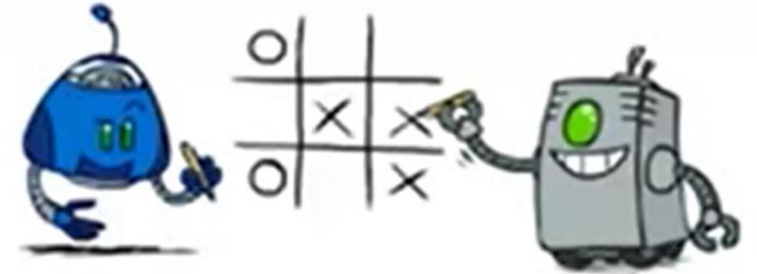
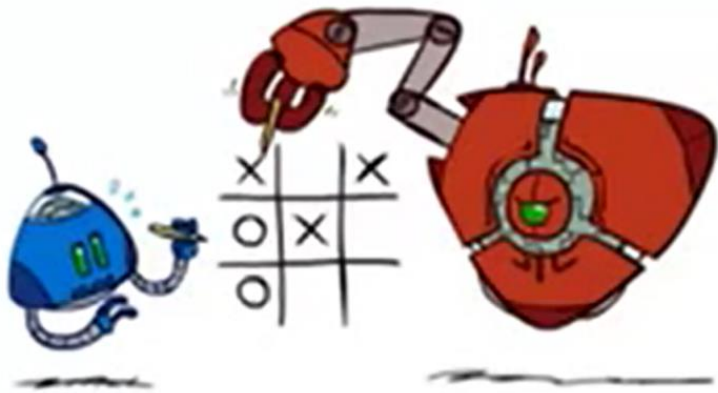


$$\hat{a} = \operatorname{argmax}_a V(s'),$$

where $s' = \text{result}(s, a)$

Modeling Assumptions

Know your opponent



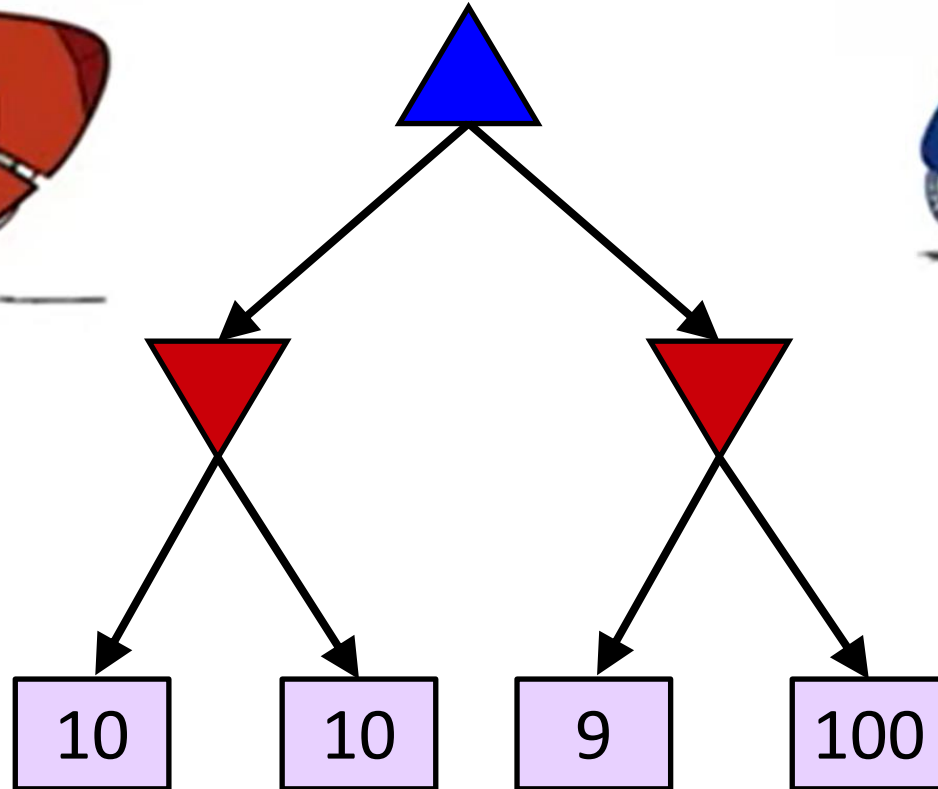
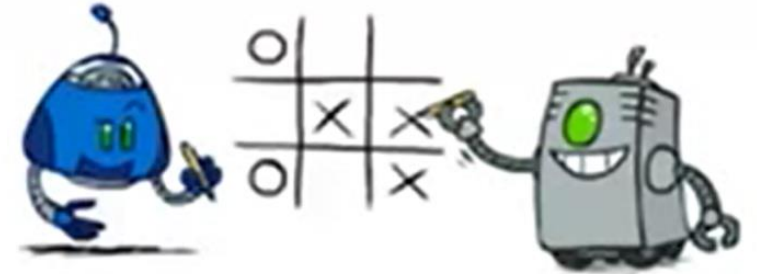
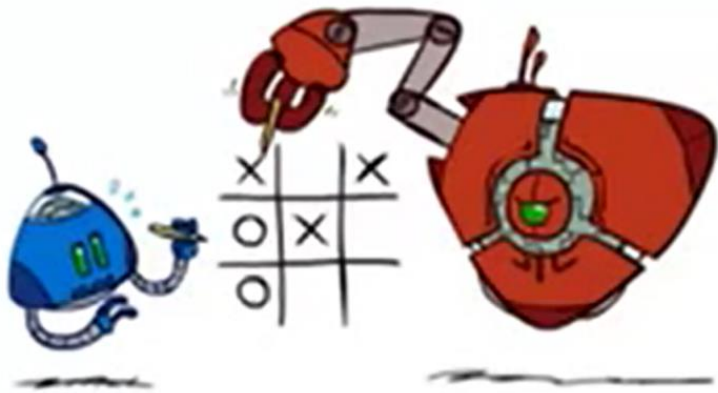
Minimax Driver?



Clip: How I Met Your Mother, CBS

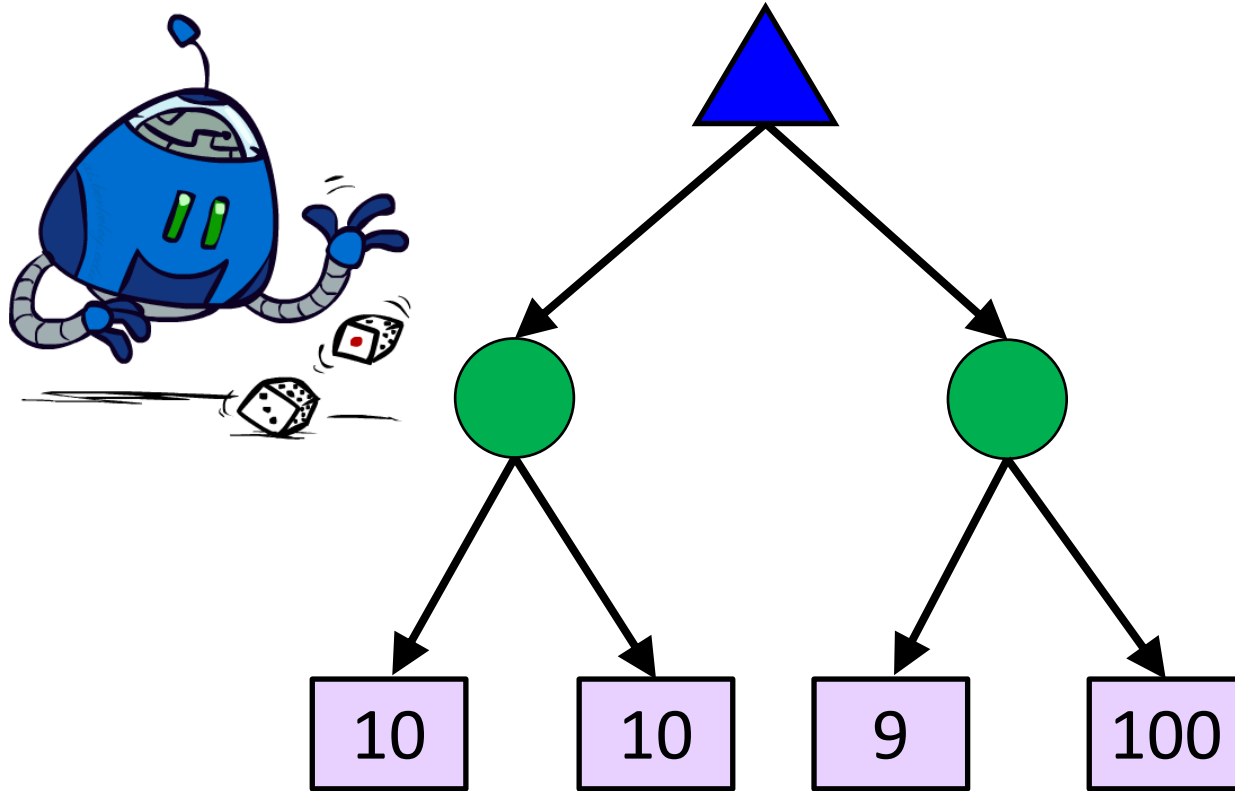
Modeling Assumptions

Know your opponent



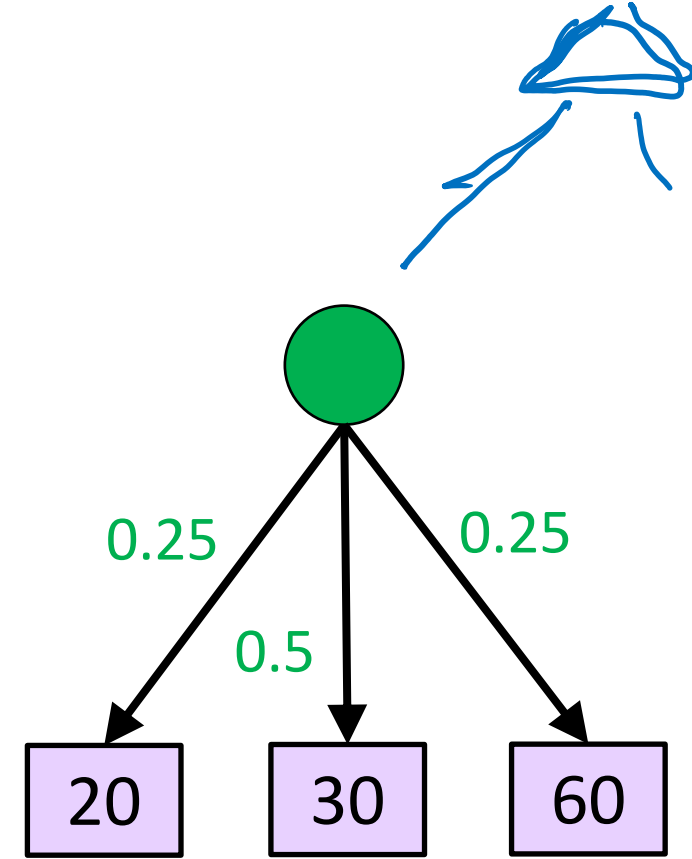
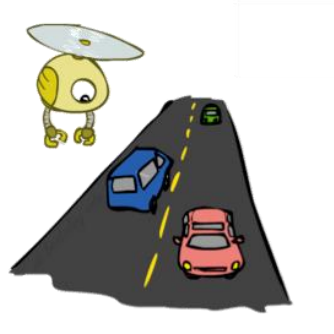
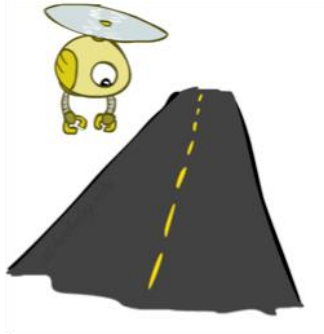
Modeling Assumptions

Chance nodes: Expectimax



Expectations

Time: 20 min + 30 min + 60 min
x x x
Probability: 0.25 0.50 0.25



Max node notation

$$V(s) = \max_a V(s'),$$

where $s' = result(s, a)$

Chance node notation

$$V(s) = \sum_{s'} P(s') V(s')$$

Piazza Poll 2

Expectimax tree search:

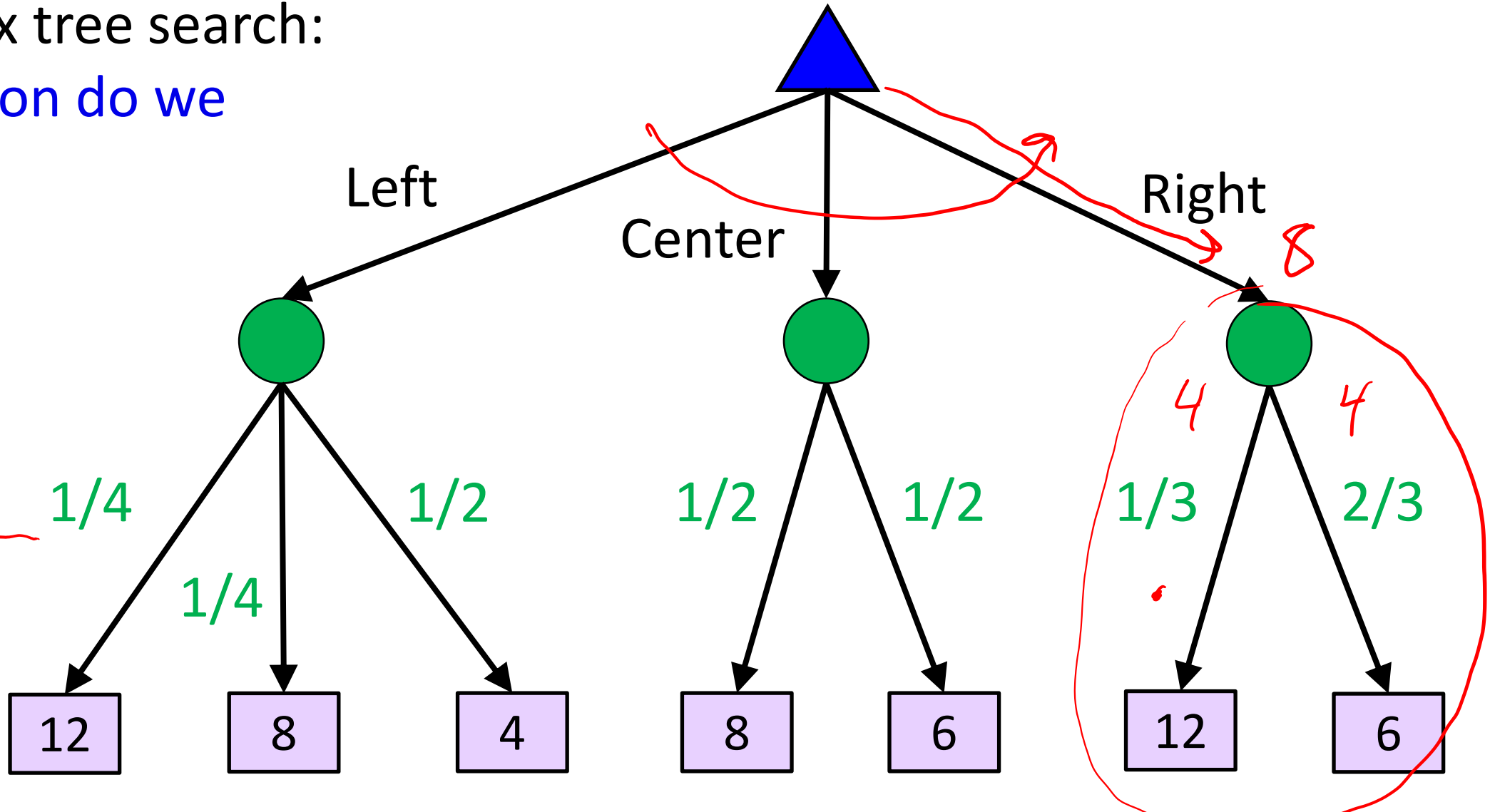
Which action do we choose?

A: Left

B: Center

C: Right

D: Eight

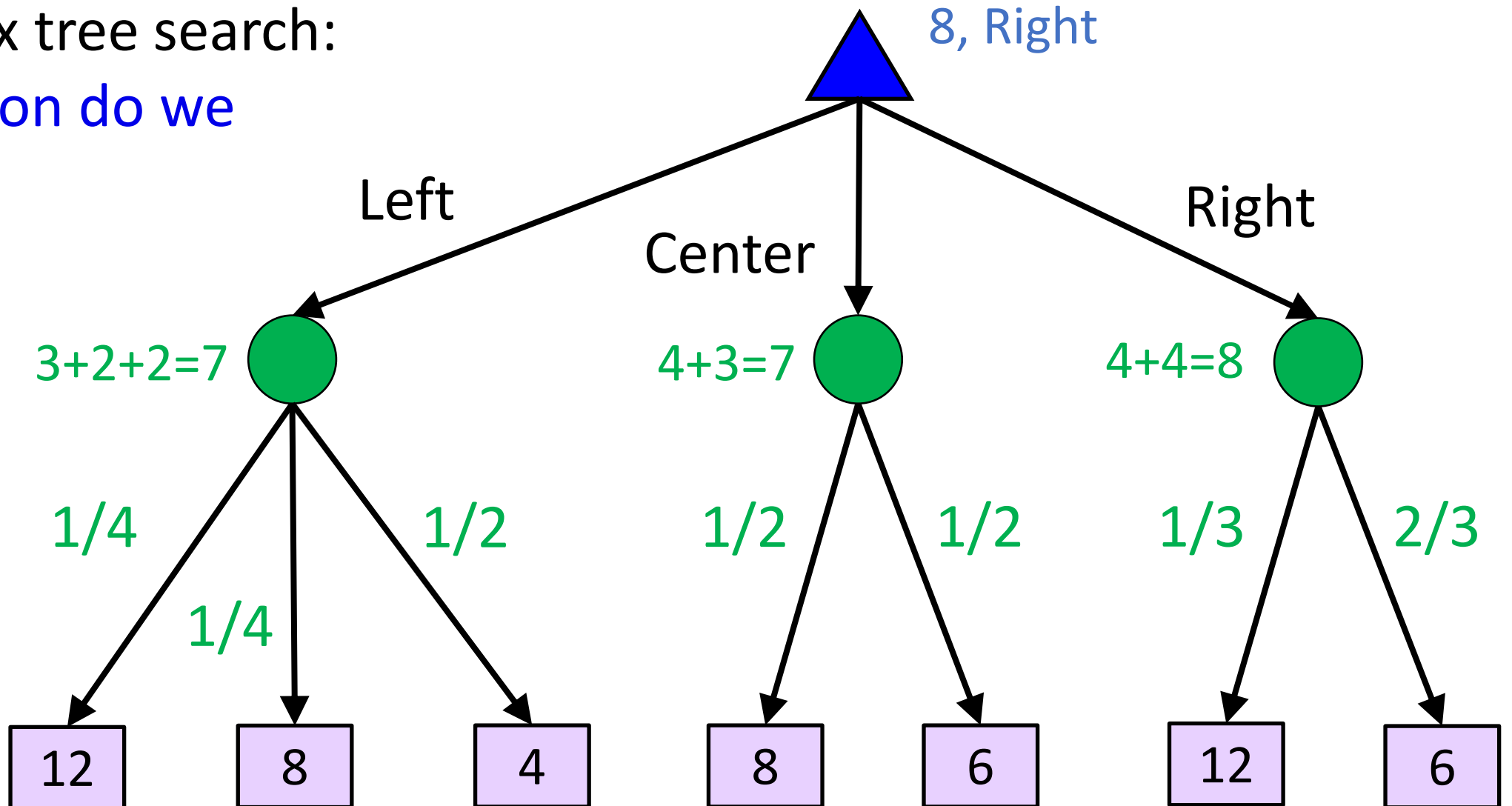


Piazza Poll 2

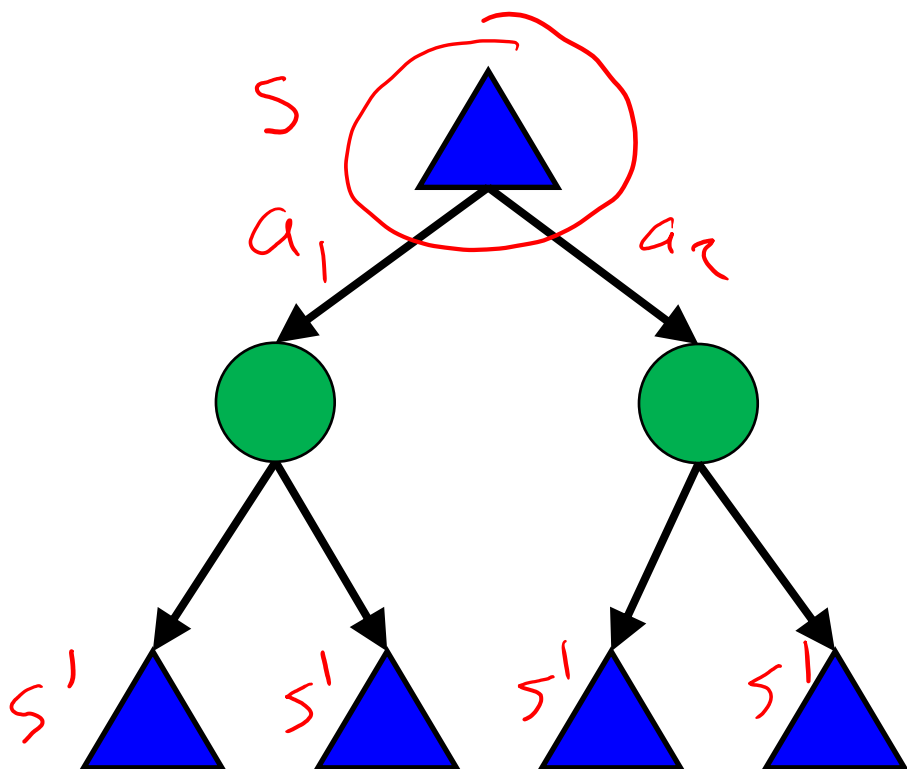
Expectimax tree search:

Which action do we choose?

- A: Left
- B: Center
- C: Right
- D: Eight

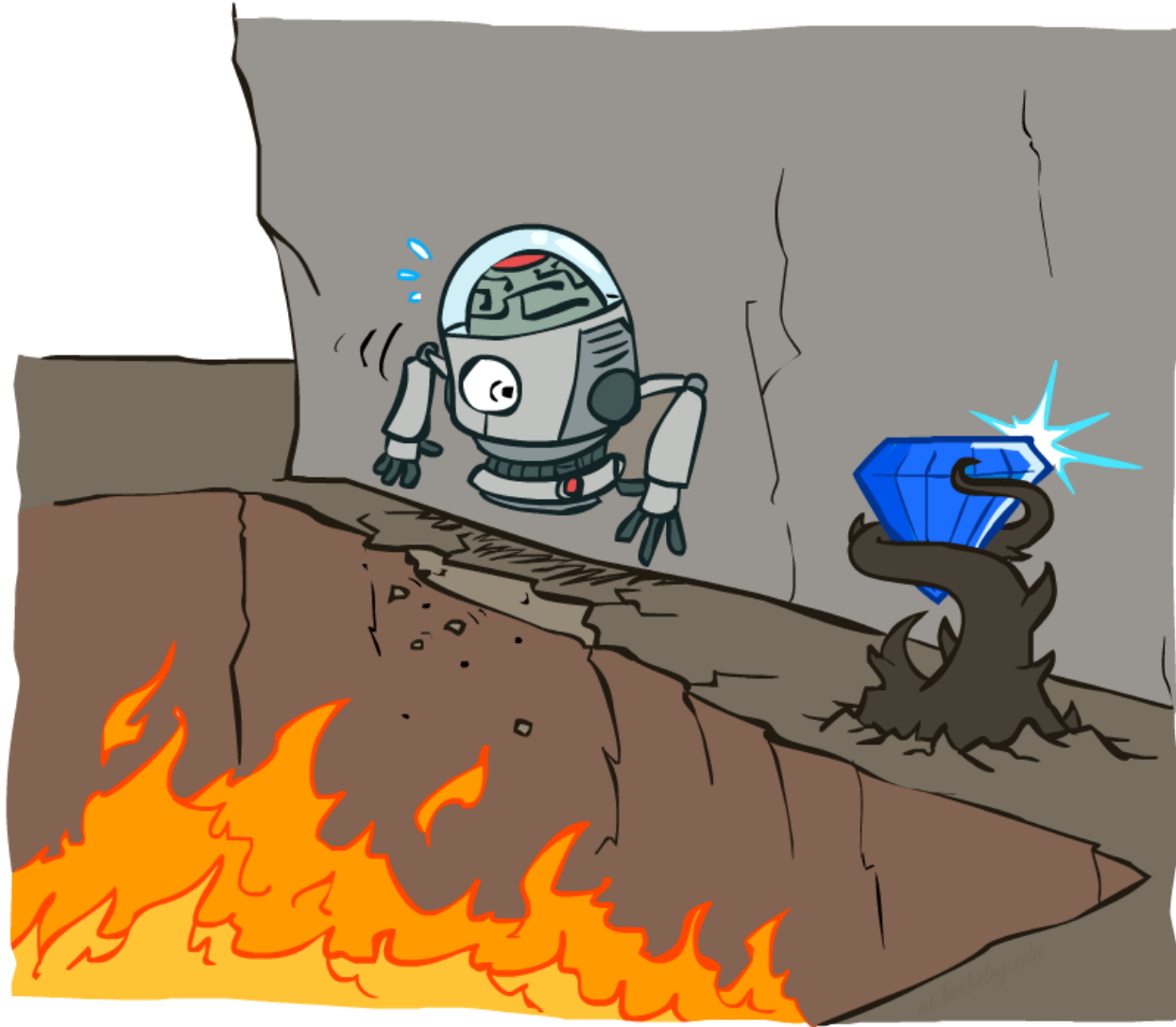


Expectimax Notation



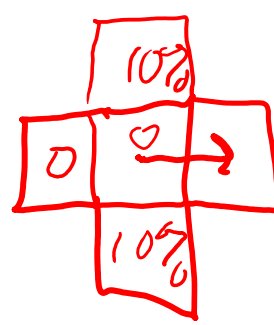
$$V(s) = \max_a \sum_{s'} P(s'|s, a) V(s')$$

Non-Deterministic Search

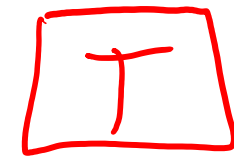


Example: Grid World

- A maze-like problem
 - The agent lives in a grid
 - Walls block the agent's path
- Noisy movement: actions do not always go as planned
 - 80% of the time, the action North takes the agent North (if there is no wall there)
 - 10% of the time, North takes the agent West; 10% East
 - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
 - Small "living" reward each step (can be negative) -0.1
 - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards



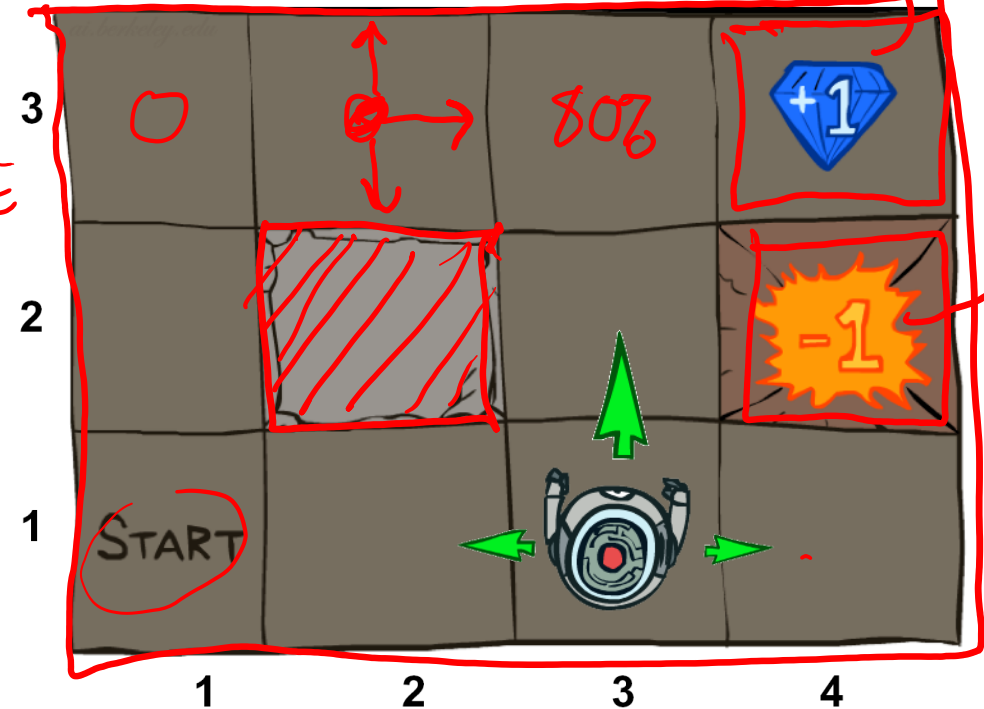
$a = \text{East}$
80%



exit

exit

N
W • E
S

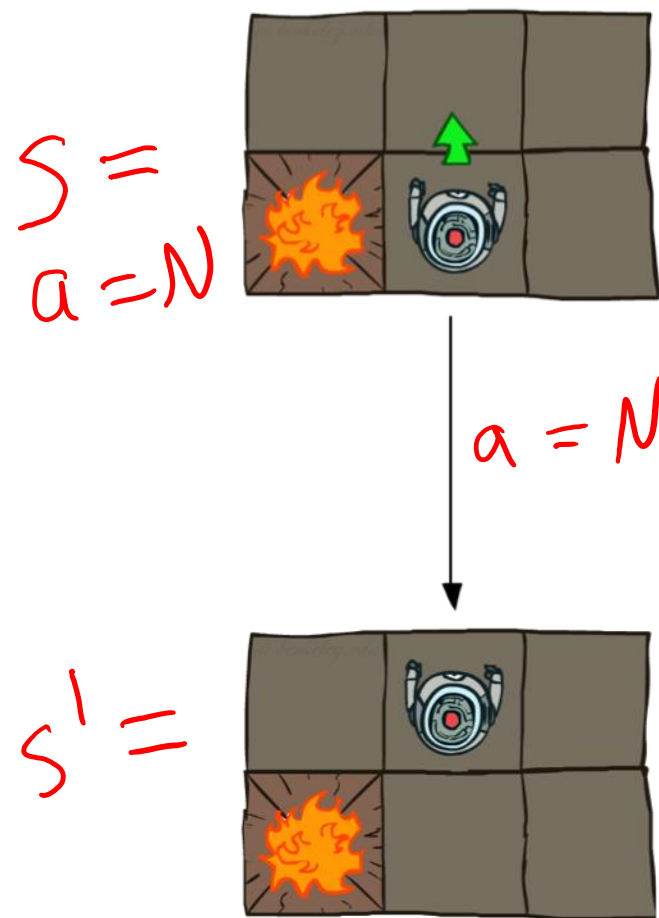


80% go the direction that
I choose

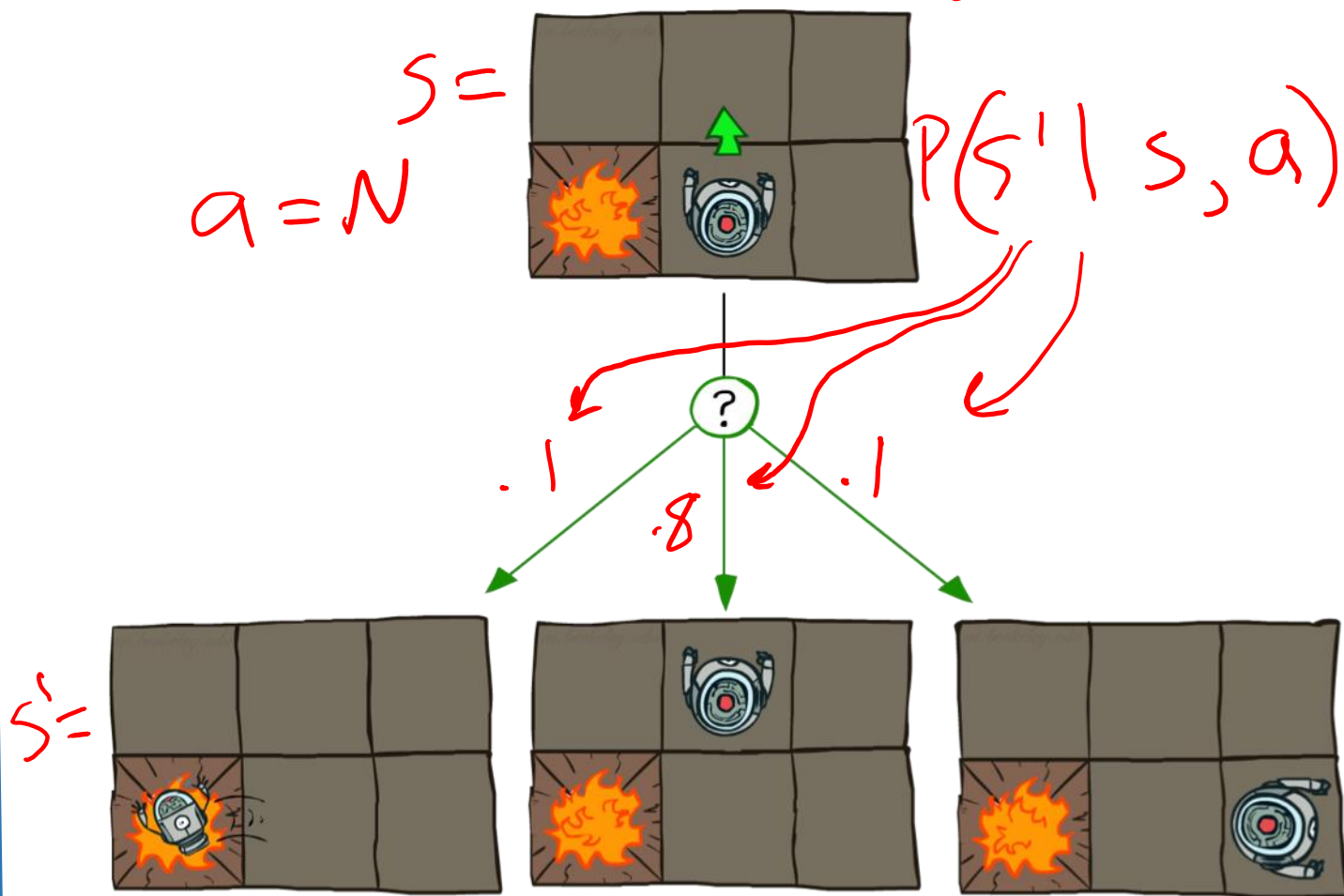
20% go in an adjacent
direction

Grid World Actions

Deterministic Grid World





Stochastic Grid World



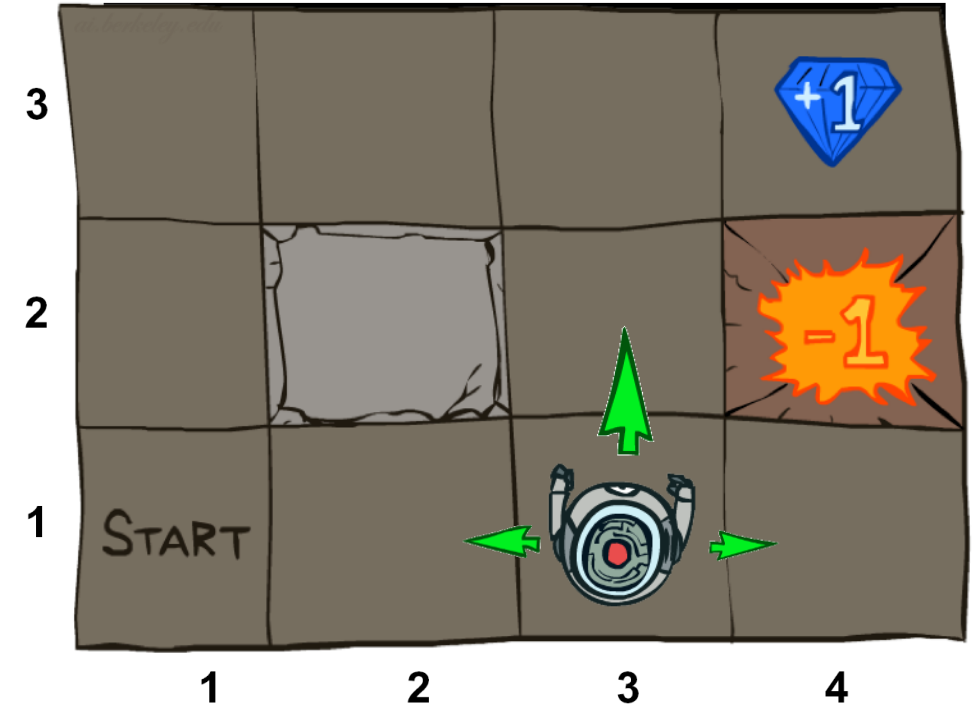
Markov Decision Processes

An MDP is defined by:

- A **set of states** $s \in S$
- A **set of actions** $a \in A$
- A **transition function** $T(s, a, s')$
 - Probability that a from s leads to s' , i.e., $P(s' | s, a)$
 - Also called the model or the dynamics
- A **reward function** $R(s, a, s')$ 
- Sometimes just $R(s)$ or $R(s')$
- A **start state** 
- Maybe a **terminal state**

MDPs are non-deterministic search problems

- One way to solve them is with expectimax search
- We'll have a new tool soon



Demo of Gridworld

What is Markov about MDPs?

“Markov” generally means that given the present state, the future and the past are independent

For Markov decision processes, “Markov” means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, \underbrace{S_{t-1} = s_{t-1}, A_{t-1}, \dots, S_0 = s_0}_{\text{past}})$$

=

$$\rightarrow P(S_{t+1} = s' | \underbrace{S_t = s_t}_{\text{current state}}, \underbrace{A_t = a_t}_{\text{action}})$$



Andrey Markov
(1856-1922)

Policies

We don't just want an optimal **plan**, or sequence of actions, from start to a goal

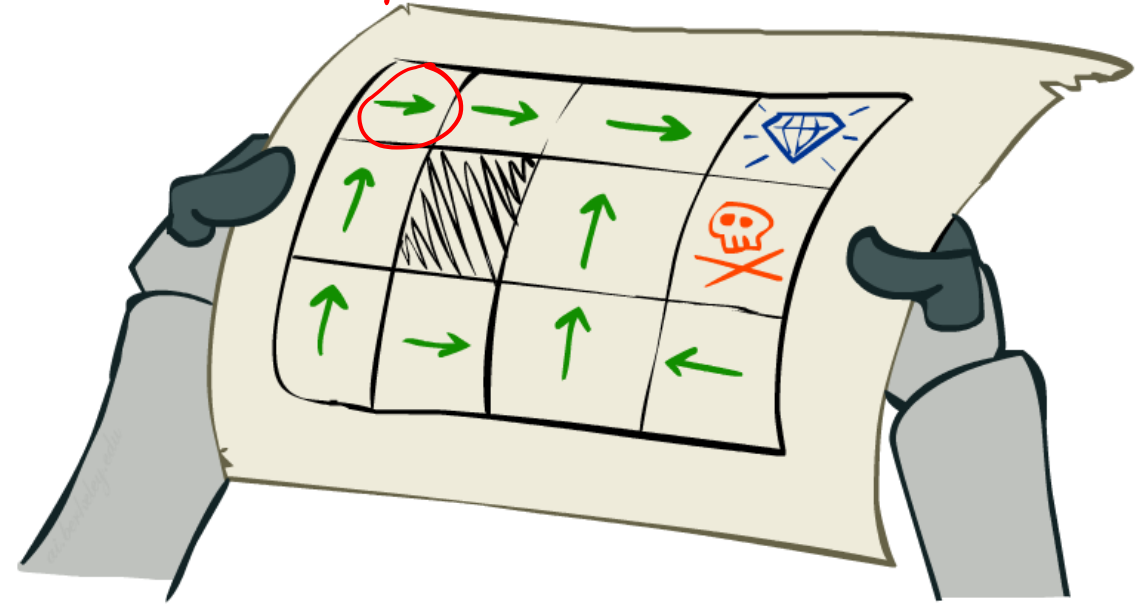
For MDPs, we want an optimal **policy** $\pi^*: S \rightarrow A$

- A policy π gives an action for each state
- An optimal policy is one that maximizes expected utility if followed

Expectimax didn't compute entire policies

- It computed the action for a single state only

$$\pi(s) \rightarrow a$$
$$h(x) \rightarrow y$$



Optimal policy when $R(s, a, s') = -0.03$
for all non-terminals s