10-315
Introduction to ML

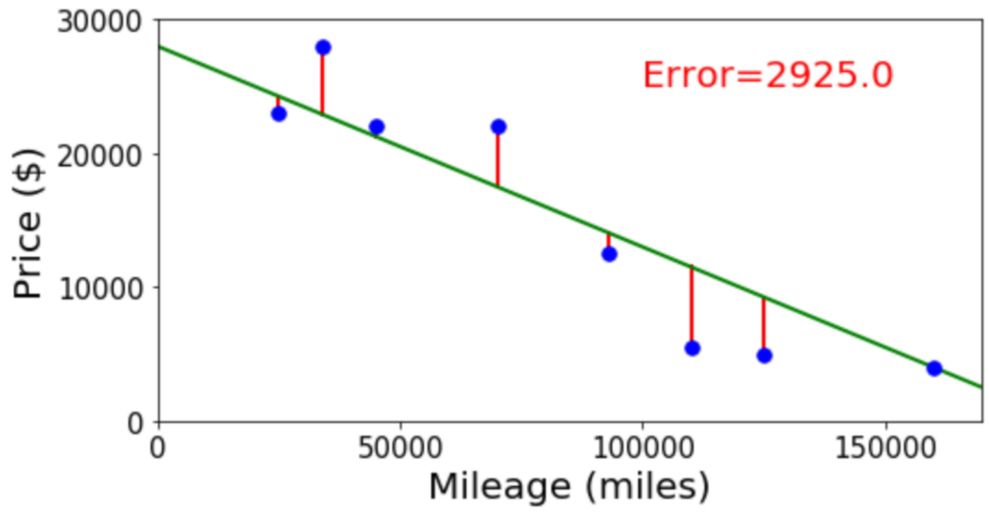
Feature Engineering

Instructor: Pat Virtue

Poll 1

Linear regression. What is linear about it?

Select all that apply.



A. Always fits a linear (or affine) shape to the data

B. Linear objective function with respect to the input

C. Linear objective function with respect to the parameters

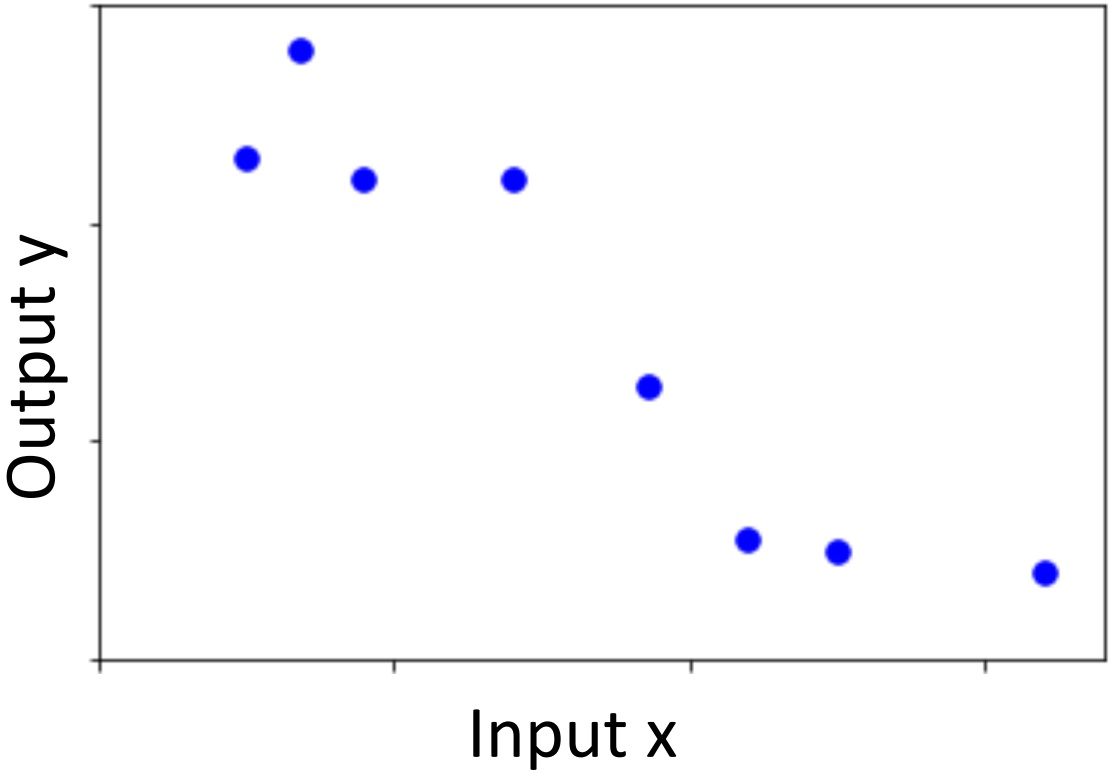
D. None of the above

Pre-reading

Polynomial Features

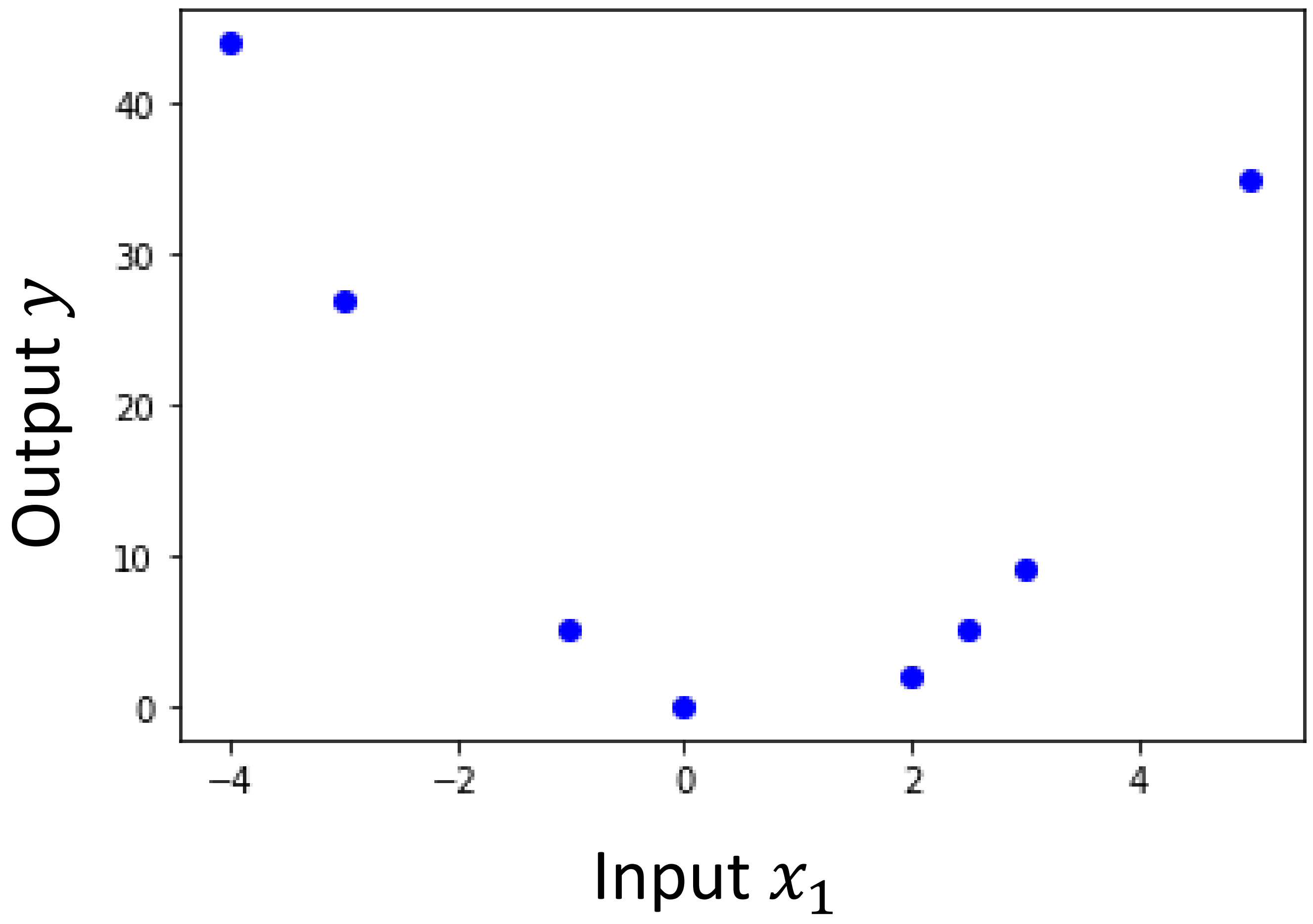
Linear Data

Regression on simple linear dataset



Non-linear Data

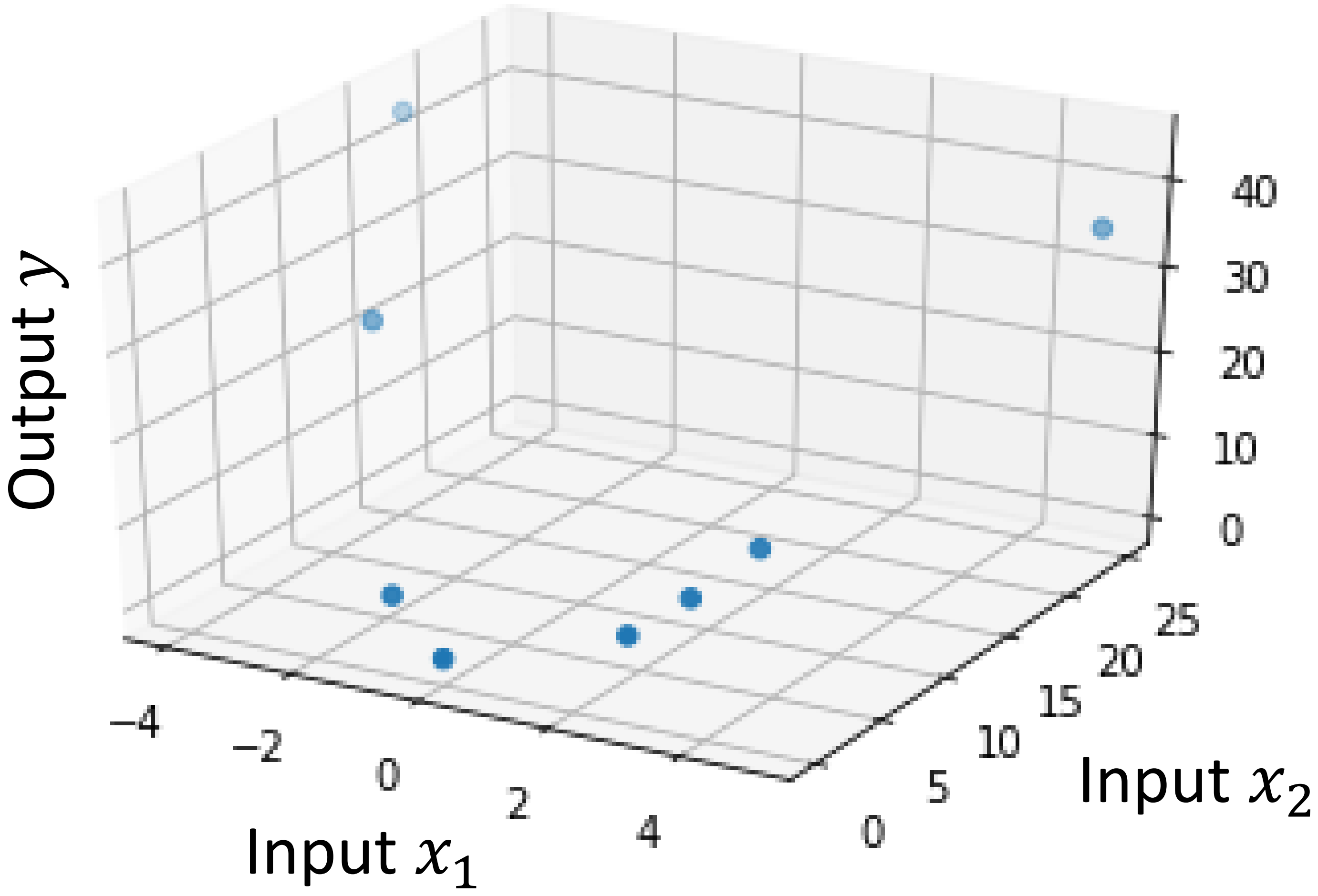
Linear regression on polynomial data?



y	x_1
44	-4
27	-3
5	-1
0	0
2	2
5	2.5
9	3
35	5

Non-linear Data

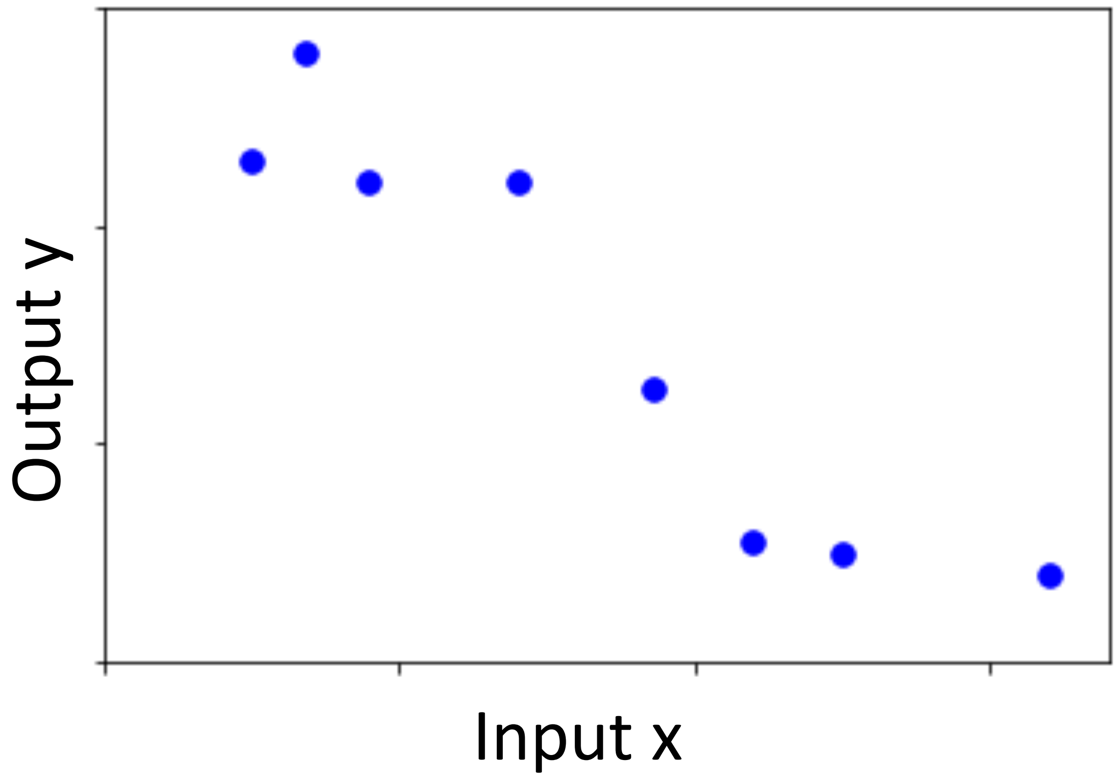
Regression on simple linear dataset



y	x_1	x_2
44	-4	16
27	-3	9
5	-1	1
0	0	0
2	2	4
5	2.5	6.25
9	3	9
35	5	25

Non-linear Data

Polynomial feature map for linear regression

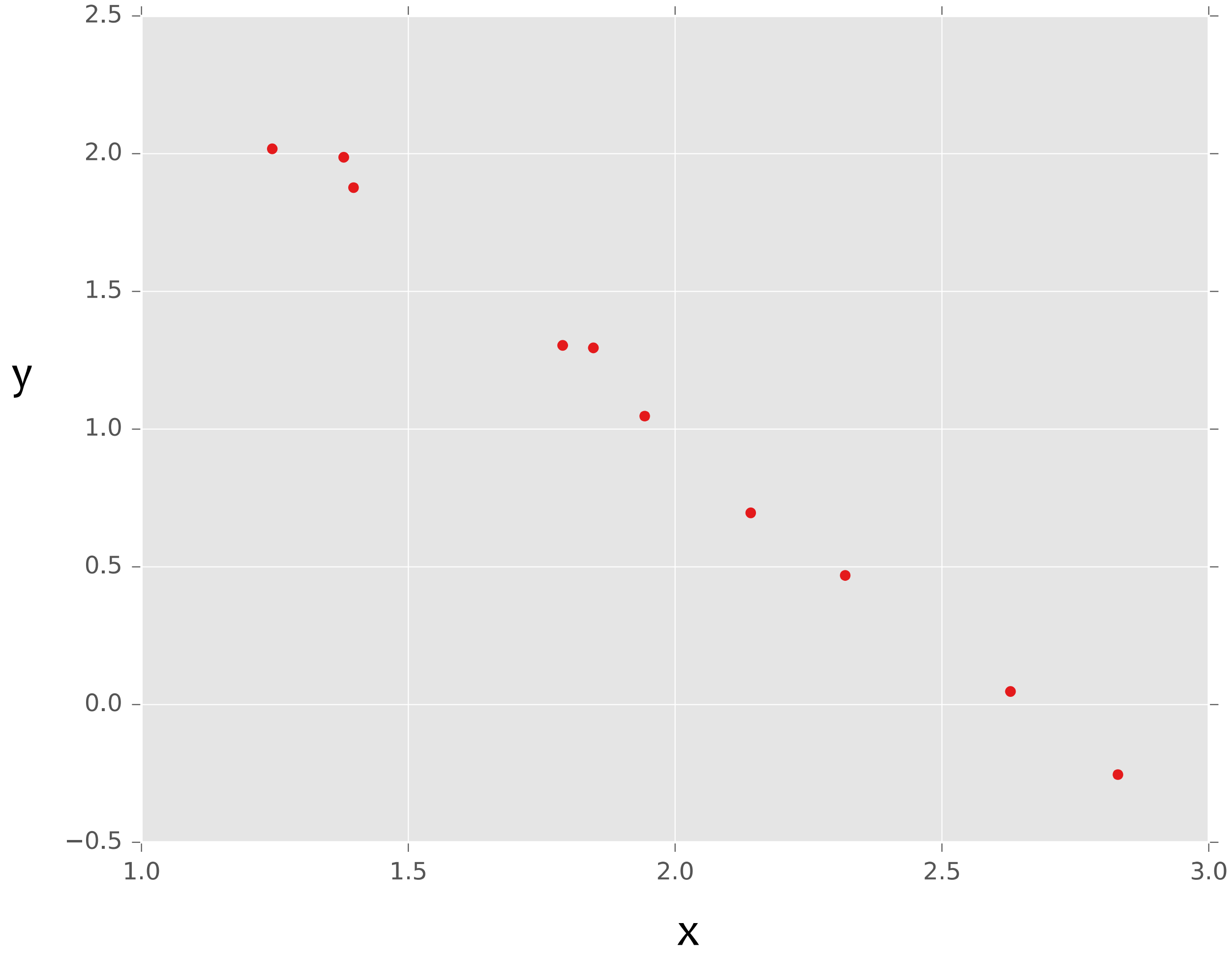


Example: Linear Regression

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$
where f(.) is a polynomial
basis function

y	x
2.0	1.2
1.3	1.7
0.1	2.7
1.1	1.9

true "unknown" target function is linear with negative slope and gaussian noise



Slide credit: CMU MLD Matt Gormley

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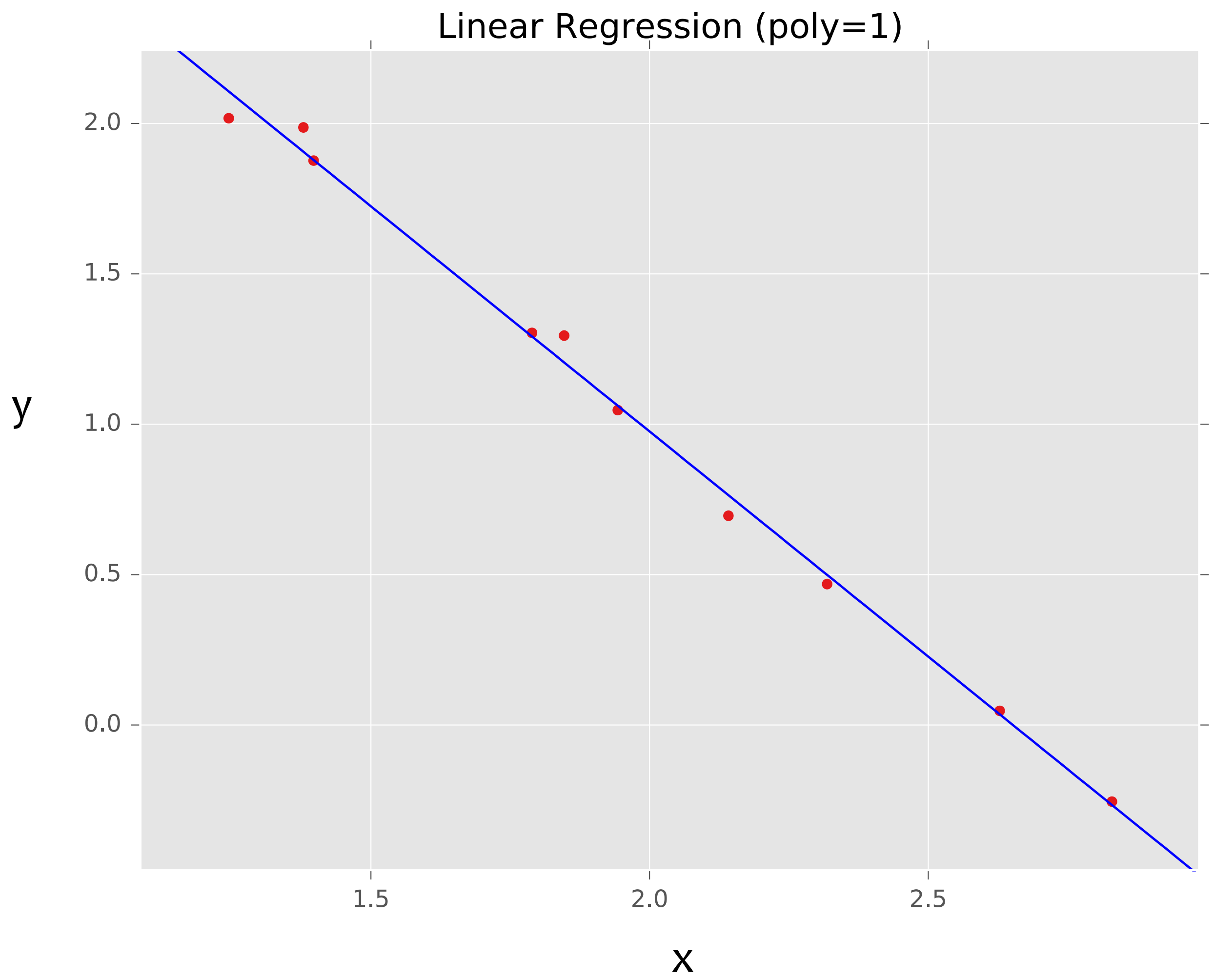

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Example: Linear Regression

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$
where f(.) is a polynomial
basis function

y	x	x^2
2.0	1.2	$(1.2)^2$
1.3	1.7	$(1.7)^2$
0.1	2.7	$(2.7)^2$
1.1	1.9	$(1.9)^2$

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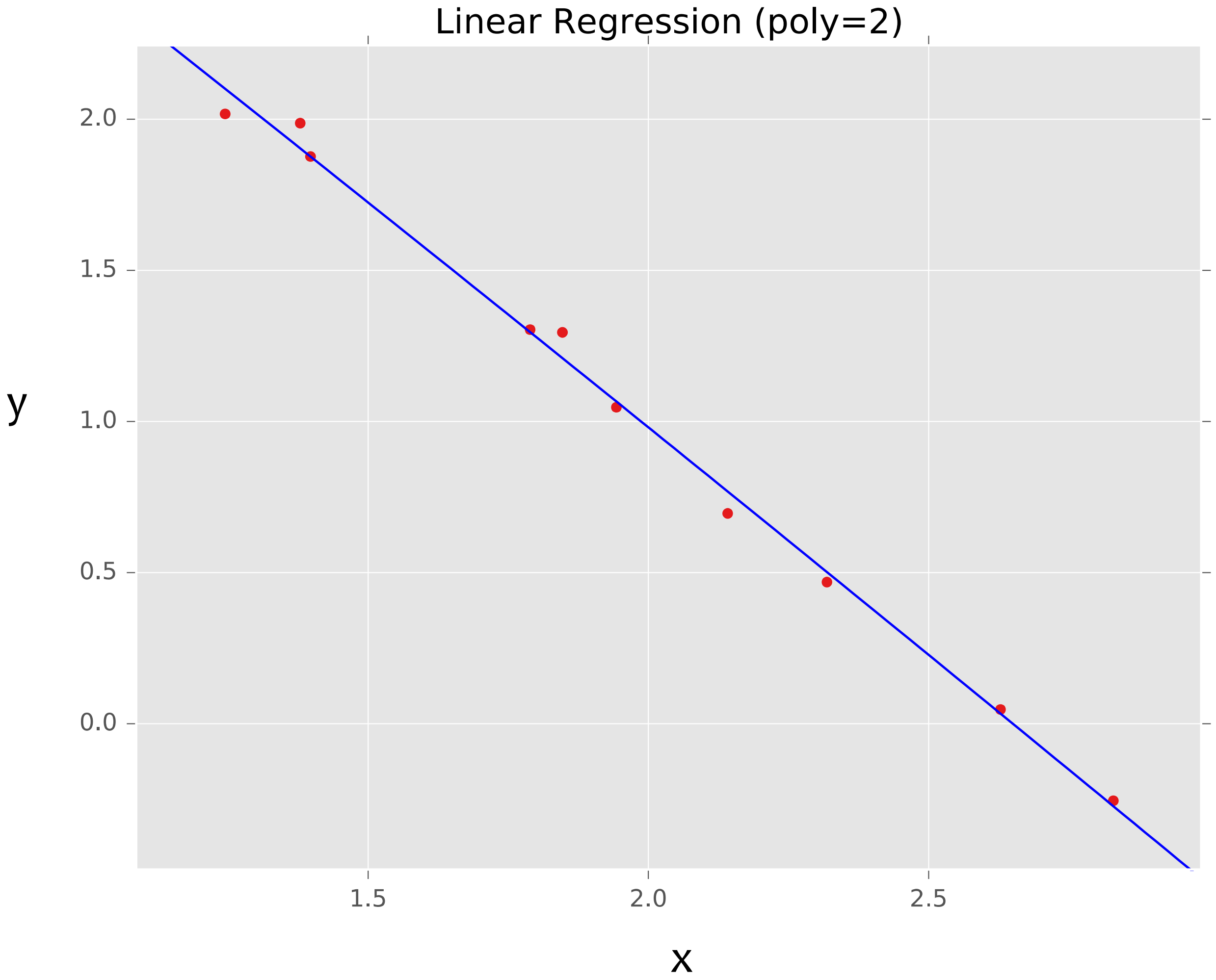

Slide credit: CMU MLD Matt Gormley

Example: Linear Regression

Goal: Learn y = $\mathbf{w}^T f(\mathbf{x}) + b$
where f(.) is a polynomial
basis function

y	x	x^2	x^3
2.0	1.2	$(1.2)^2$	$(1.2)^3$
1.3	1.7	$(1.7)^2$	$(1.7)^3$
0.1	2.7	$(2.7)^2$	$(2.7)^3$
1.1	1.9	$(1.9)^2$	$(1.9)^3$

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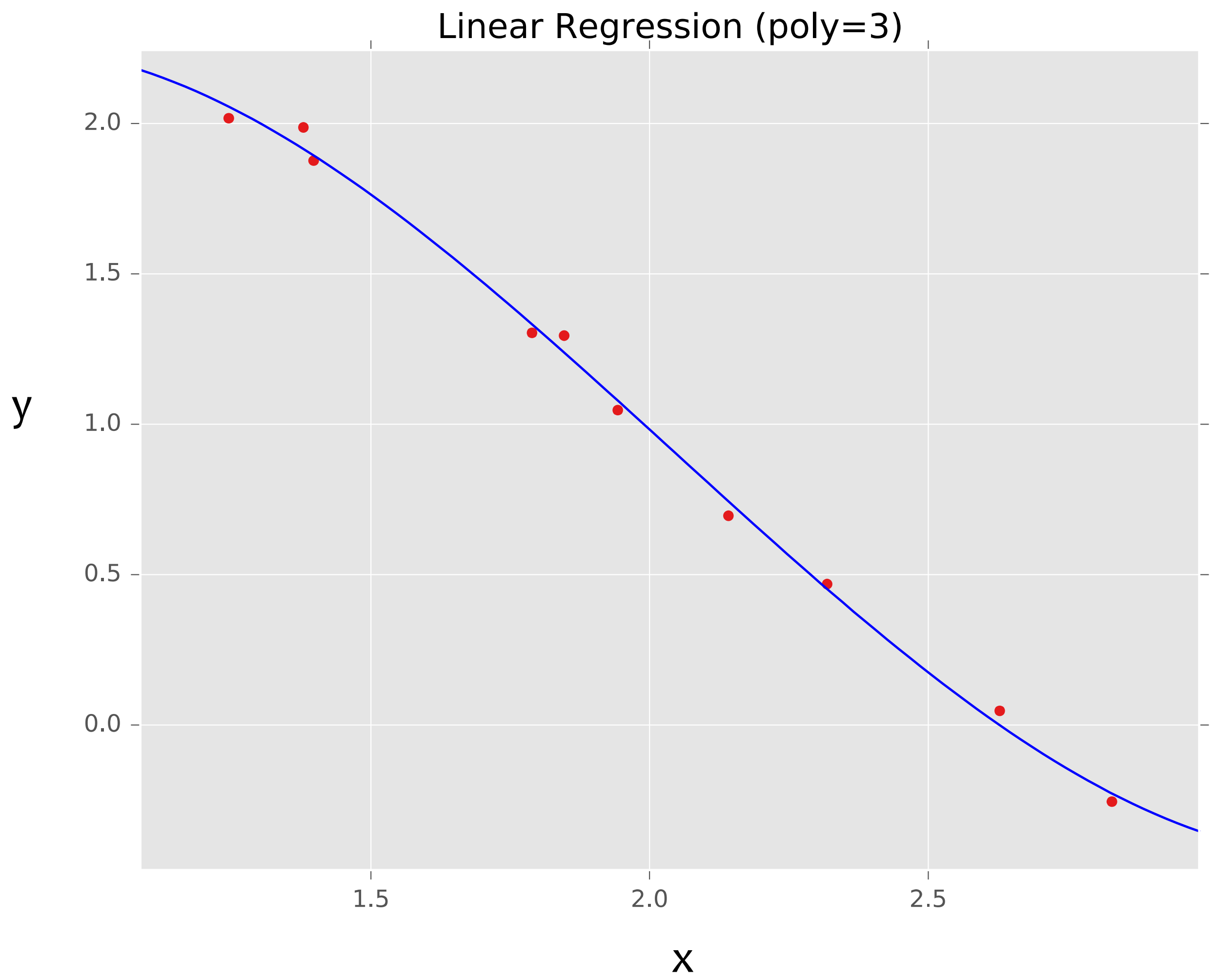


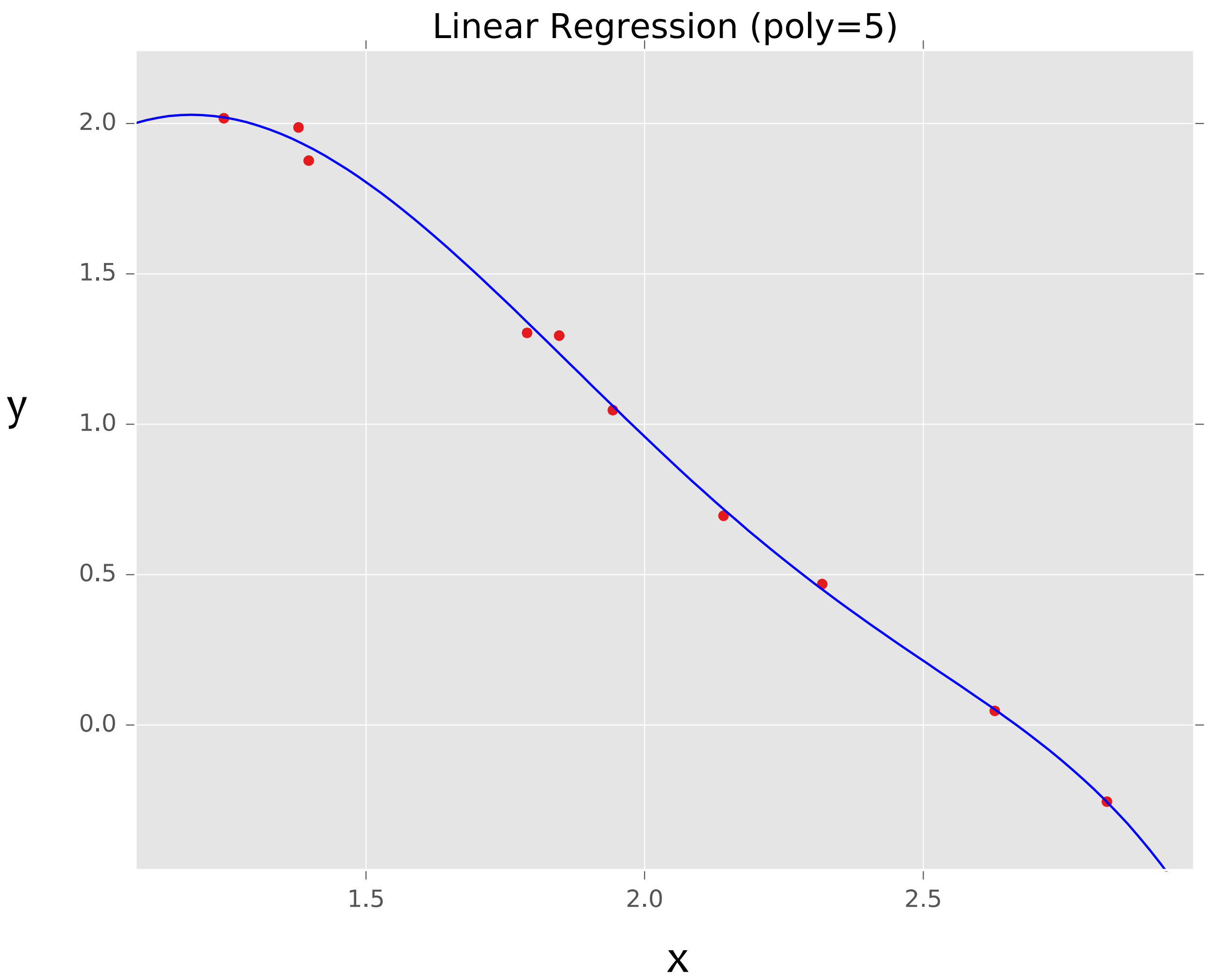
Slide credit: CMU MLD Matt Gormley

Example: Linear Regression

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$
where f(.) is a polynomial
basis function

y	x	x^2	…	x^5
2.0	1.2	$(1.2)^2$	…	$(1.2)^5$
1.3	1.7	$(1.7)^2$	…	$(1.7)^5$
0.1	2.7	$(2.7)^2$	…	$(2.7)^5$
1.1	1.9	$(1.9)^2$	…	$(1.9)^5$

true "unknown" target function is linear with negative slope and gaussian noise



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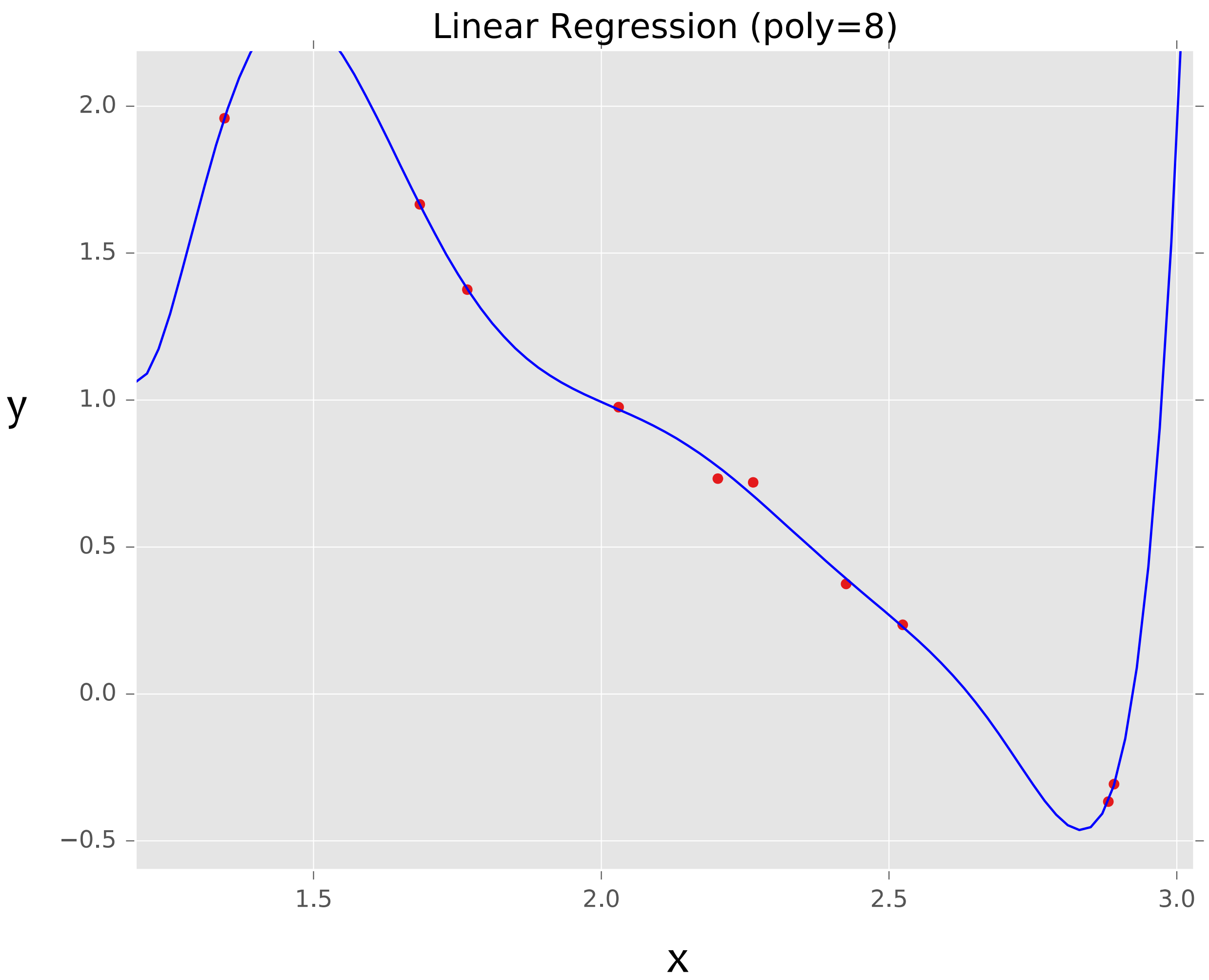
Example: Linear Regression

Goal: Learn y = $\mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

y	x	x^2	…	x^8
2.0	1.2	$(1.2)^2$	…	$(1.2)^8$
1.3	1.7	$(1.7)^2$	…	$(1.7)^8$
0.1	2.7	$(2.7)^2$	…	$(2.7)^8$
1.1	1.9	$(1.9)^2$	…	$(1.9)^8$

true "unknown" target function is linear with negative slope and gaussian noise



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Example: Linear Regression

Goal: Learn y = $\mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

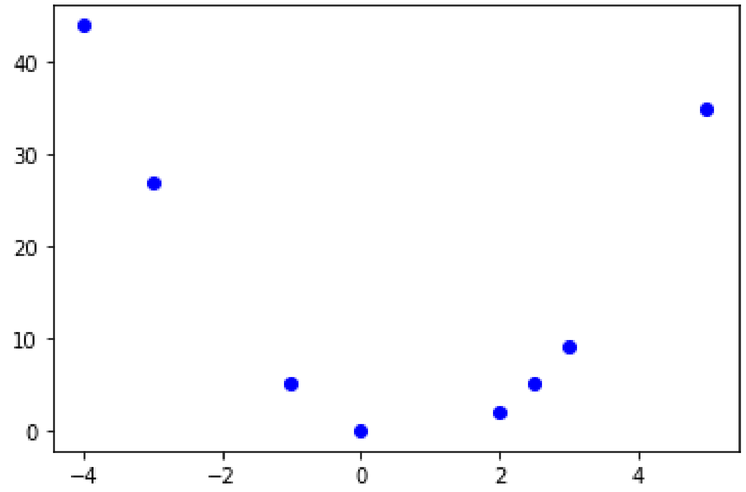
y	x	x^2	…	x^9
2.0	1.2	$(1.2)^2$	…	$(1.2)^9$
1.3	1.7	$(1.7)^2$	…	$(1.7)^9$
0.1	2.7	$(2.7)^2$	…	$(2.7)^9$
1.1	1.9	$(1.9)^2$	…	$(1.9)^9$

true "unknown" target function is linear with negative slope and gaussian noise

Slide credit: CMU MLD Matt Gormley

Polynomial Features

Design Matrix



y	x_1
44	-4
27	-3
5	-1
0	0
2	2
5	2.5
9	3
35	5

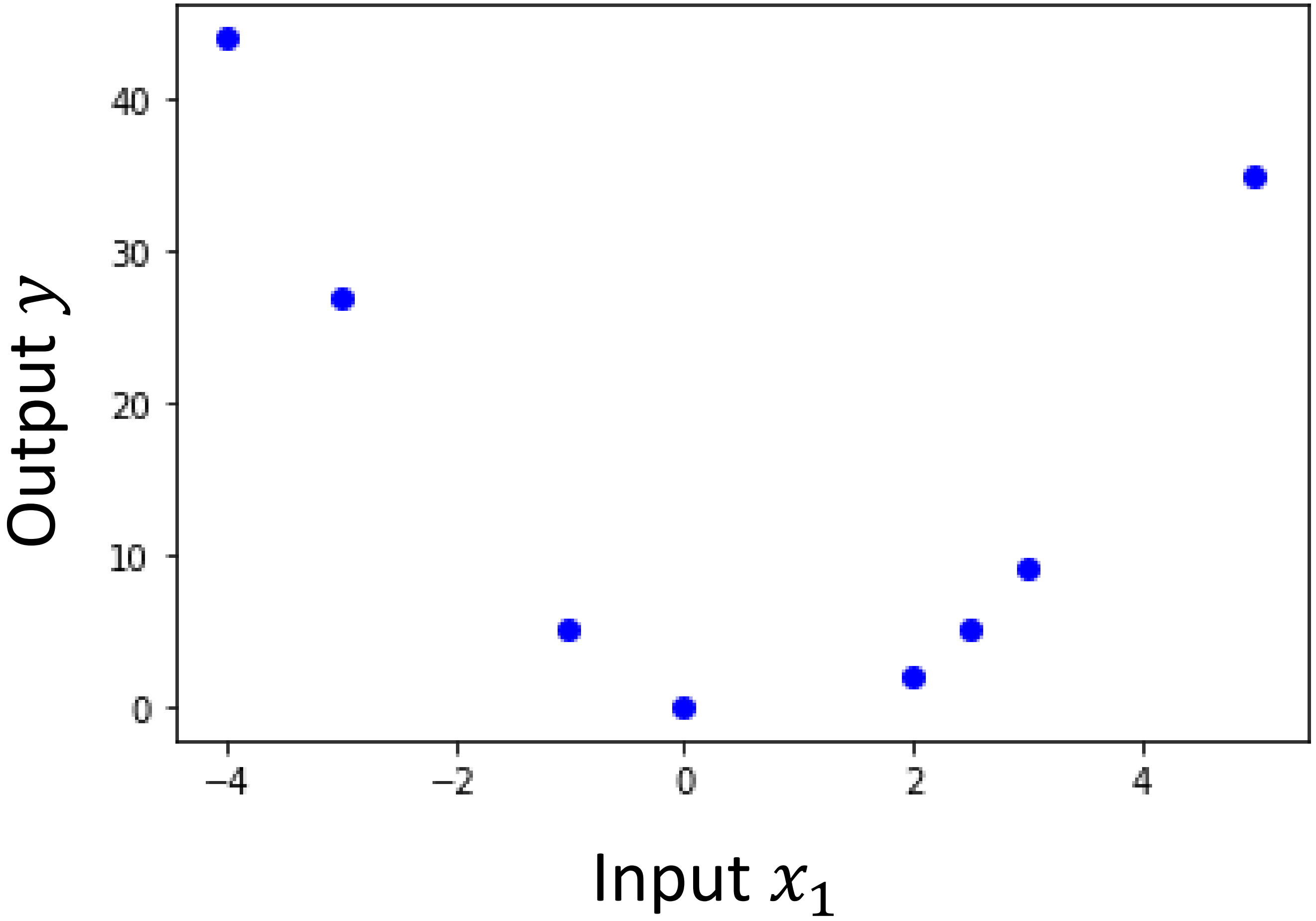
$$X = \begin{bmatrix} — & \mathbf{x}^{(1)\top} & — \\ — & \mathbf{x}^{(2)\top} & — \\ — & \mathbf{x}^{(3)\top} & — \\ & \cdots & \\ — & \mathbf{x}^{(N)\top} & — \end{bmatrix}$$

$$\phi(X) = \begin{bmatrix} — & \phi\left(\mathbf{x}^{(1)}\right)^\top & — \\ — & \phi\left(\mathbf{x}^{(2)}\right)^\top & — \\ — & \phi\left(\mathbf{x}^{(3)}\right)^\top & — \\ & \cdots & \\ — & \phi\left(\mathbf{x}^{(N)}\right)^\top & — \end{bmatrix}$$

Polynomial Features

Non-linear Data

Linear regression on polynomial data?



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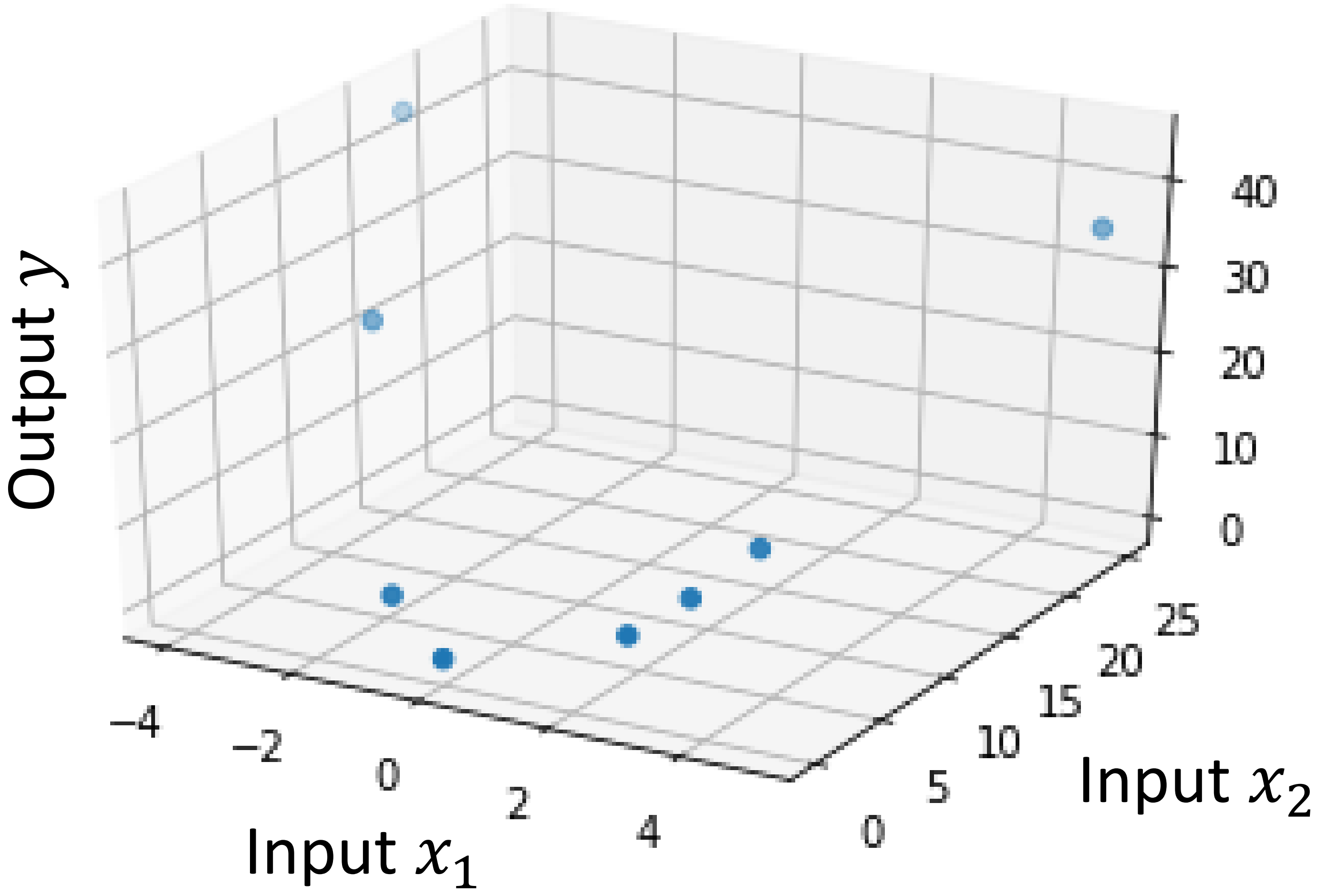
Non-linear Data

Regression on simple linear dataset



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0	0	0
2	2	4
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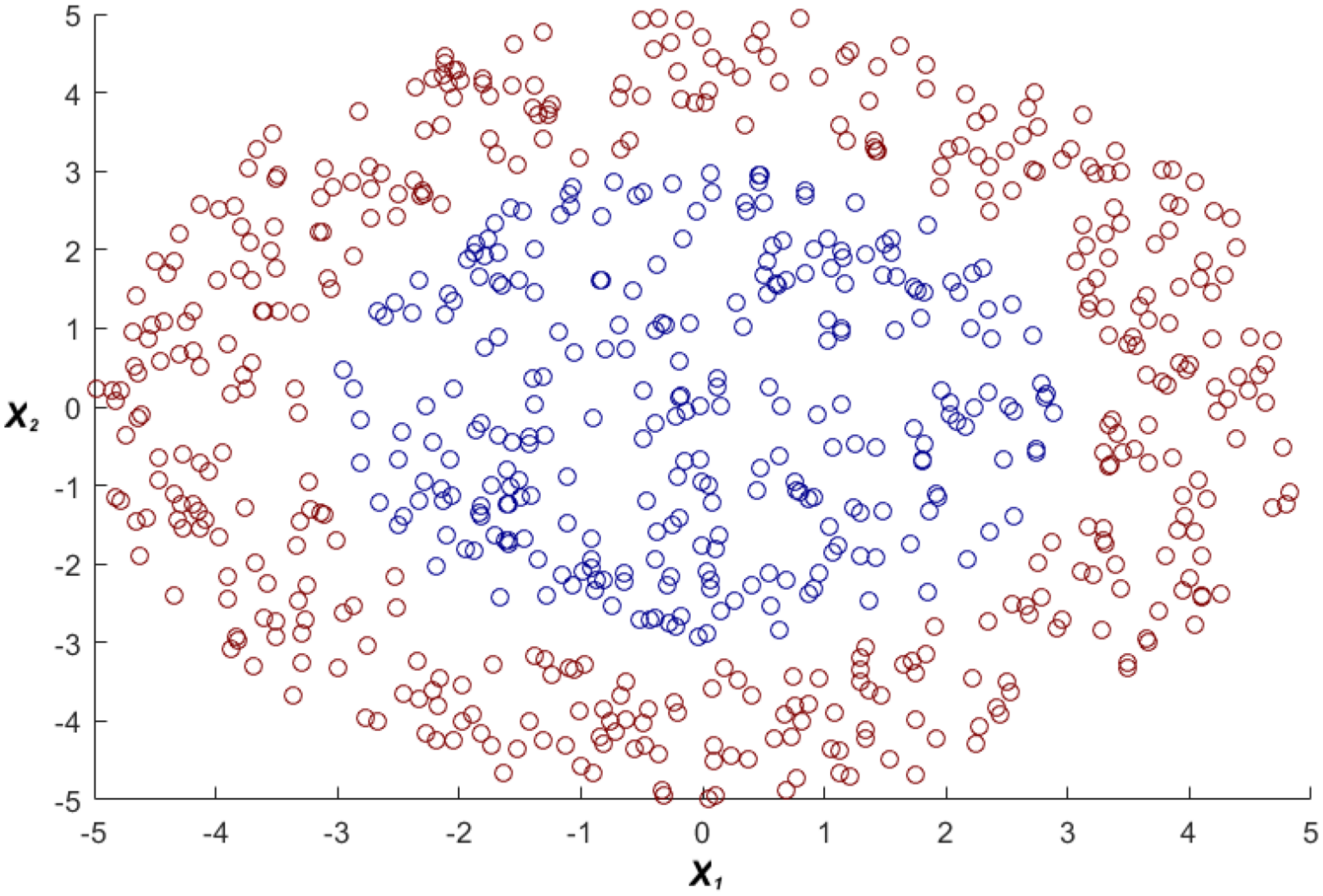
Non-linear Data

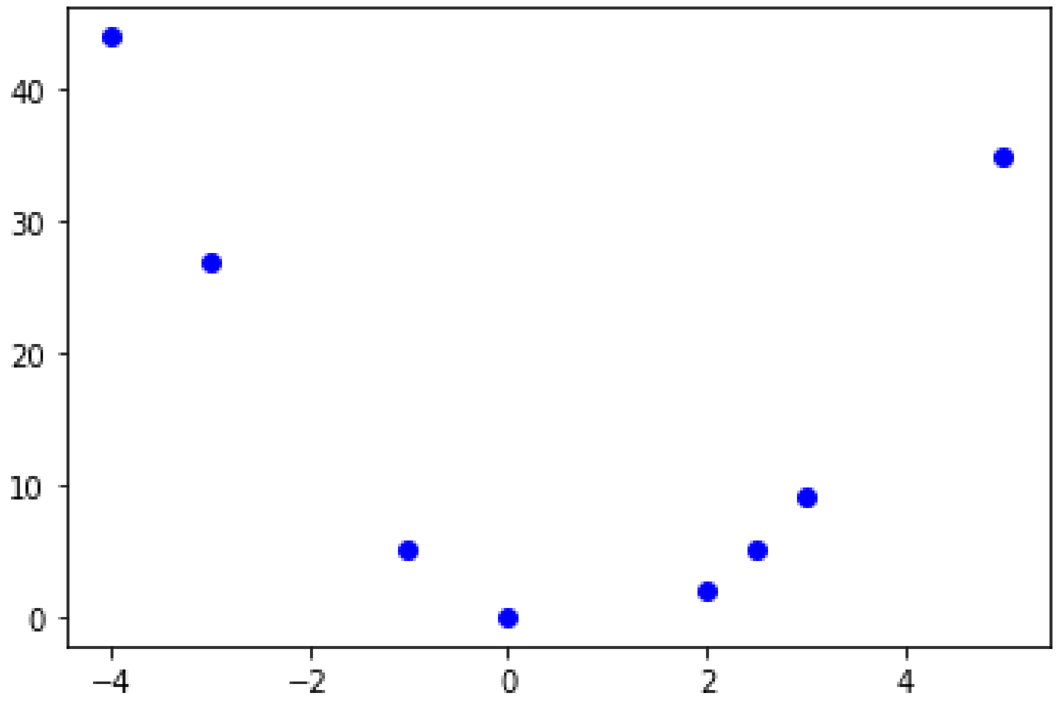
Polynomial feature map for linear classification

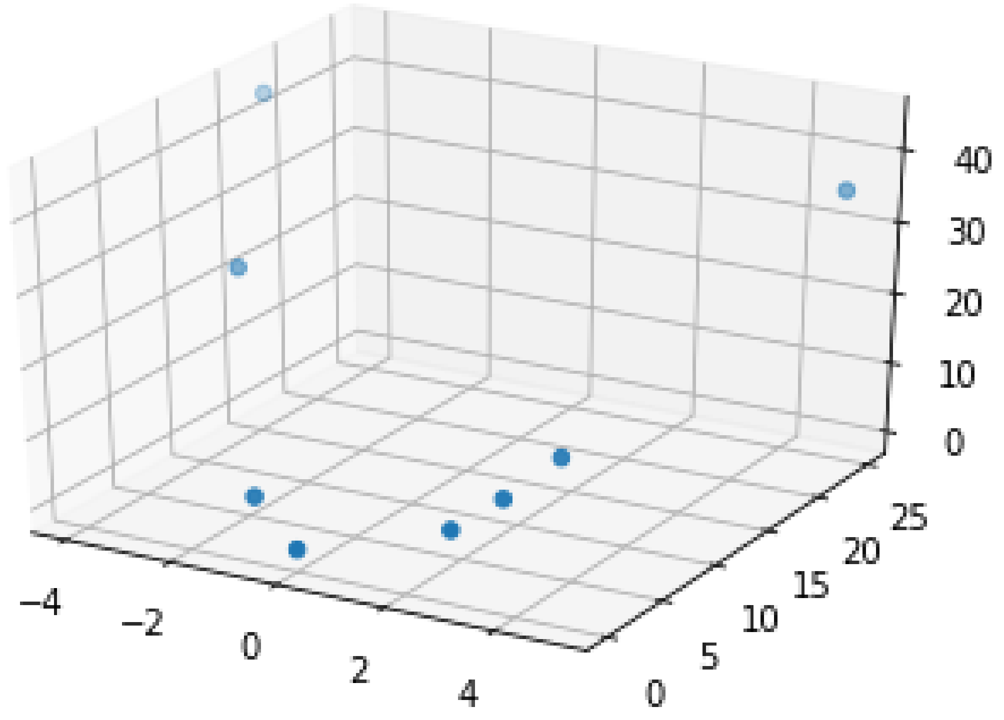


https://www.youtube.com/watch?v=3liCbRZPrZA

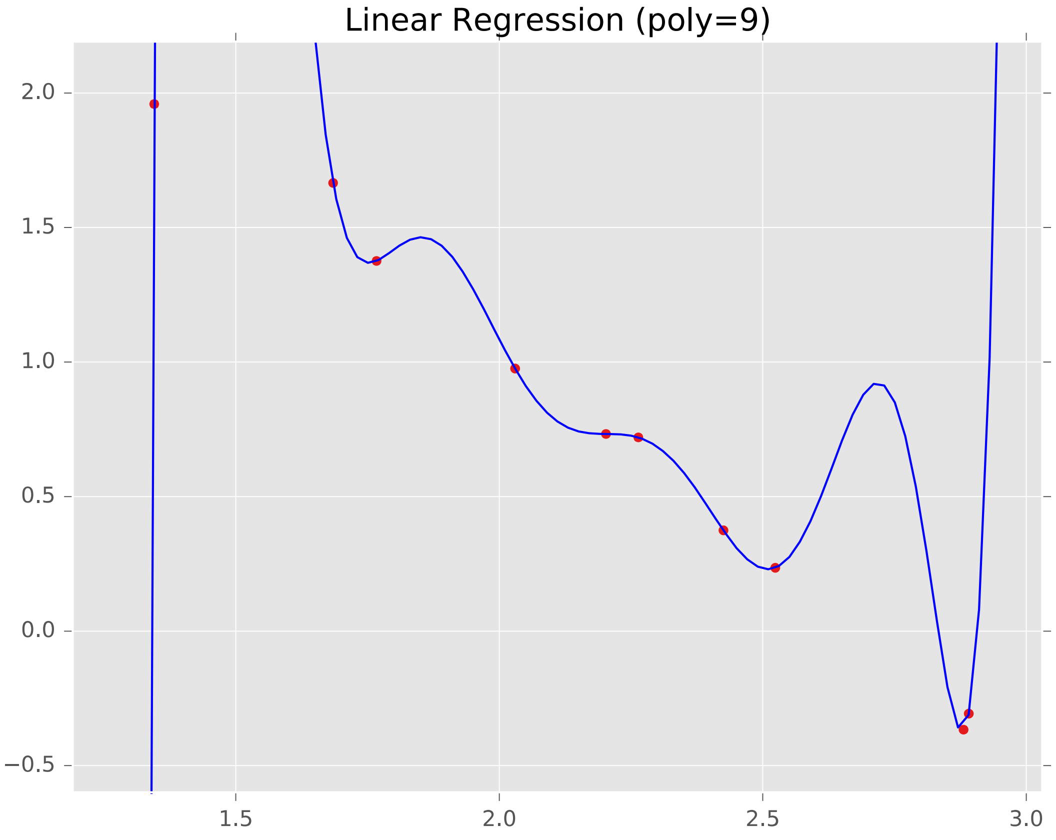
Feature Maps

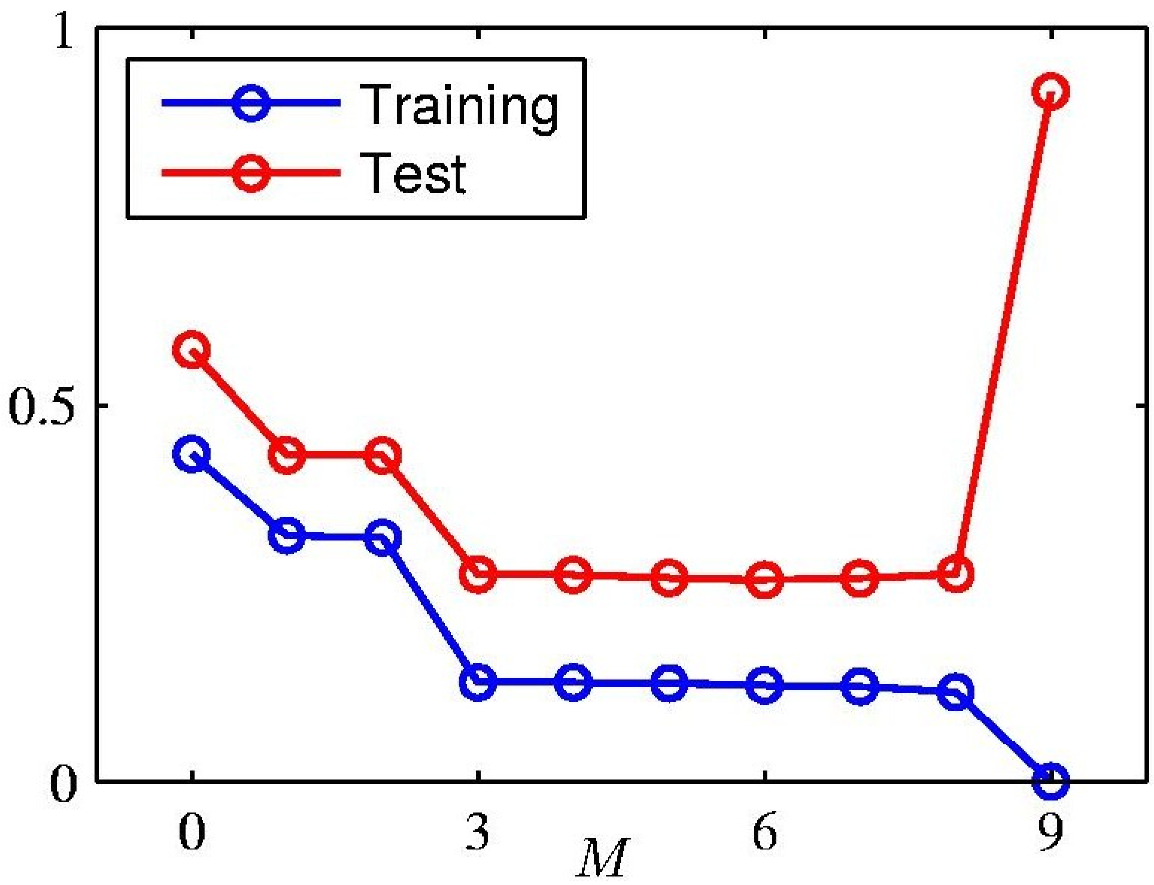
Two ways to think about it





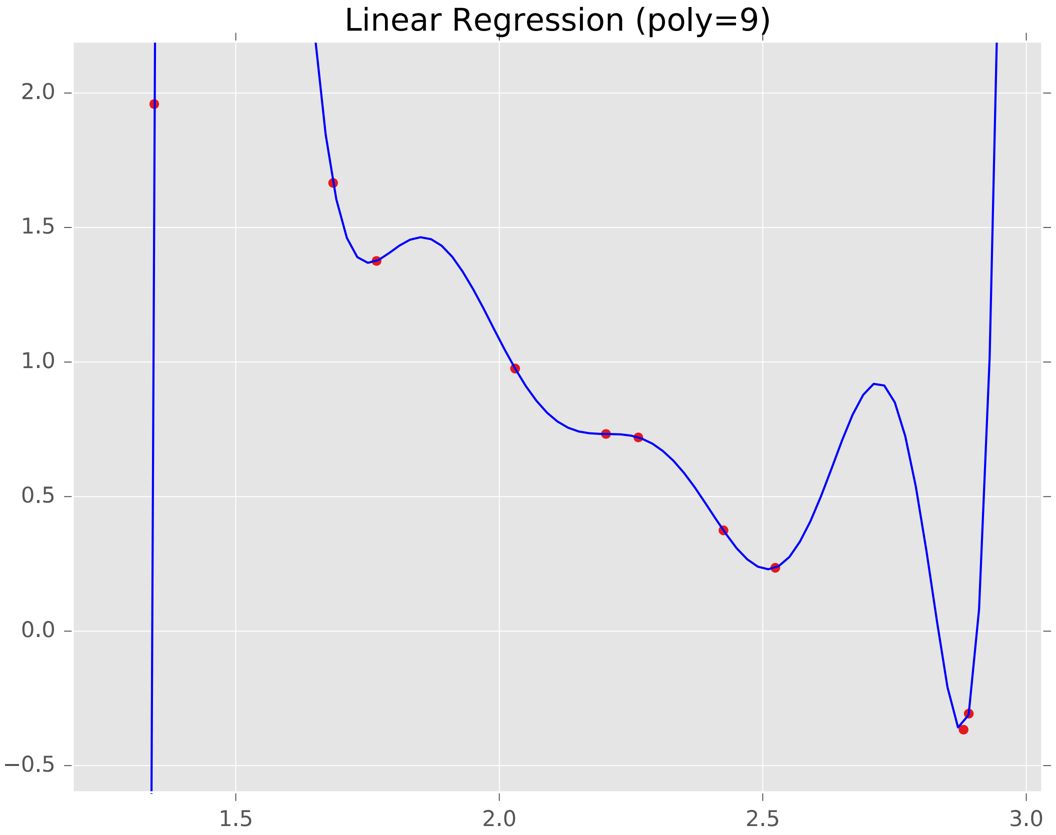
Overfitting: Symptoms





Slide credit: CMU MLD William Cohen

Overfitting: Symptoms



	$M=0$	$M=1$	$M=3$	$M=9$
θ_0	0.19	0.82	0.31	0.35
θ_1		-1.27	7.99	232.37
θ_2			-25.43	-5321.83
θ_3			17.37	48568.31
θ_4				-231639.30
θ_5				640042.26
θ_6				-1061800.52
θ_7				1042400.18
θ_8				-557682.99
θ_9				125201.43

Slide credit: CMU MLD William Cohen

Overfitting: Fix?

N = 10 points we overfit!

Goal: Learn y = $\mathbf{w}^T f(\mathbf{x}) + b$
where f(.) is a polynomial basis function

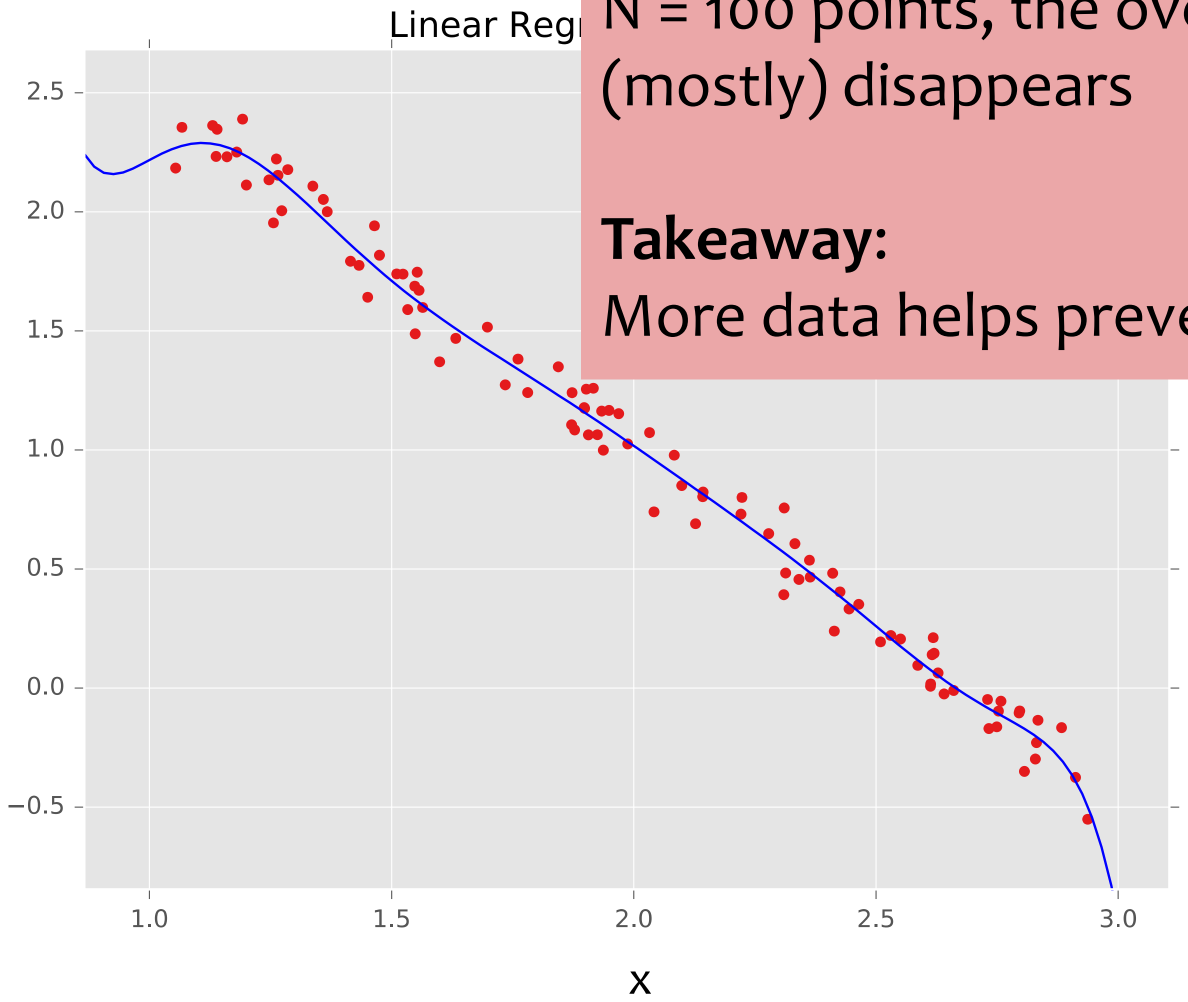
i	y	x	…	x^9
1	2.0	1.2	…	$(1.2)^9$
2	1.3	1.7	…	$(1.7)^9$
…	…	…	…	…
10	1.1	1.9	…	$(1.9)^9$

Slide credit: CMU MLD Matt Gormley

Overfitting: More training data!

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$
where f(.) is a polynomial
basis function

i	y	x	…	x^9
1	2.0	1.2	…	$(1.2)^9$
2	1.3	1.7	…	$(1.7)^9$
3	0.1	2.7	…	$(2.7)^9$
4	1.1	1.9	…	$(1.9)^9$
…	…	…	…	…
…	…	…	…	…
…	…	…	…	…
98	…	…	…	…
99	…	…	…	…
100	0.9	1.5	…	$(1.5)^9$



N = 10 points we overfit!

N = 100 points, the overfitting (mostly) disappears

Takeaway:
More data helps prevent overfitting

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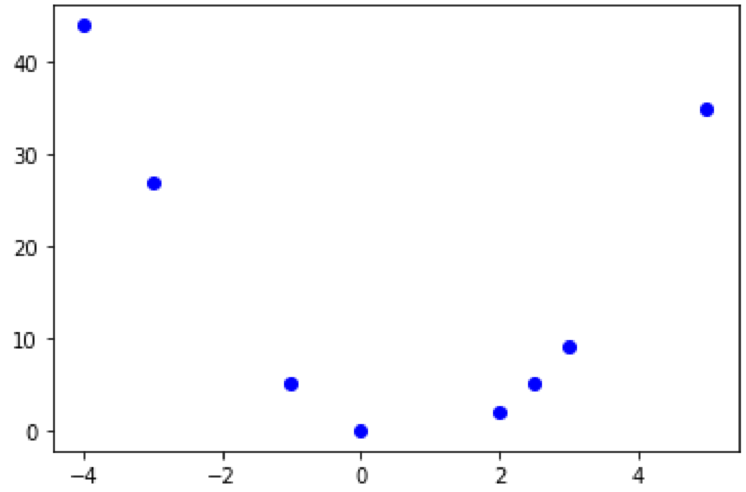
Polynomial Features

Design Matrix



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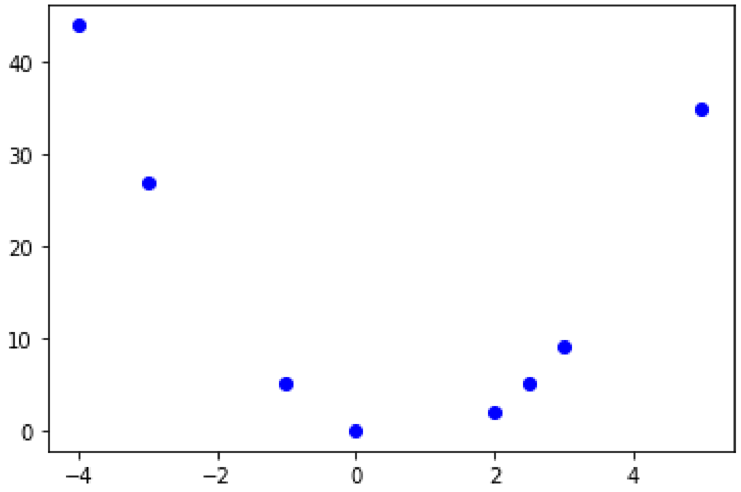
$$X = \begin{bmatrix} — \mathbf{x}^{(1)\top} — \\ — \mathbf{x}^{(2)\top} — \\ — \mathbf{x}^{(3)\top} — \\ \cdots \\ — \mathbf{x}^{(N)\top} — \end{bmatrix}$$

$$\phi(X) = \begin{bmatrix} — \phi\left(\mathbf{x}^{(1)}\right)^\top — \\ — \phi\left(\mathbf{x}^{(2)}\right)^\top — \\ — \phi\left(\mathbf{x}^{(3)}\right)^\top — \\ \cdots \\ — \phi\left(\mathbf{x}^{(N)}\right)^\top — \end{bmatrix}$$

Polynomial Features

Solving linear regression



y	x_1
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Feature Engineering vs Feature Learning

Feature engineering

- Humans decide what the features may be useful
- Humans implement feature extraction algorithms

Feature learning

- Humans choose data and performance measure
- Humans decide on structure/algorithm to learn features
  - Features are just intermediate values between input and output
  - Structure defines number of feature values
- Allow machine to map data to feature values as needed

One-hot encoding

Need to convert text for categories into numerical values for input and output

Categorical

color	color	r	g	b
red	1	1	0	0
green	2	0	1	0
blue	3	0	0	1
red	1	1	0	0
red	1	1	0	0

Categorical variables

- Enumeration
- One-hot vector (or one-hot columns)

One-hot vector

- Vector of length M with exactly one 1 and M-1 zeros
- Series of M binary values, exactly one of which is 1

More to come later in the semester

✅ Polynomial feature engineering

✅ Categorical features and one-hot encoding

Images

- Raw data, feature engineering
- Feature learning

Audio

- Raw data, feature engineering
- Feature learning

Text

- Raw data, feature engineering
- Feature learning