

10-315

Introduction to ML

Decision Trees

Instructor: Pat Virtue

Today

Autoencoders

More course info

Announcements

ML data and notation

Using data to learn

Decision trees



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Machine Learning Systems

Three components $\langle T, P, E \rangle$:

1. Task, T
2. Performance measure, P
3. Experience, E

Definition of learning:

A computer program **learns** if its performance at tasks in T , as measured by P , improves with experience E .

Input/Output Tasks *Task*

numerical data → Iris classification → category

hour/time → Traffic prediction → num

img → Image classification → category

img → Image denoising → img

img → Medical image recon → img

text → Text to image generation → img

rand num → Face generation → img

Machine Learning

Using (training) data to learn a model that we'll later use for prediction

Training Data

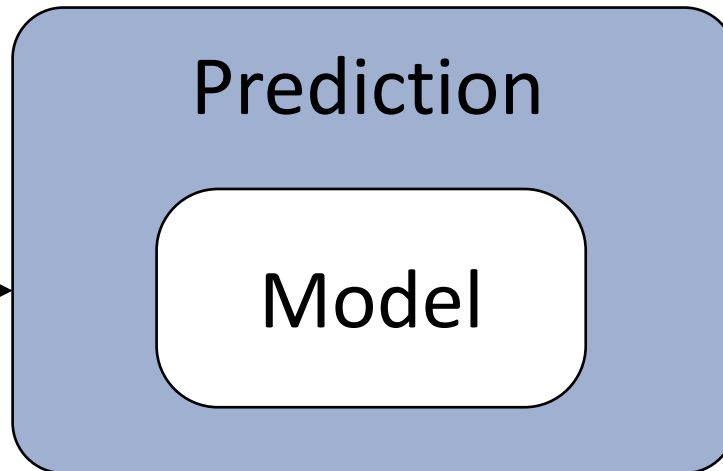
Input and
Measured Output



Model

Structure and
Parameters

Input



Predicted
Output

Example Dataset: Fisher Iris Dataset

Fisher (1936) used 150 measurements of flowers from 3 different species: *Iris setosa* (0), *Iris virginica* (1), *Iris versicolor* (2) collected by Anderson (1936)

Species	Sepal Length	Sepal Width	Petal Length	Petal Width
0	4.3	3.0	1.1	0.1
0	4.9	3.6	1.4	0.1
0	5.3	3.7	1.5	0.2
1	4.9	2.4	3.3	1.0
1	5.7	2.8	4.1	1.3
1	6.3	3.3	4.7	1.6
2	5.9	3.0	5.1	1.8



Images and full dataset: https://en.wikipedia.org/wiki/Iris_flower_data_set

Example Dataset: Fisher Iris Dataset

Assume samples in data are i.i.d.

```
from sklearn import datasets  
  
iris = datasets.load_iris()  
X = iris.data  
y = iris.target
```

Dataset notation

$$\mathcal{D} = \left\{ \left(y^{(i)}, \vec{x}^{(i)} \right) \right\}_{i=1}^{N=7}$$
$$= \left\{ \left(y^{(i)}, x_1^{(i)}, x_2^{(i)}, x_3^{(i)}, x_4^{(i)} \right) \right\}_{i=1}^N$$

Linear algebra can represent all data

$$\mathbf{y} \in \{0, 1, 2\}^N$$

$$\mathbf{X} \in \mathbb{R}^{N \times 4} \quad (\text{design matrix})$$

$$\vec{x} \in \mathbb{R}^4$$

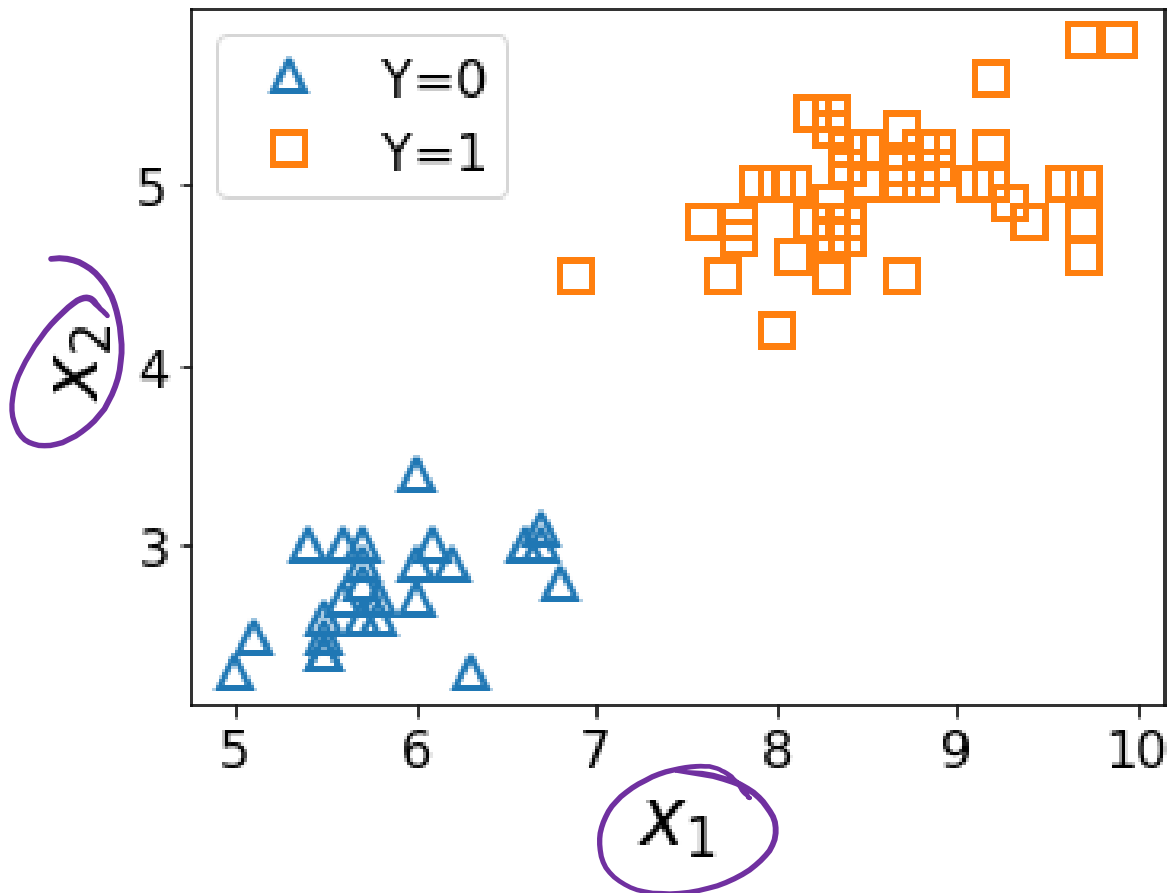
$$\vec{x}^{(i)}$$

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ML Task: Classification

Predict species label from first two input measurements

$$h(\mathbf{x}) \rightarrow \hat{y}$$



$(y^{(i)}, \vec{x}^{(i)})$

Species	x_1 Sepal Length	x_2 Sepal Width
0	4.3	3.0
0	4.9	3.6
0	5.3	3.7
1	4.9	2.4
1	5.7	2.8
1	6.3	3.3
2	5.9	3.0

ML Tasks

Supervised learning

$$\mathcal{D} = \{(y^{(i)}, \mathbf{x}^{(i)})\}_{i=1}^N \quad h(\mathbf{x}) \rightarrow \hat{y}$$

Classification

- Output labels
- $y \in \mathcal{Y}$, where \mathcal{Y} is discrete and order of values has no meaning

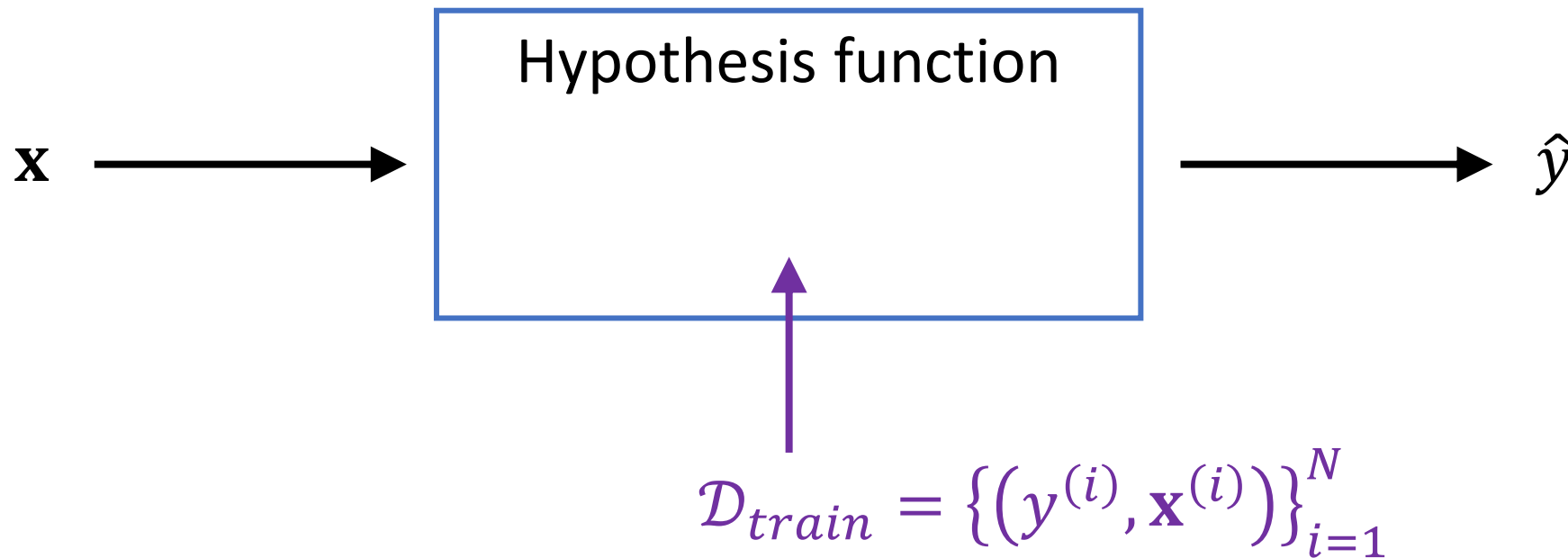
Regression

- Output values
- $y \in \mathcal{Y}$, where \mathcal{Y} is usually continuous, order of values has meaning

Supervised Learning

Use training dataset to learn “best” hypothesis function

Predict output from input features

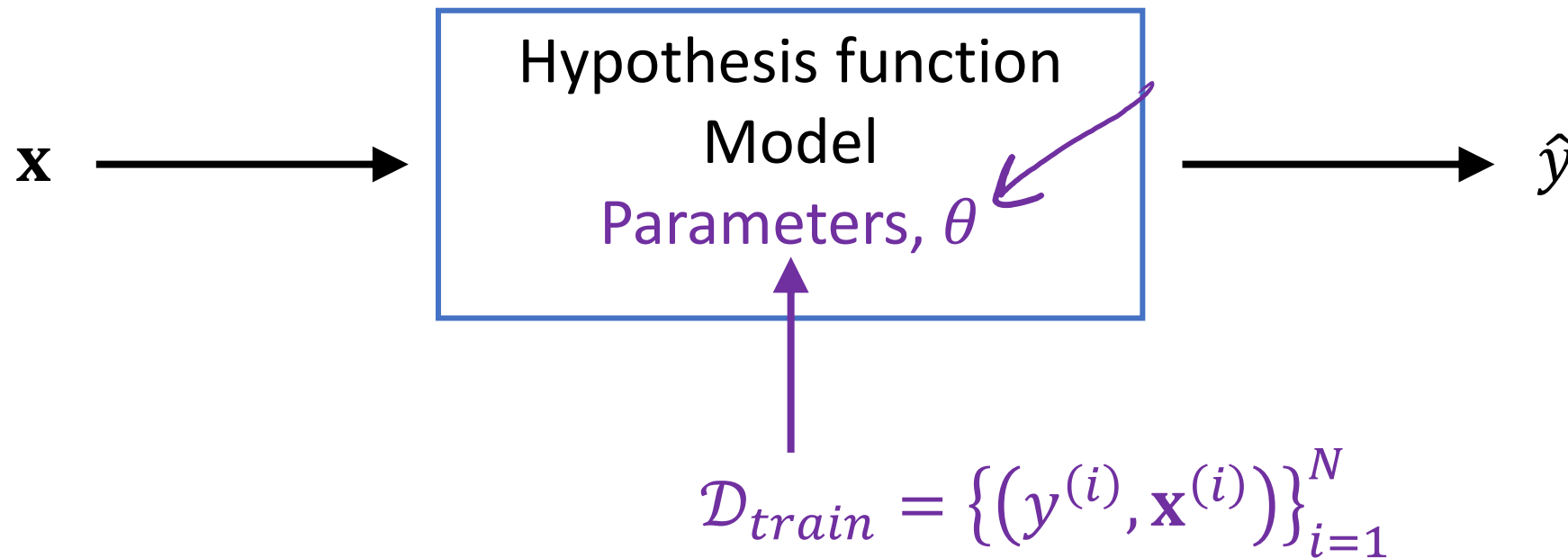


Need **performance measure** to determine meaning of “best” function

Supervised Learning

Use training dataset to learn “best” hypothesis function

Predict output from input features



Need **performance measure** to determine meaning of “best” function

Classification

Iris data example

$$\mathcal{D} = \{(y^{(i)}, \mathbf{x}^{(i)})\}_{i=1}^N, \text{ where } \mathbf{x}^{(i)} \in \mathbb{R}^4, y^{(i)} \in \{0, 1, 2\}$$

Predict species label from input measurements

$$h(\mathbf{x}) \rightarrow \hat{y}$$

Performance measure?

Classification error rate

- Fraction of times $y \neq \hat{y}$ in a given dataset

Species	Sepal Length	Sepal Width	Petal Length	Petal Width
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Classification

Iris data example

$$\mathcal{D} = \{(y^{(i)}, \mathbf{x}^{(i)})\}_{i=1}^N, \text{ where } \mathbf{x}^{(i)} \in \mathbb{R}^4, y^{(i)} \in \{0, 1, 2\}$$

Notation alert: Indicator function

$$\mathbb{1}(z) = \mathbf{1}(z) = \begin{cases} 1 & \text{if } z \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

Predict species label from input measurements

$$h(\mathbf{x}) \rightarrow \hat{y}$$

Performance measure?

Classification error rate

- Fraction of times $y \neq \hat{y}$ in a given dataset

- $\frac{1}{N} \sum_{i=1}^N \mathbb{1}(y^{(i)} \neq \hat{y}^{(i)})$

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Problem Formulation

Medical Prediction

z Outcome	x_1 Fetal Position	x_2 Fetal Distress	x_3 Previous C-sec
Natural	Vertex	N	N
C-section	Breech	N	N
Natural	Vertex	Y	Y
C-section	Vertex	N	Y
Natural	Abnormal	N	N

Problem Formulation

Medical Prediction

Y	X_1	X_2	X_3
Outcome	Fetal Position	Fetal Distress	Previous C-sec
Natural	Vertex	N	N
C-section	Breech	N	N
Natural	Vertex	Y	Y
C-section	Vertex	N	Y
Natural	Abnormal	N	N

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [x_1, x_2, x_3]^T$$

$$x_1 \in \{Vertex, Breech, Abn\}$$

$$x_2 \in \{Y, N\}$$

$$x_3 \in \{Y, N\}$$

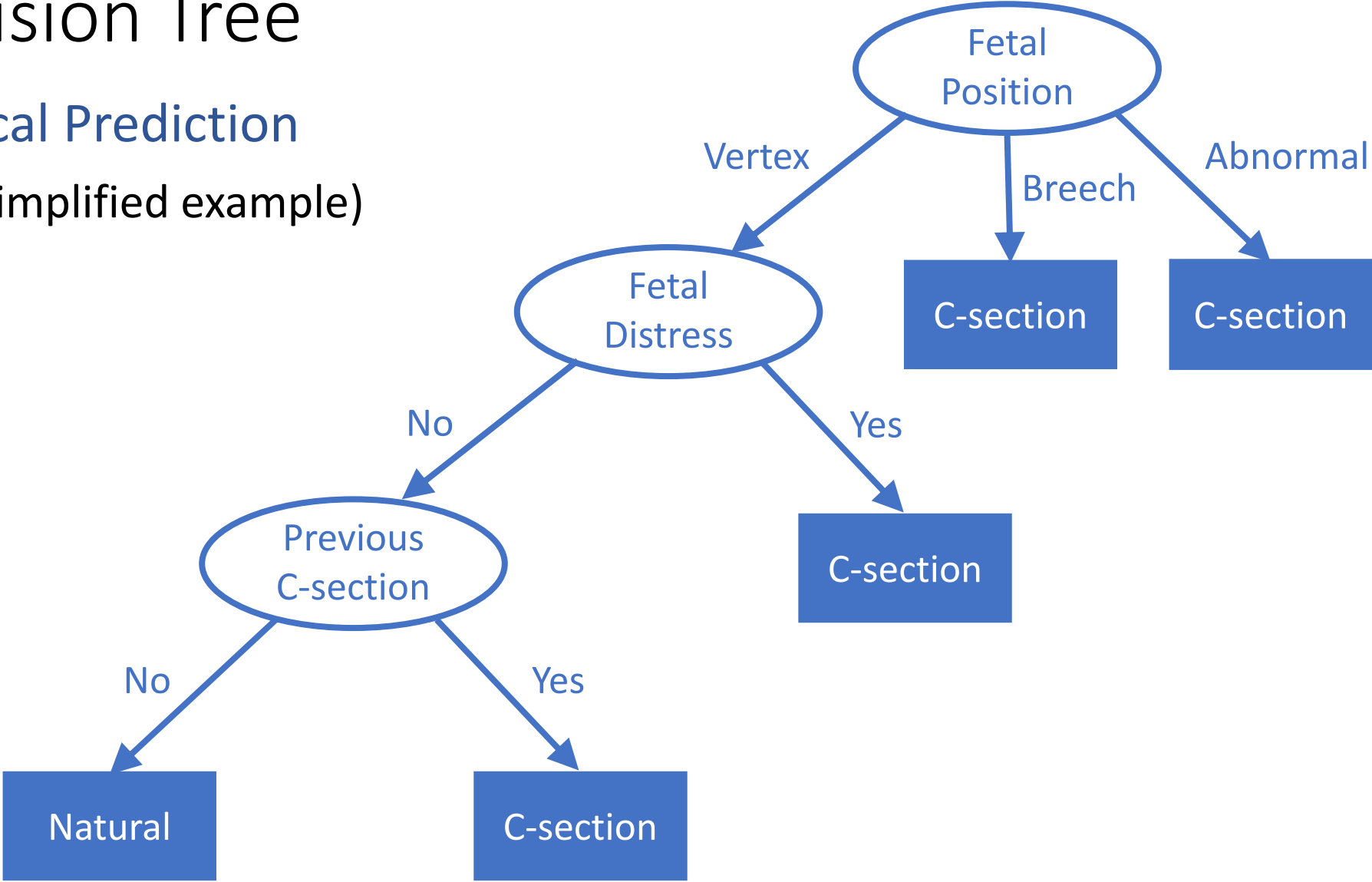
$$y \in \{Csection, Natural\}$$

$$\hat{y} = h(\mathbf{x})$$

Decision Tree

Medical Prediction

(Oversimplified example)



How could we implement training and prediction?

Algorithm 0: Memorization algorithm

def train (\mathcal{D}) \rightarrow "model"
store ρ

def predict (\vec{x})
for i in $1..N$:
if $\vec{x}^{(i)} = \vec{x}$
return $y^{(i)}$

\mathcal{D}

Piazza Poll 1

Does the memorization algorithm learn?

A. Yes

B. No

C. I have no clue

Piazza Poll 1

Does the memorization algorithm learn?

A. Yes *But not terribly efficient/effective*

B. No

C. I have no clue