

Course Update *Out*

Current Plan (updated) *Due*

- HW 8 (online)
- Mini-project proposal
- HW 9 (online)
- HW 10 (written/prog)
- Midterm 2
- Mini-project



Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	3	4	5 HW8 Proj	6	7	8
9 HW8	10 HW9 HW10	11	12	13	14	15
16	17 HW9	18	19 Prop	20	21	22 HW10
23	24	25	26 MT2	27	28	29
30	1	2	3	4	5 Proj	6

# Plan

## Last time

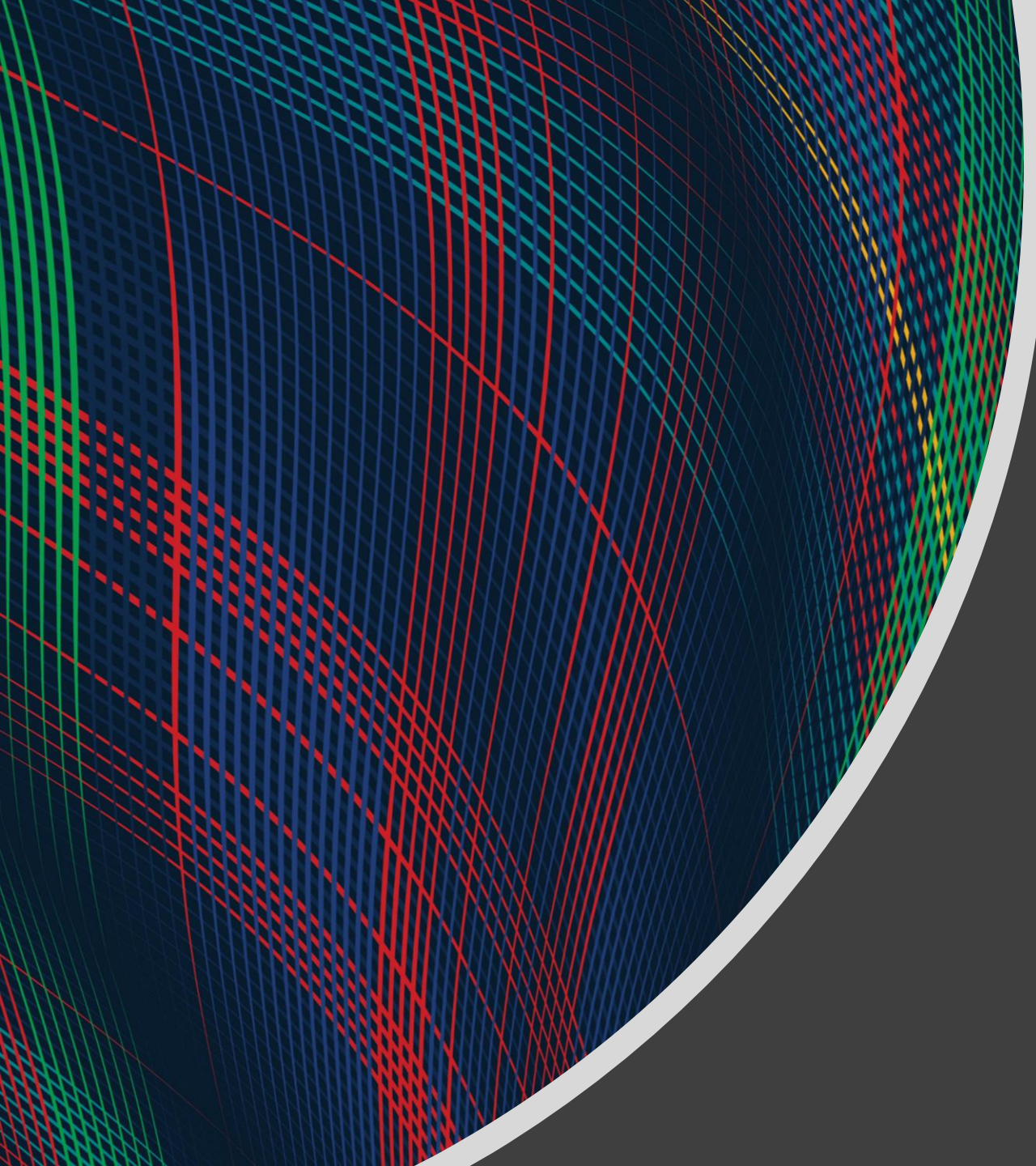
- Unsupervised Learning: Dimensionality Reduction

## Today

- Recommender Systems 
- Unsupervised Learning: Clustering
-  ■ K-means

## Next time

- Unsupervised Learning: Clustering
  - Gaussian mixture models and expectation maximization 

An abstract graphic on the left side of the slide, featuring a sphere-like shape composed of a dense grid of intersecting red, green, and blue lines. The lines are curved and follow the contours of the sphere, creating a complex, woven pattern. The sphere is set against a dark gray background.

10-315

Introduction to ML

Recommender Systems

Instructor: Pat Virtue

# Recommender Systems

## A Common Challenge:

- Assume you're a company selling items of some sort: movies, songs, products, etc.
- Company collects millions of ratings from users of their items
- To maximize profit / user happiness, you want to recommend items that users are likely to want

# Recommender Systems

The screenshot shows the Amazon homepage with a personalized recommendation section for a user named Matt. The header includes the Amazon Prime logo, a search bar, and a 'CYBER MONDAY DEALS WEEK' banner. Below the header, there's a navigation bar with links like 'Departments', 'Browsing History', 'Matt's Amazon.com', 'Cyber Monday', 'Gift Cards & Registry', 'Sell', and 'Help'. A 'Hello, Matt' greeting is followed by links to 'Your Account', 'Prime', 'Lists', and a shopping cart icon.

The main recommendation section is titled 'Recommended for you, Matt'. It features four categories of products:

- Buy It Again in Grocery**: 14 ITEMS. Products include Jif peanut butter, maple syrup, and various snack boxes.
- Buy It Again in Pets**: 6 ITEMS. Products include petsh-p, advantage II cat flea treatment, and World's Best cat food.
- Buy It Again in Baby Products**: 5 ITEMS. Products include Crayola crayons, baby wipes, and baby bibs.
- Engineering Books**: 6 ITEMS. The book shown is 'PROBABILISTIC GRAPHICAL MODELS: PRINCIPLES AND TECHNIQUES' by DAPHNE KOLLER AND NIR FRIEDMAN.

Each category has a green underline under the title and a green checkmark next to the item count.



# Recommender Systems

The image is a screenshot of the Netflix website during the Netflix Prize competition. A large yellow banner at the top reads "Netflix Prize" and "COMPLETED". Below the banner is a navigation bar with links: Home, Rules, Leaderboard, and Update. The main content area shows a "Movies For You" section with a list of recommended movies. A large white box on the right side of the page contains the "Congratulations!" message. The background of the page features a silhouette of two people looking at a screen, with a faint image of a movie poster visible behind them.

**NETFLIX**

**Netflix Prize**

**COMPLETED**

Home Rules Leaderboard Update

NETFLIX

Browse Recommendations Friends Queue Buy DVDs

Home Games New Releases Previews Netflix Top 100 Crit

**Movies For You**

Ready, the following movies were chosen based on your interest in: [The Godfather Part 2](#) [The Godfather Part 3](#) [The Godfather](#)

**The Big One**

★★★★★

For subversive

by from

OTI

**You really liked it...**

Now only for just \$5.99

**Shop as low as**

Original an

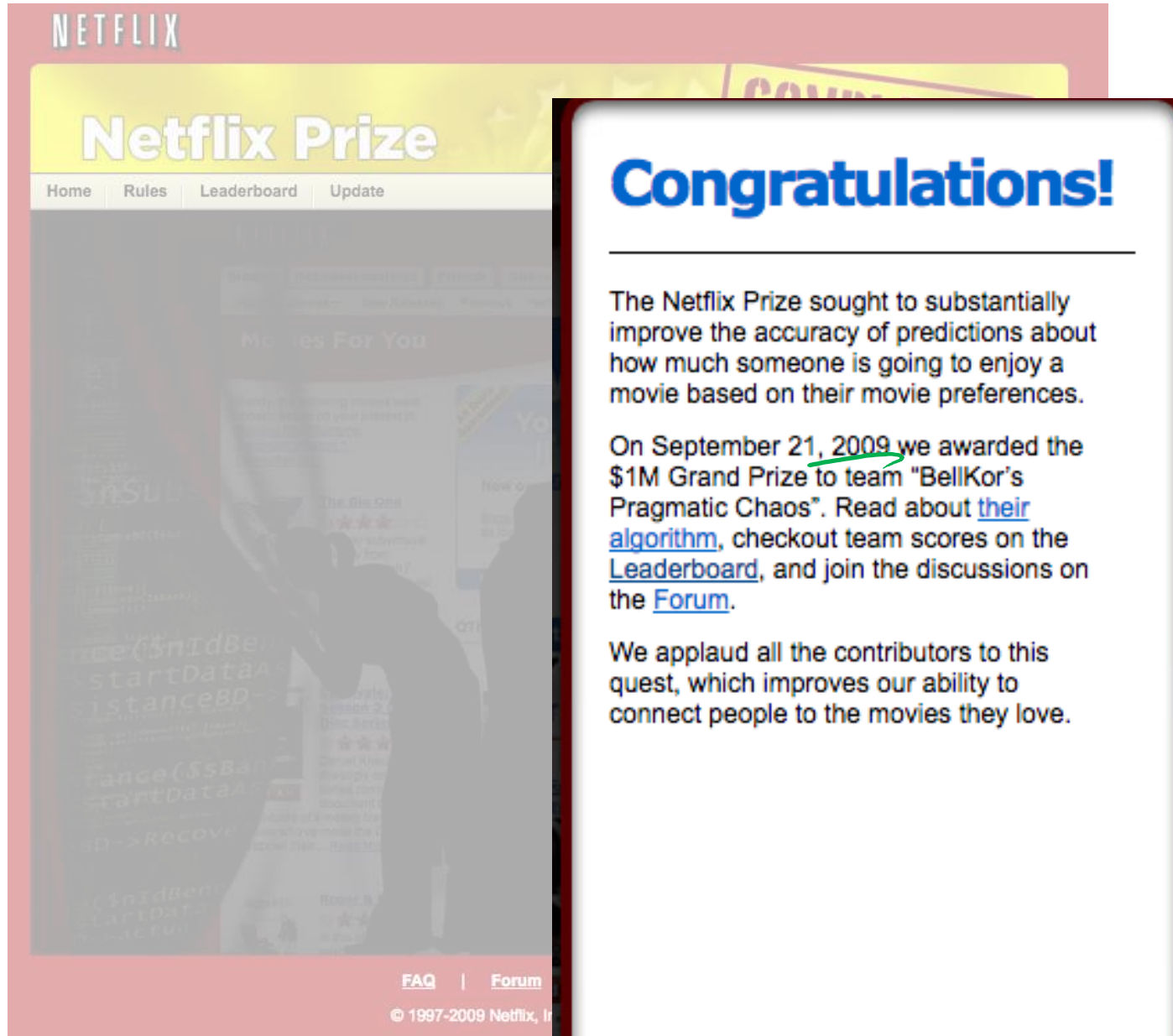
**James Page, Jr.**

**Add**

**FAQ | Forum | Netflix Home**

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# Recommender Systems



The image shows a screenshot of the Netflix Prize announcement page. The page has a yellow header with the Netflix logo and the title "Netflix Prize". Below the header is a navigation bar with links: Home, Rules, Leaderboard, and Update. The main content area is titled "Congratulations!" in large blue letters. Below this, the text reads: "The Netflix Prize sought to substantially improve the accuracy of predictions about how much someone is going to enjoy a movie based on their movie preferences." It then states: "On September 21, 2009 we awarded the \$1M Grand Prize to team 'BellKor's Pragmatic Chaos'. Read about [their algorithm](#), checkout team scores on the [Leaderboard](#), and join the discussions on the [Forum](#)." The text concludes with: "We applaud all the contributors to this quest, which improves our ability to connect people to the movies they love." The background of the page is a faded image of a person holding a movie. At the bottom, there are links for "FAQ" and "Forum" and a copyright notice "© 1997-2009 Netflix, Inc."

NETFLIX

## Netflix Prize

Home Rules Leaderboard Update

### Congratulations!

The Netflix Prize sought to substantially improve the accuracy of predictions about how much someone is going to enjoy a movie based on their movie preferences.

On September 21, 2009 we awarded the \$1M Grand Prize to team "BellKor's Pragmatic Chaos". Read about [their algorithm](#), checkout team scores on the [Leaderboard](#), and join the discussions on the [Forum](#).

We applaud all the contributors to this quest, which improves our ability to connect people to the movies they love.

FAQ | Forum

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\$1M

# AI System Design: Movie Recommendation

## Task

$h(\text{input}) \rightarrow \text{output}$

user  $\rightarrow$   $\leftarrow$  item

rating  
1-5

## Experience

Data (training)

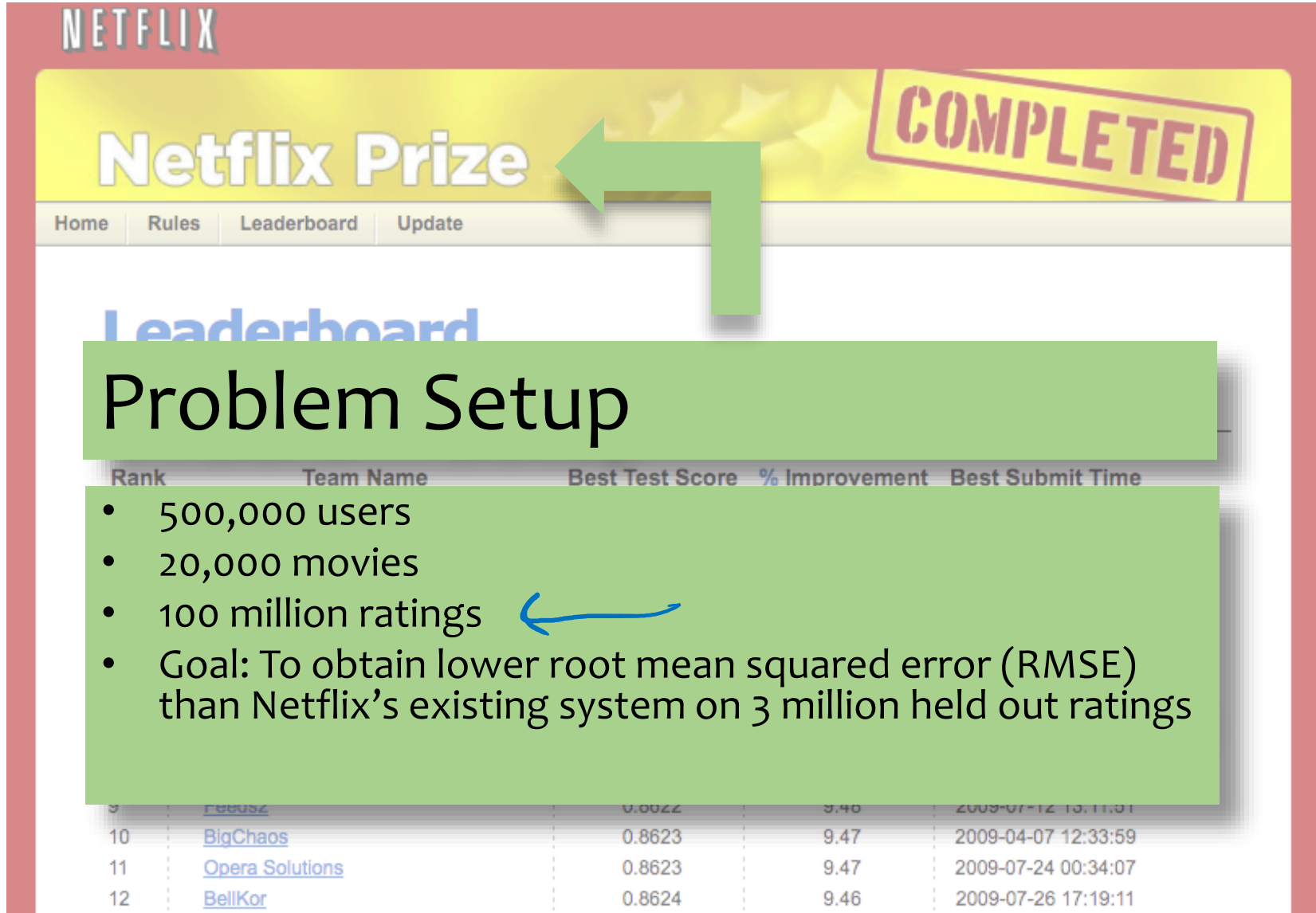
$N \quad (i, j, \underline{r})$

## Performance measure

output  
 $\hat{y} \quad y$



# Recommender Systems



The image is a screenshot of the Netflix Prize website. At the top, the Netflix logo is visible. Below it, a yellow banner reads "Netflix Prize" with a green arrow pointing to it from the left. To the right of the banner is a red stamp that says "COMPLETED". Below the banner is a navigation bar with links: Home, Rules, Leaderboard, and Update. The "Leaderboard" link is highlighted. Below the navigation bar, the word "Leaderboard" is written in large blue letters. A green box with the text "Problem Setup" is overlaid on the page. Below this box is a table with the following columns: Rank, Team Name, Best Test Score, % Improvement, and Best Submit Time. The table lists the top 12 teams. A blue arrow points to the "100 million ratings" item in the list.

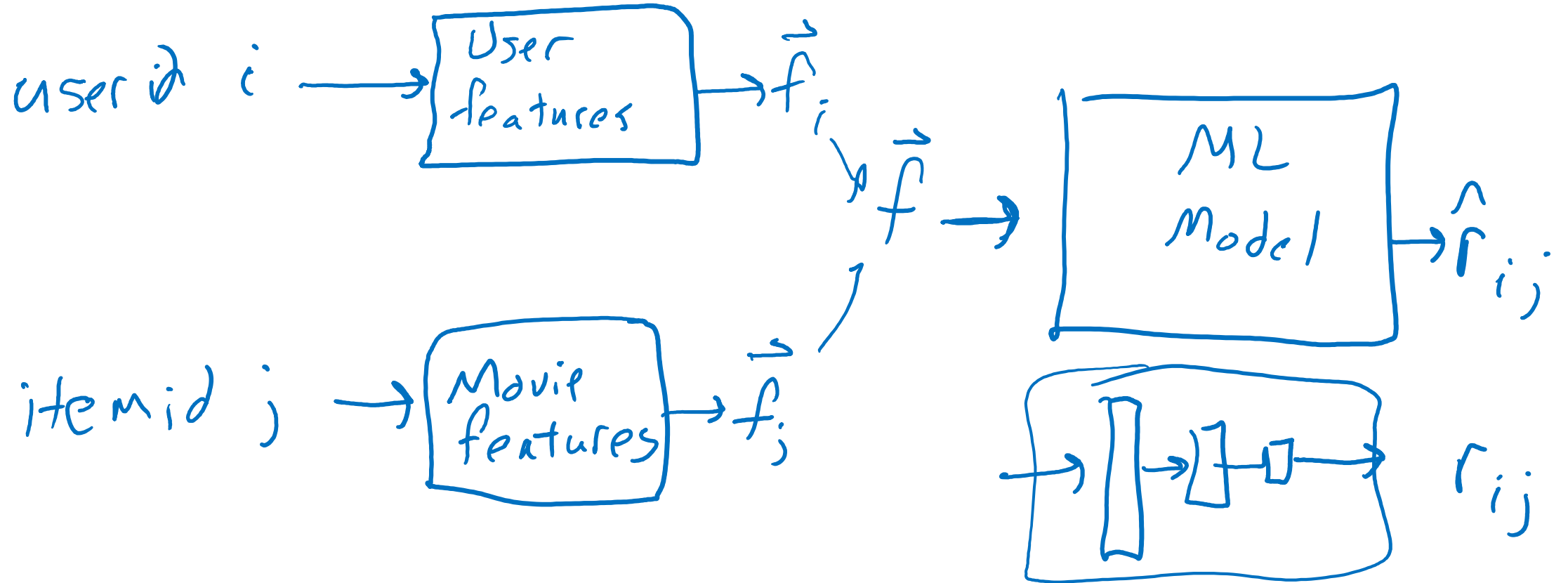
## Problem Setup

- 500,000 users
- 20,000 movies
- 100 million ratings
- Goal: To obtain lower root mean squared error (RMSE) than Netflix's existing system on 3 million held out ratings

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
9	<a href="#">Feus2</a>	0.8622	9.48	2009-07-12 15:11:51
10	<a href="#">BigChaos</a>	0.8623	9.47	2009-04-07 12:33:59
11	<a href="#">Opera Solutions</a>	0.8623	9.47	2009-07-24 00:34:07
12	<a href="#">BellKor</a>	0.8624	9.46	2009-07-26 17:19:11

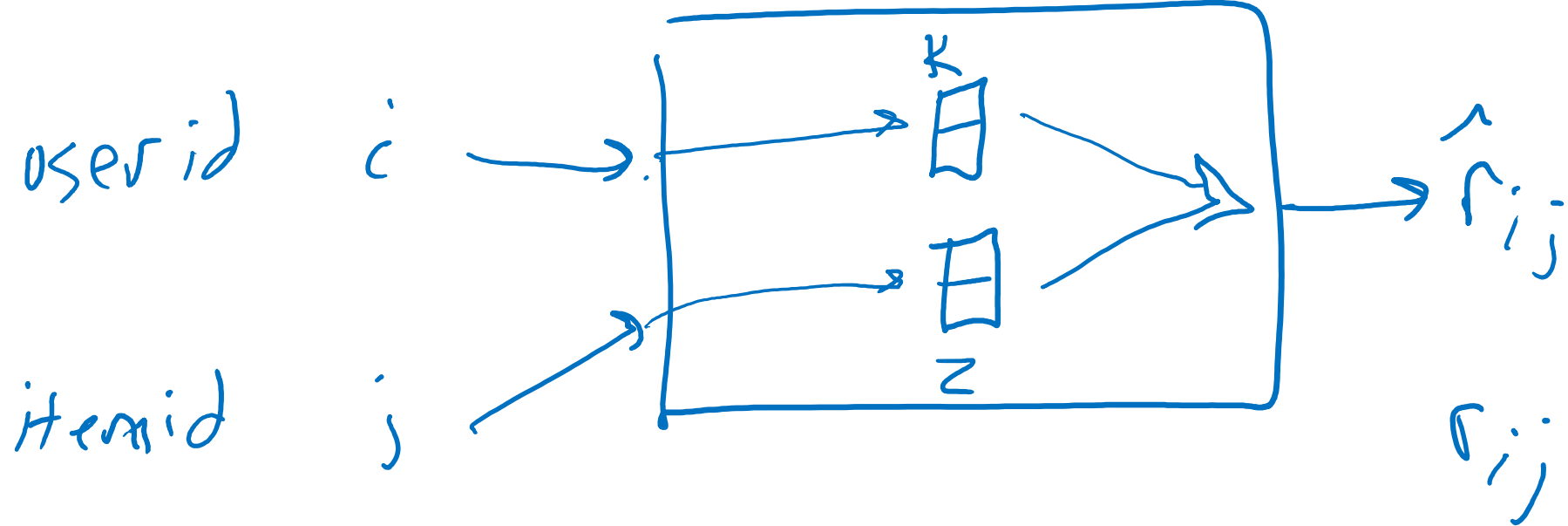
# ML System Design: Movie Recommendation

## Model



# ML System Design: Movie Recommendation

Model



# Recommender Systems

## Setup:


- Items:  
movies, songs, products, etc.  
(often many thousands)
- Users:  
watchers, listeners, purchasers, etc.  
(often many millions)
- Feedback:  
5-star ratings, not-clicking 'next', purchases,  
etc.

## Challenge:

- Users only rate a small number of items  
(the user/item rating data is sparse)

	Doctor Strange	Star Trek: Beyond	Zootopia
Alita	1		5
BB-8	3	4	
C-3PO	3	5	2

Pat



# Different Approaches

## Item-based (*Content filtering*)

- Features about each item
- Given an item, other “close” items have similar values
- e.g. Pandora.com, music genome project



# Different Approaches

## Item-based (*Content filtering*)

- Features about each item
- Given an item, other “close” items have similar values
- e.g. Pandora.com, music genome project

## User-based

- Features about each user
- Given a user, other “close” users have similar preferences
- *Market segmentation*

## Learning user-item relationship

- Can be done without features on either user or item
- Collaborative filtering techniques

# Collaborative Filtering

# Collaborative Filtering

## Everyday Examples of Collaborative Filtering...

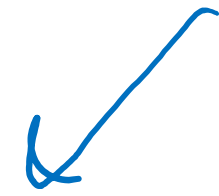
- Bestseller lists
- Top 40 music lists
- The “recent returns” shelf at the library
- Unmarked but well-used paths thru the woods
- The printer room at work
- “Read any good books lately?”
- ...



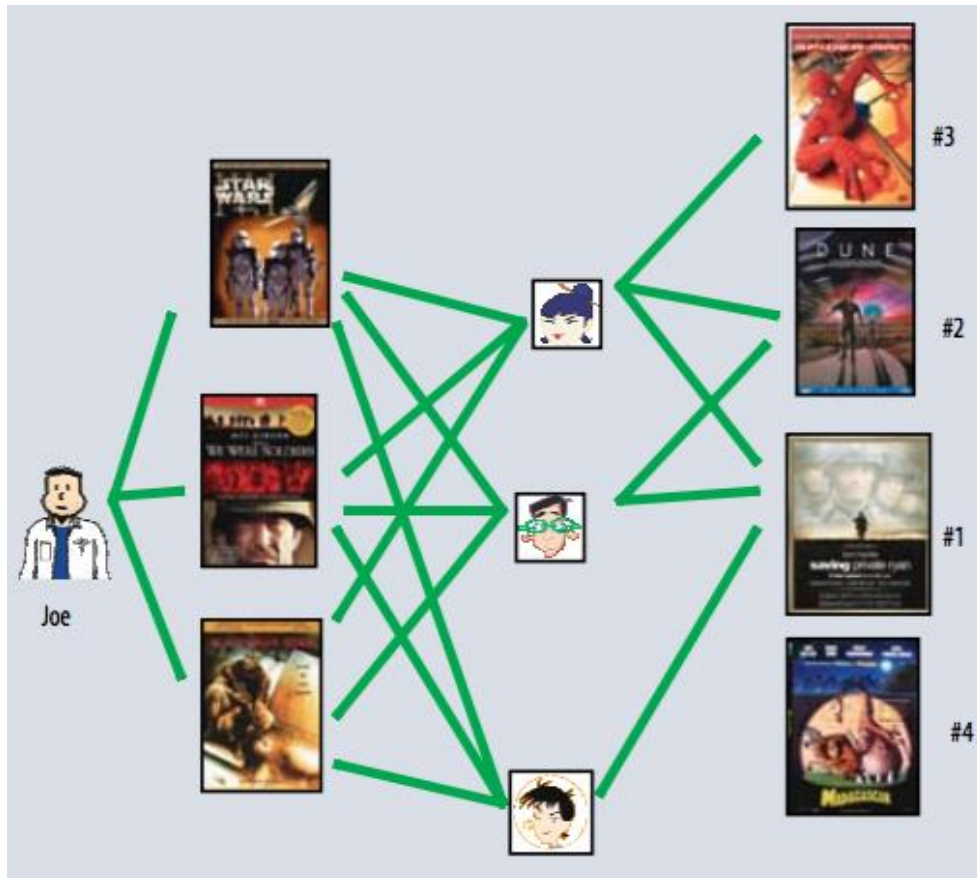
## Common insight: personal tastes are correlated

- If Alita and BB-8 both like X and Alita likes Y then BB-8 is more likely to like Y
- especially (perhaps) if BB-8 knows Alita

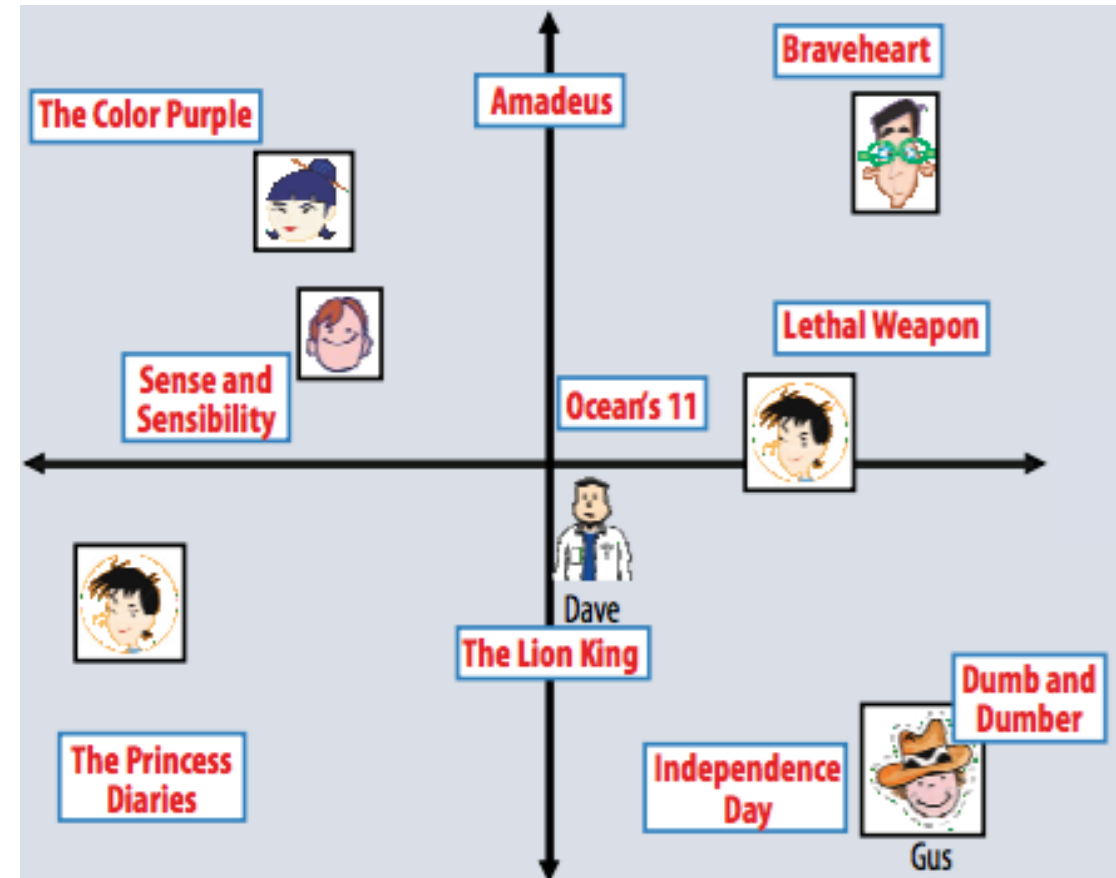
# Two Types of Collaborative Filtering



## 1. Neighborhood Methods

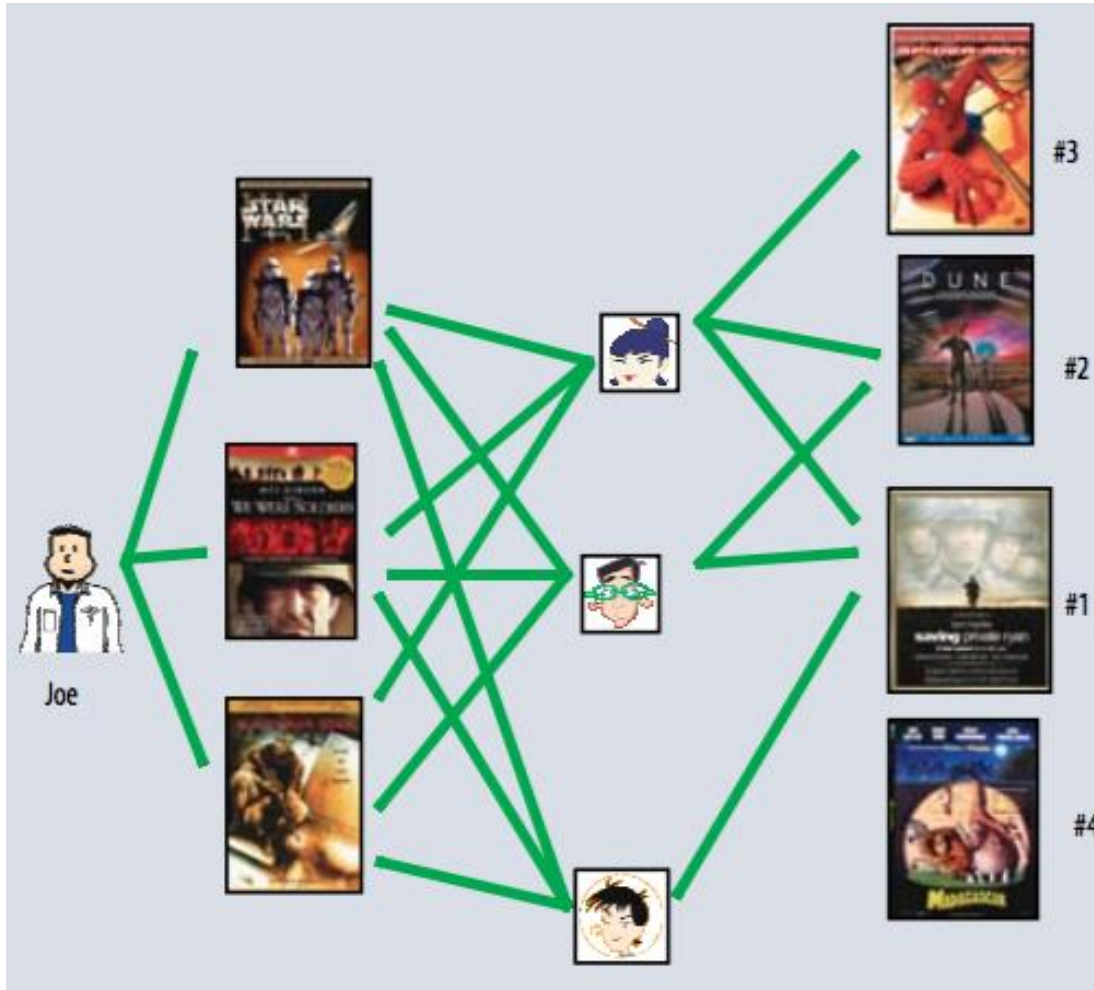


## 2. Latent Factor Methods



# Two Types of Collaborative Filtering

## 1. Neighborhood Methods



In the figure, assume that a green line indicates the movie was **watched**

### Algorithm:

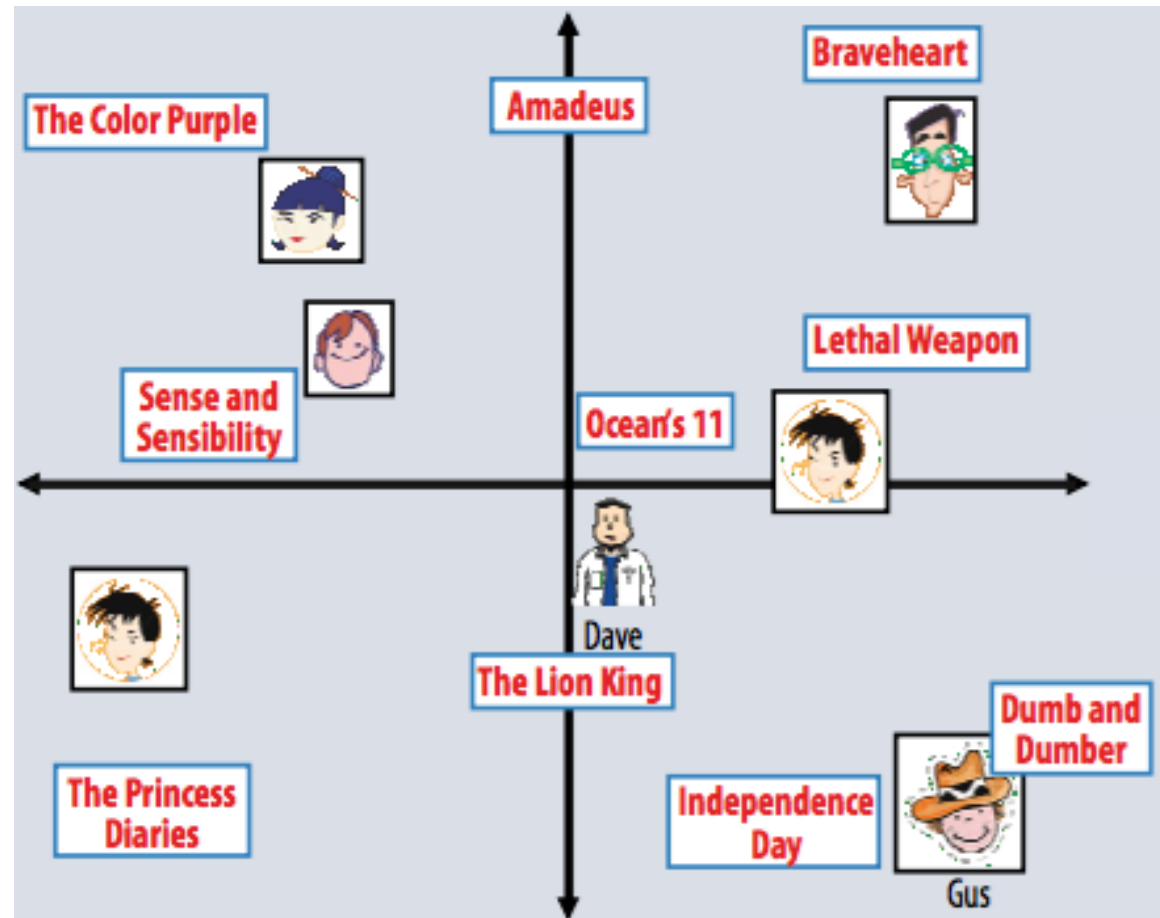
1. **Find neighbors** based on similarity of movie preferences
2. **Recommend** movies that those neighbors watched



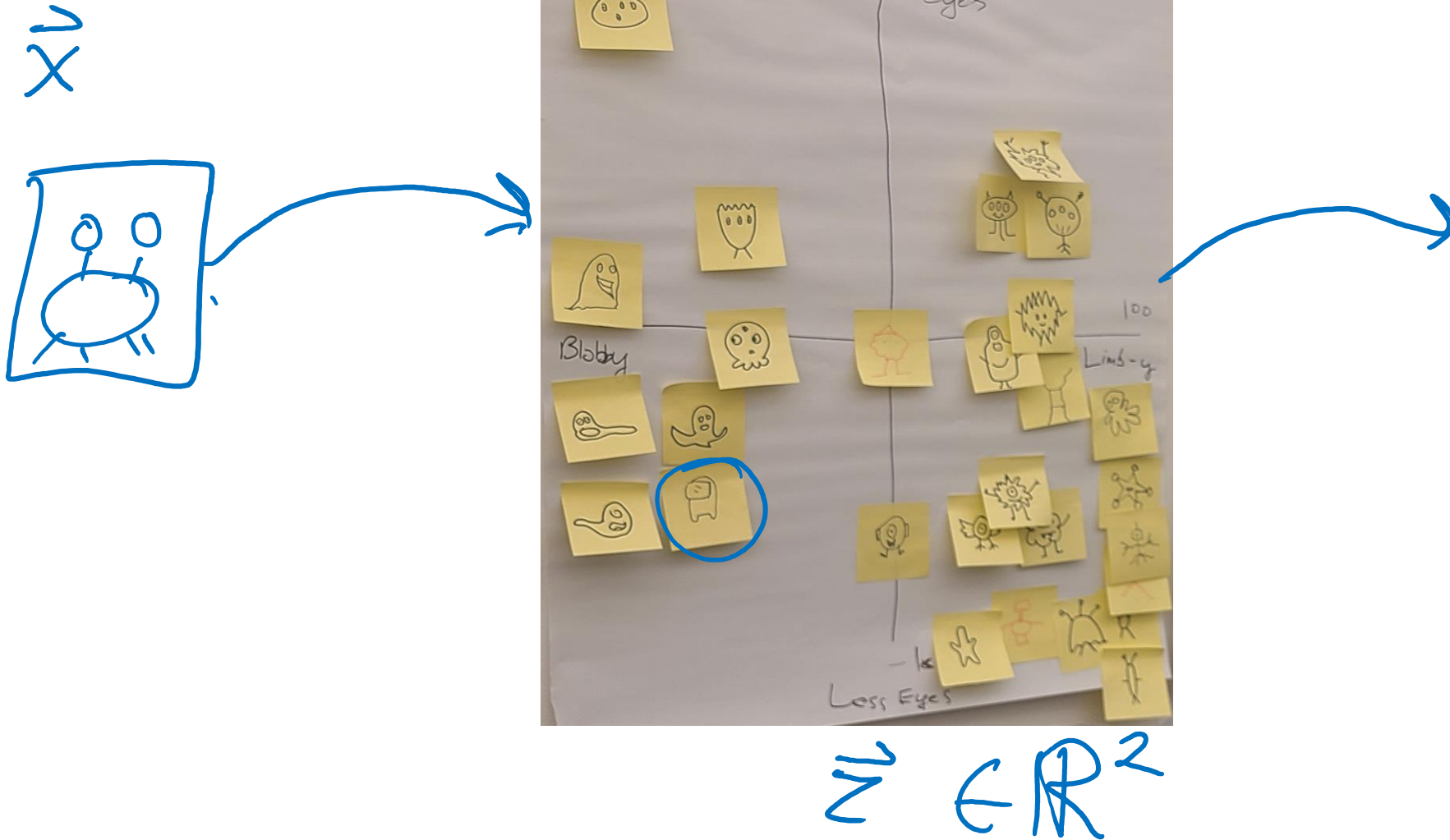
# Two Types of Collaborative Filtering

## 2. Latent Factor Methods

- Assume that both movies and users can be mapped to the same **feature space**
- **Recommend** a movie based on its **proximity** to the user in the feature (latent) space
- **Example algorithm:**  
Matrix Factorization



# Background: Learn a Feature Space



# Background: Learn a Feature Space

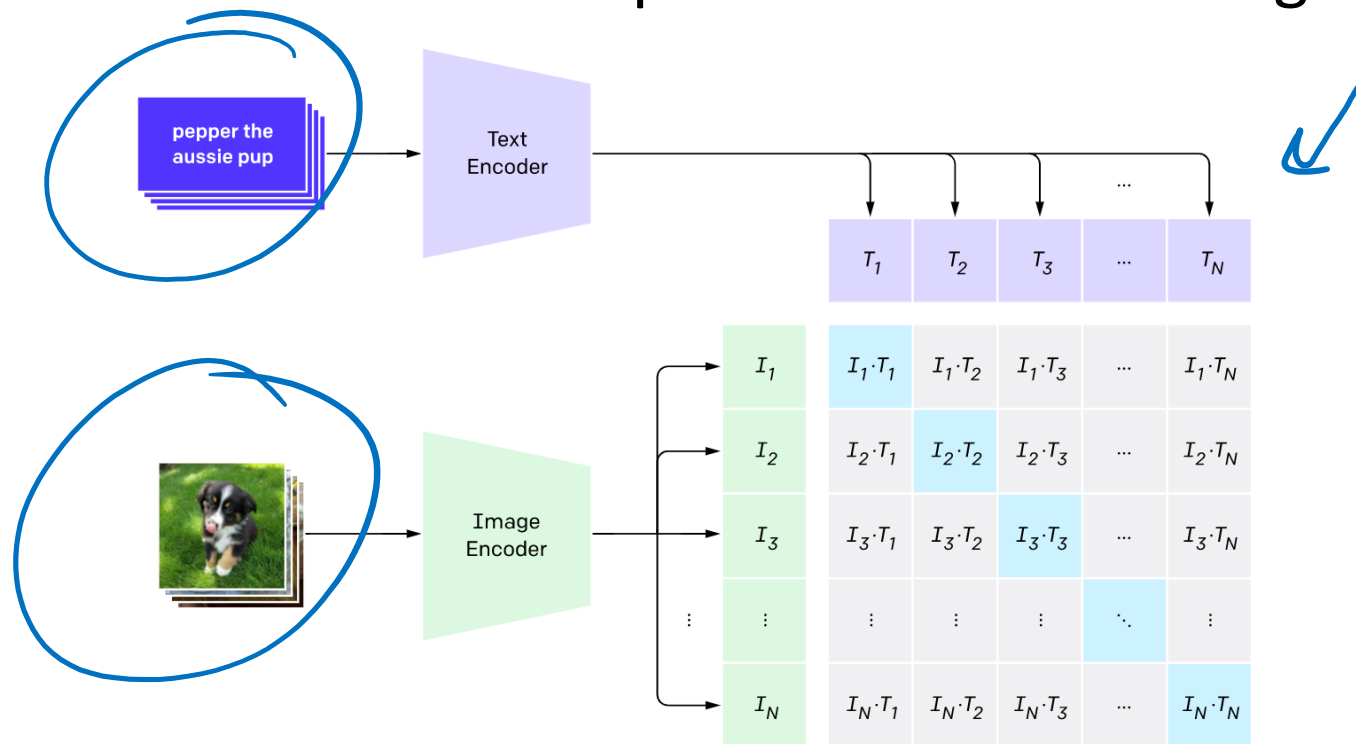
## Word2vec:

Feature space for words



## CLIP:

Common feature space for text and images



# Background: Learn a Feature Space

Why might low dimensional embeddings be useful?

- Example: MNIST digit classification with nearest neighbor

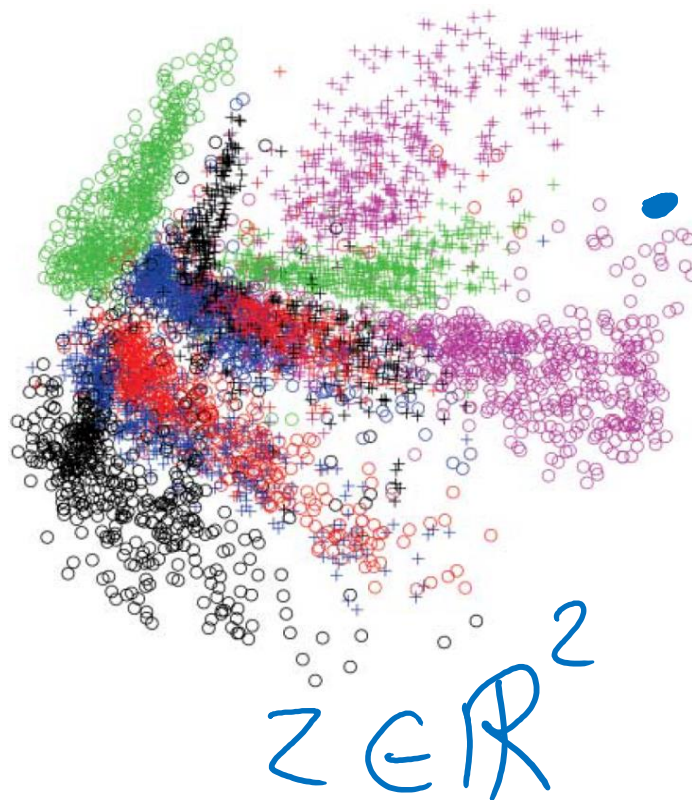
2



# Background: Learn a Feature Space

Why might low dimensional embeddings be useful?

- Example: MNIST digit classification with nearest neighbor



$$z \in \mathbb{R}^2$$



# Background: Measure of Similarity

$$\|u - v\|_2$$

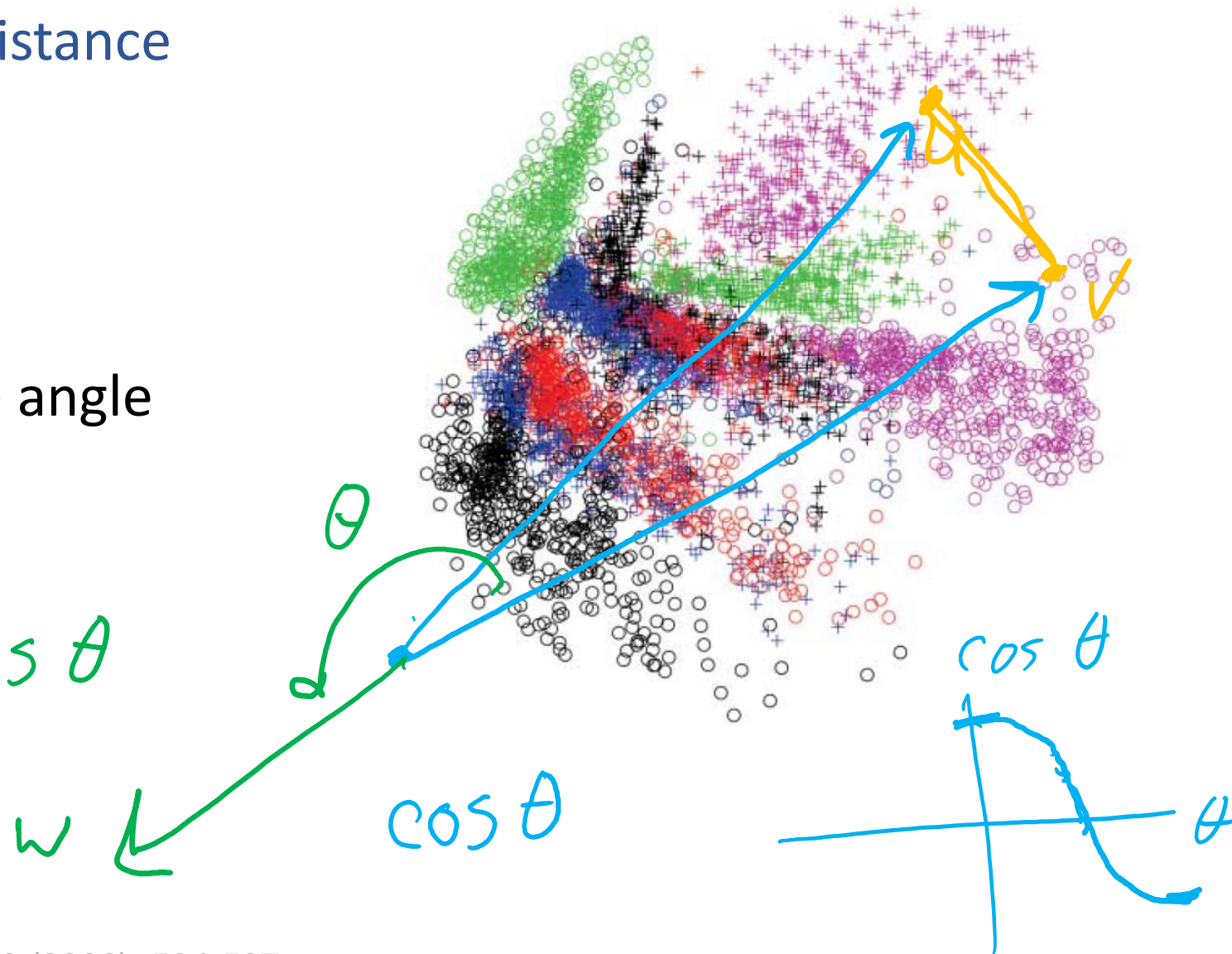
We've been using Euclidean distance

- $d(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|_2$

Cosine similarity

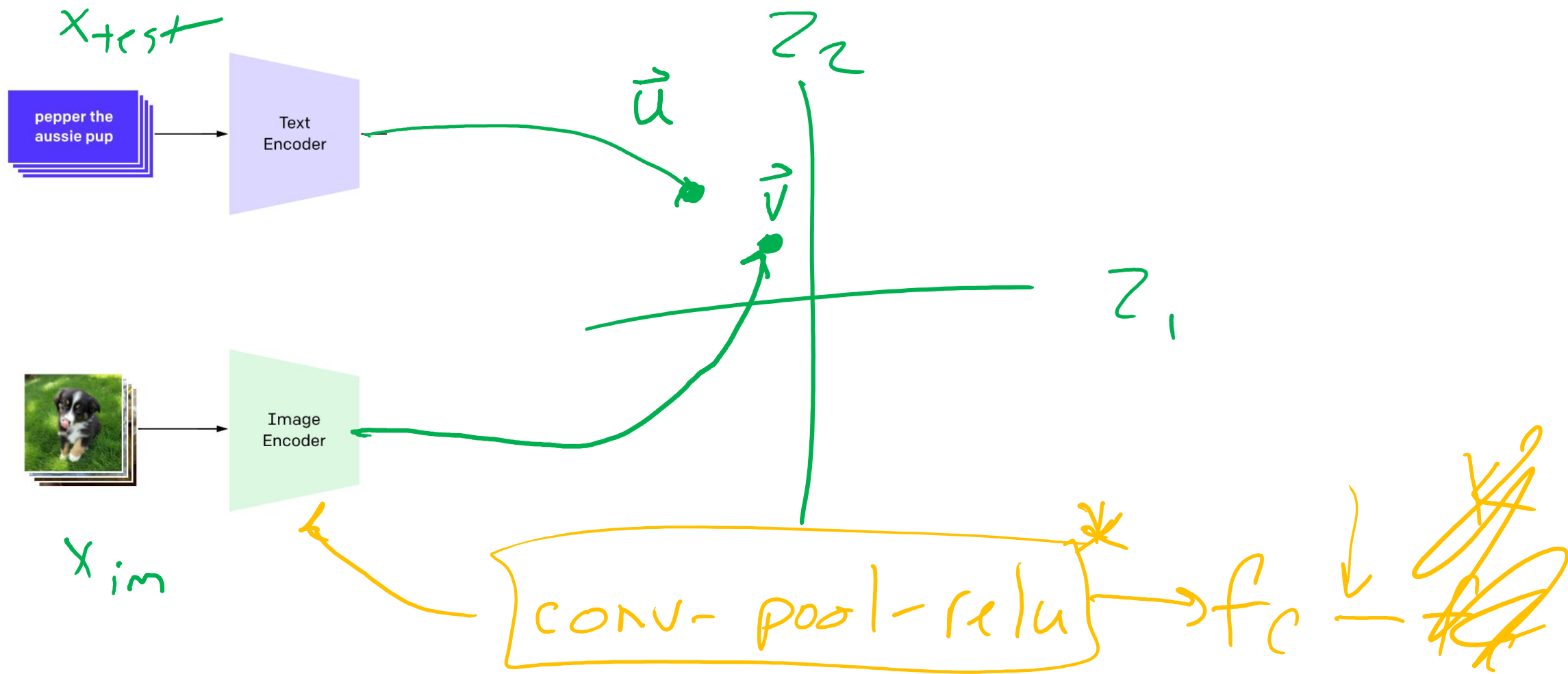
- Two vectors are similar if the angle between them is small

- $\underset{S}{d}(\mathbf{u}, \mathbf{v}) = \mathbf{u}^T \mathbf{v}$   
 $= \|\vec{u}\|_2 \|\vec{v}\|_2 \cos \theta$



# Background: Learn a Feature Space

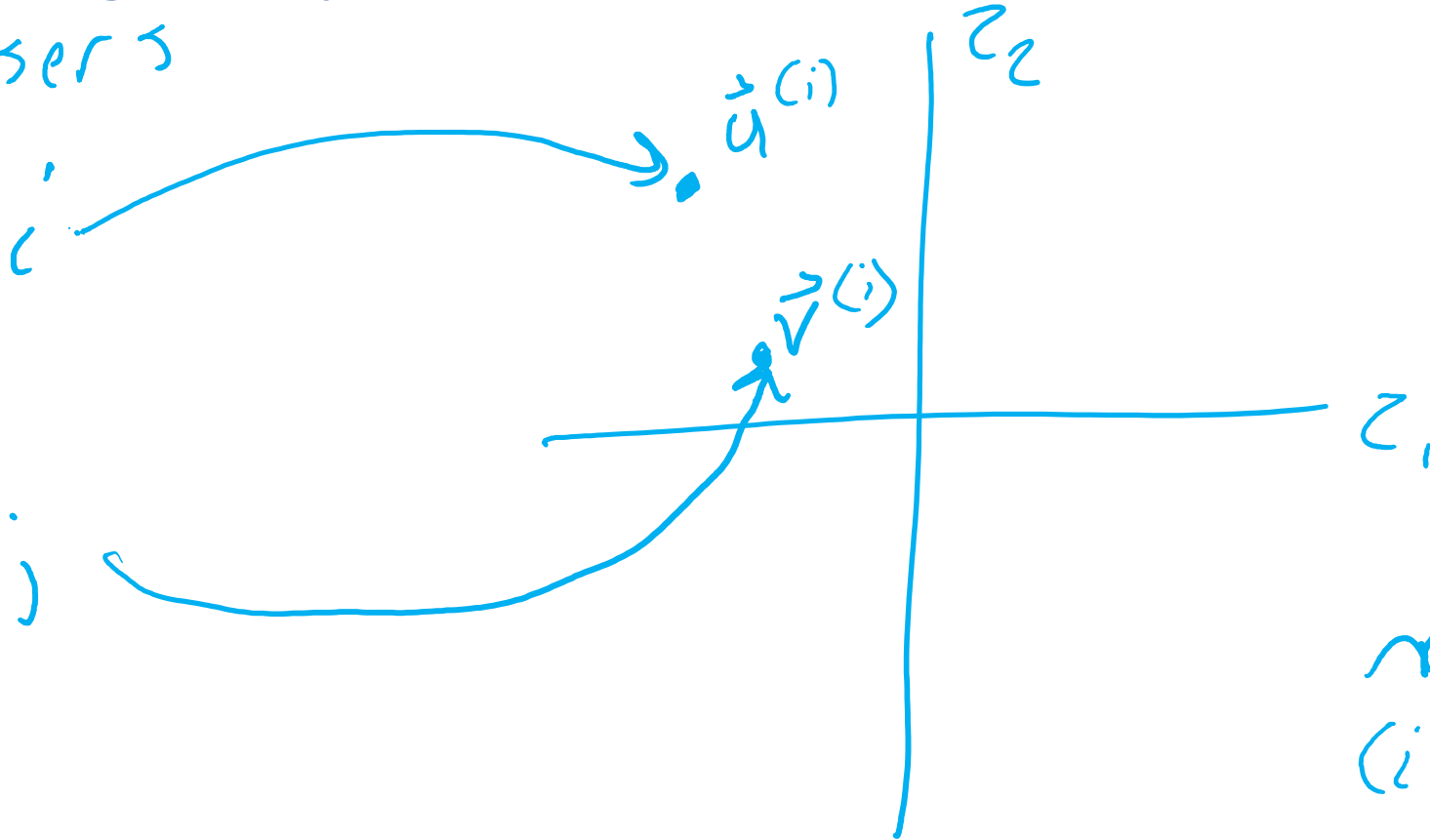
CLIP: Common feature space for text and images



# Recommender System: Matrix Factorization

Learning to map items and users to the same feature space

$N$  users



$$\vec{u}^{(i)T} \vec{v}^{(j)} = \hat{r}_{ij}$$

$$\min_{(i,j,r)} \sum (r_{ij} - \hat{r}_{ij})^2$$

$M$  items

# Recommender System: Matrix Factorization

Learning to map items and users to the same feature space

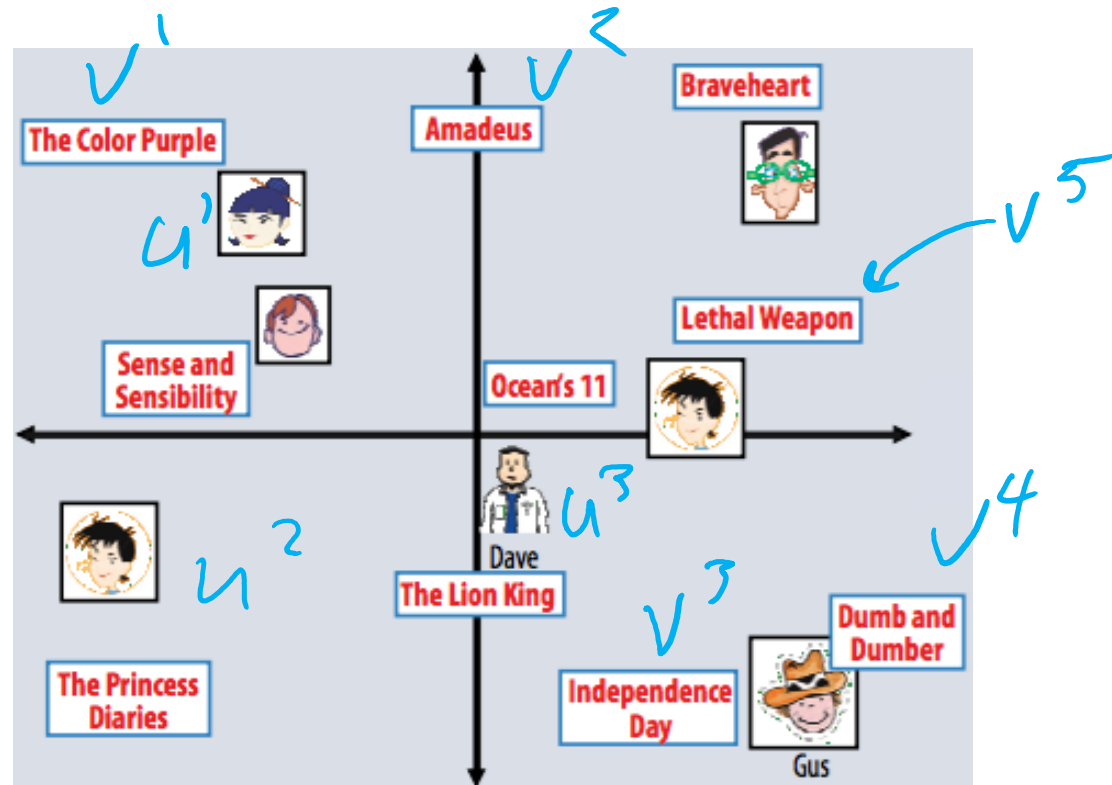
# Recommender System: Matrix Factorization

Learning to map items and users to the same lower dimensional space

$N$  user  
 $M$  item

$$U \in \mathbb{R}^{\underline{N} \times K}$$

$$V \in \mathbb{R}^{M \times K}$$



$$K=2$$



# Recommender System: Matrix Factorization

Optimization: Objective function using only the labels we have

$$\underset{U, V}{\operatorname{argmin}} \sum_{(i, j), r_{ij}} \left( r_{ij} - \frac{u^{(i)\top} v^{(j)}}{\|u^{(i)}\| \|v^{(j)}\|} \right)^2$$

# Poll 1

Is the following optimization a quadratic optimization?

$$\min_{\mathbf{U}, \mathbf{V}} \sum_{i,j \in \mathcal{S}} \left( r_{ij} - \mathbf{u}^{(i)T} \mathbf{v}^{(j)} \right)^2$$

A. ~~Yes~~ AIV

B. Yes

C. No

D. ~~No~~ AIV

# Matrix Factorization

Method of *alternating minimization*

$$\min_{U,V} J(U,V) \quad J(U,V) = \sum_{i,j \in \mathcal{S}} \left( r_{ij} - \mathbf{u}^{(i)T} \mathbf{v}^{(j)} \right)^2$$

do

①

$$\min_U J(U,V)$$

②

$$\min_V J(U,V)$$

$$U^{(t+1)} \leftarrow U^{(t)} - \alpha \nabla_U J(U^{(t)}, V^{(t)})$$

---

$$V^{(t+1)} \leftarrow V^{(t)} - \alpha \nabla_V J(U^{(t+1)}, V^{(t)})$$

# Matrix Factorization

Method of *alternating minimization*

$$\min_{U,V} J(U, V) \qquad J(U, V) = \sum_{i,j \in \mathcal{S}} (r_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2$$

- 1)  $\operatorname{argmin}_U J(U, V)$
- 2)  $\operatorname{argmin}_V J(U, V)$

# Matrix Factorization

Method of *alternating minimization*

$$\min_{U,V} J(U,V)$$

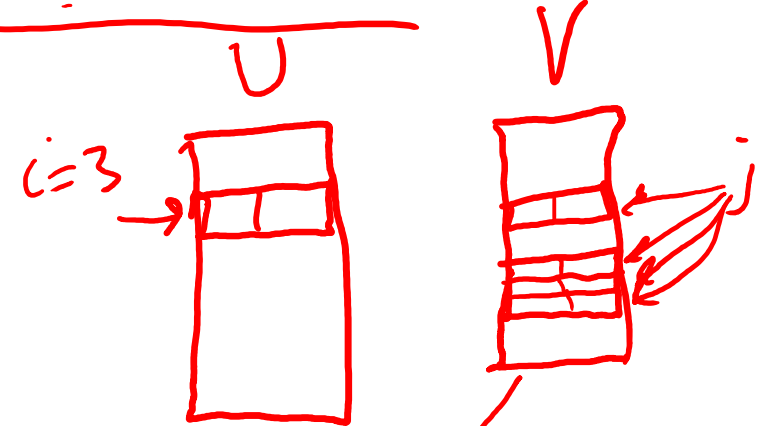
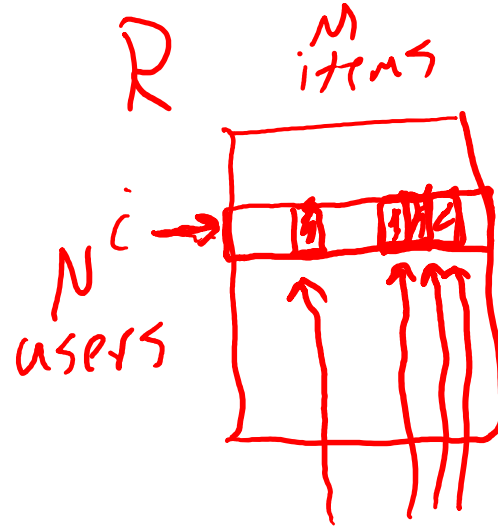
$$J(U,V) = \sum_{i,j \in \mathcal{S}} (r_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2$$

$K=2$

$i=3$  update  $\vec{u}_i$

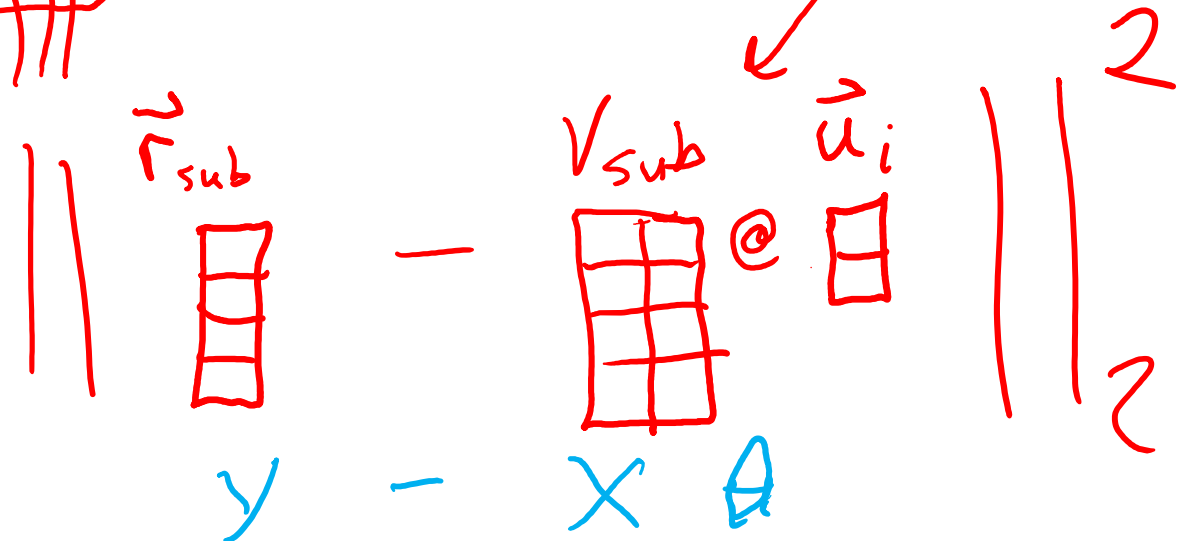
1)  $\operatorname{argmin}_U J(U,V)$

2)  $\operatorname{argmin}_V J(U,V)$



1) For all  $i$ ,  $\operatorname{argmin}_{\vec{u}_i} J(U,V)$

2) For all  $j$ ,  $\operatorname{argmin}_{\vec{v}_j} J(U,V)$



# Matrix Factorization

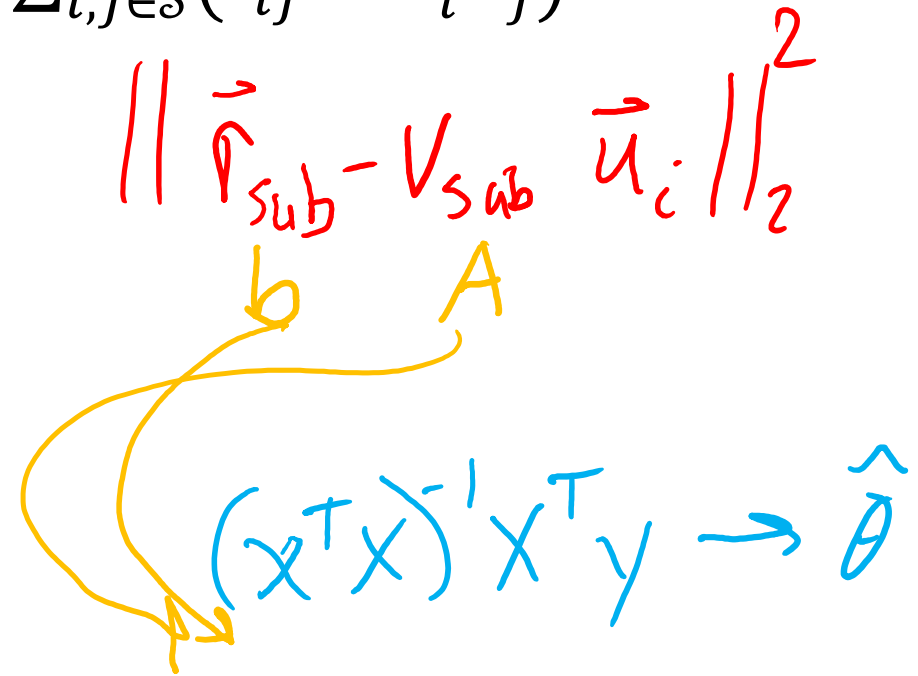
Method of *alternating minimization*

$$\min_{U,V} J(U,V)$$

$$J(U,V) = \sum_{i,j \in \mathcal{S}} (r_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2$$

$$\| \vec{r}_{sub} - V_{sub} \vec{u}_i \|_2^2$$

$$(X^T X)^{-1} X^T y \rightarrow \hat{\theta}$$



1) For all  $i$ ,  $\underset{\mathbf{u}_i}{\operatorname{argmin}} J(U,V)$

2) For all  $j$ ,  $\underset{\mathbf{v}_j}{\operatorname{argmin}} J(U,V)$

$$\mathbf{u}_i^{(t+1)} = (A^T A)^{-1} A^T \mathbf{b}$$

$$\mathbf{v}_j^{(t+1)} = (C^T C)^{-1} C^T \mathbf{d}$$

# Matrix Factorization

Method of *alternating minimization*

$$\min_{U,V} J(U,V) \qquad J(U,V) = \sum_{i,j \in \mathcal{S}} (r_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2$$

- 1) For all  $i$ ,  $\operatorname{argmin}_{\mathbf{u}_i} J(U,V)$
  - 2) For all  $j$ ,  $\operatorname{argmin}_{\mathbf{v}_j} J(U,V)$
- $$\mathbf{u}_i^{(t+1)} = (A^T A)^{-1} A^T \mathbf{b}$$
- $$\mathbf{v}_j^{(t+1)} = (C^T C)^{-1} C^T \mathbf{d}$$

# Matrix Factorization

First solve:

$$\min_{U,V} J(U,V) \quad J(U,V) = \sum_{i,j \in \mathcal{S}} \left( r_{ij} - \mathbf{u}^{(i)T} \mathbf{v}^{(j)} \right)^2$$

$U V$

What then?

predict

$$h(i,j) \rightarrow \hat{r}_{ij}$$

$$\mathbf{u}^{(i)T} \mathbf{v}^{(j)} \rightarrow \hat{r}_{ij}$$

	Doctor Strange	Star Trek: Beyond	Zootopia
Alita	1	6.4	5
BB-8	3	4	2.5
C-3Po	3	5	2



# Matrix Factorization

First solve:

$$\min_{U,V} J(U,V) \quad J(\underline{U}, V) = \sum_{i,j \in \mathcal{S}} \left( r_{ij} - \mathbf{u}^{(i)T} \mathbf{v}^{(j)} \right)^2$$

*items (j)*

What then?

*users (i)*

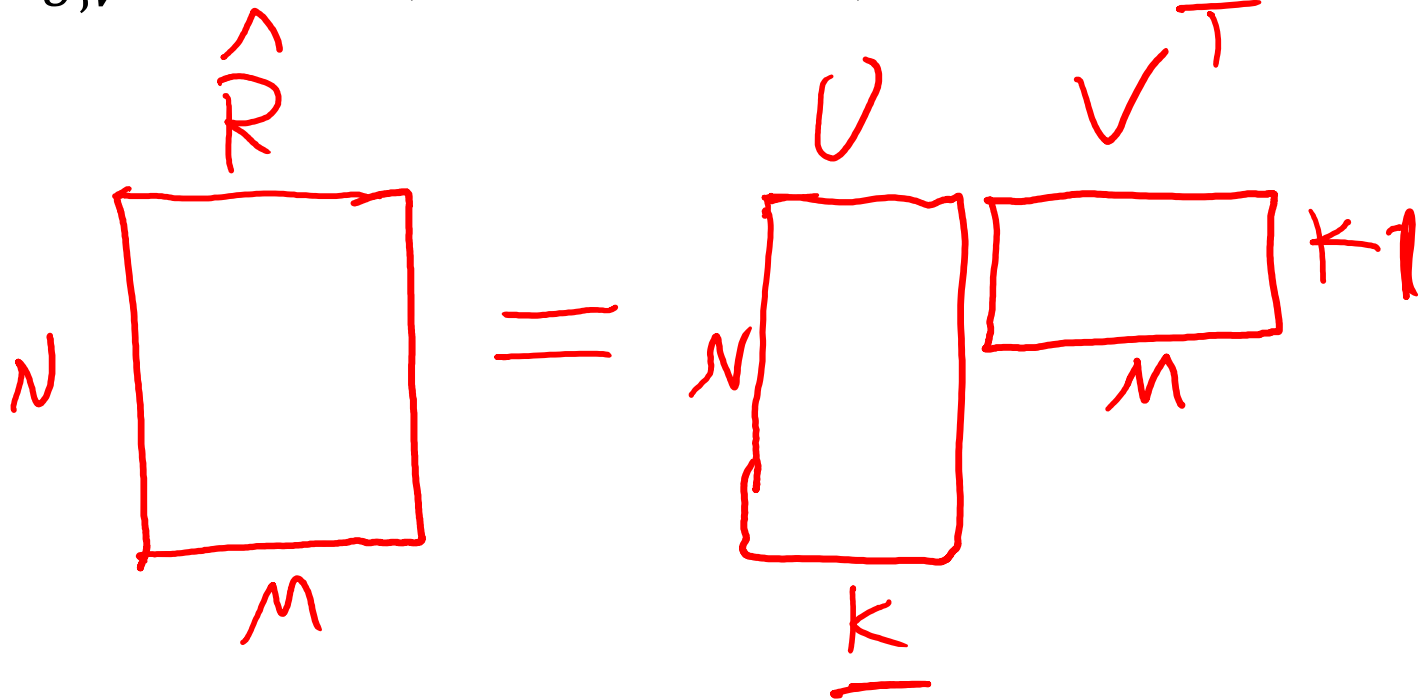
→


? ? ? ✓ ? ✓

# Recommender System: Matrix Factorization

Why is it called matrix factorization?

$$\min_{U,V} \sum_{i,j \in \mathcal{S}} \left( r_{ij} - \mathbf{u}^{(i)T} \mathbf{v}^{(j)} \right)^2$$



SVD

$$X = U S V^T$$

$$R = U \underbrace{V^T}$$

matrix factorization

# Recommender System: Matrix Factorization

Why is it called matrix factorization?

$$\min_{U,V} \|R - UV^T\|_F^2$$


Sparse labels ☹️



	Doctor Strange	Star Trek: Beyond	Zootopia
Alita	1		5
BB-8	3	4	
C-3Po	3	5	2

# Matrix Factorization

Add regularization to avoid overfitting

$$\min_{U,V} J(U,V) + \lambda_1 \|U\|_F^2 + \lambda_2 \|V\|_F^2$$

$$J(U,V) = \sum_{i,j \in \mathcal{S}} \left( r_{ij} - \mathbf{u}^{(i)T} \mathbf{v}^{(j)} \right)^2$$

# Bias in Recommender Systems

What high-level problems can occur from recommender systems?

# Summary

Recommender systems solve many **real-world** (\*large-scale) **problems**

Collaborative filtering by matrix factorization (MF) is an **efficient** and **effective** approach

(Sparse matrix makes MF more challenging)

MF is just another example of a **common recipe**:

1. define a model
2. define an objective function
3. optimize

## Optimization

- Use alternating minimization
- Add regularization to avoid overfitting