315 Lecture 25

Wednesday, April 22, 2020 10:14 AM

Theorem 1:

$$N \geq \frac{1}{\epsilon}\left[\log |H| + \log \frac{1}{\delta}\right]$$

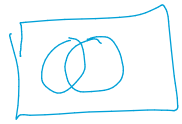
N examples is sufficient to ensure that

with prob $1-\delta$

all $h \in H$ with $\hat{R}(h)=0$ have $R(h) \leq \epsilon$

Union bound

$P(A \cup B) \leq P(A) + P(B)$



(1) Assume k "bad" hypotheses $h_1 h_2 \ldots h_k$ with $R(h_i) > \epsilon$

(2) Pick bad h_i: Prob h_i is consistent w/ first training point $\leq 1-\epsilon$

Prob h_i is $\underbrace{\text{consistent w/ first } N \text{ training}}_{R(h_i)=0}$ points $\leq (1-\epsilon)^N$

(3) Prob at least one bad h_i is consistent with w/first N points $\leq k(1-\epsilon)^N$

$\leq |H|(1-\epsilon)^N$

(4) $1-x \leq e^{-x} \Rightarrow |H|(1-\epsilon)^N \leq |H|e^{-\epsilon N}$

(5) Calc N and δ s.t. $\boxed{|H|e^{-\epsilon N} \leq \delta}$

(6) Solve N

$\Rightarrow |H| \, 1/\delta \leq \frac{1}{e^{-\epsilon N}}$

$\boxed{\log |H| + \log 1/\delta \leq \epsilon N}$

Assume (1)

with prob δ "bad"

$\exists h$ s.t. $R(h) > \epsilon$ and $\hat{R}(h)=0$

with prob $1-\delta$ "good"

all $h \in H$ with $\underset{A}{R(h) > \epsilon}$ $\Rightarrow$ have $\underset{B}{\hat{R}(h) > 0}$

$\Rightarrow$ all $h \in H$ with $\underset{\neg B}{\hat{R}(h)=0}$ have $\underset{\neg A}{R(h) \leq \epsilon}$ $\Leftarrow$

$\Rightarrow$

Contrapositive

$A \Rightarrow B$

$\neg B \Rightarrow \neg A$