Announcements

Assignments:

- HW4
  - Due date Mon, 2/24, 11:59 pm

Midterm

- Monday the 2nd of March from 5:00pm-6:30pm

Midterm Conflicts

- See Piazza post
- Due 11:59pm on Wednesday the 19th of February

Plan

Last time

- Naïve Bayes Assumptions
- Naïve Bayes MLE and MAP
- MLE vs MAP
- Generative vs Discriminative Models

Today

- Decision Boundaries
- Gaussian Generative Models
- Neural Networks

Introduction to Machine Learning

Generative Models
then
Intro to Neural Networks

Instructor: Pat Virtue

Decision Boundaries

Decision boundary

- The set of points in the domain of the input ($\boldsymbol{x}$) where the predicted classification changes

Two class decision boundary

- So far, we have decided to let the decision boundary be all $\boldsymbol{x}$ such that:

$$p(y = 0 \mid \boldsymbol{x}) = p(y = 1 \mid \boldsymbol{x})$$

- What assumptions are we making here?
  - This assumes that the cost of predicting it wrong is the same for both classes

Piazza Poll 1

Which of the following also define the decision boundary for two classes when we just want $p(Y = 0 \mid \boldsymbol{x}) = p(Y = 1 \mid \boldsymbol{x})$?

A. All $\boldsymbol{x}$, s.t. $p(\boldsymbol{x} \mid Y = 0) = p(\boldsymbol{x} \mid Y = 1)$
B. All $\boldsymbol{x}$, s.t. $p(\boldsymbol{x}, Y = 0) = p(\boldsymbol{x}, Y = 1)$
C. All $\boldsymbol{x}$, s.t. $p(Y = 0) = p(Y = 1)$
D. All $\boldsymbol{x}$, s.t. $p(Y = 1 \mid \boldsymbol{x}) = 0.5$
E. All $\boldsymbol{x}$, s.t. $p(\boldsymbol{x} \mid Y = 1) = 0.5$
F. All $\boldsymbol{x}$, s.t. $p(\boldsymbol{x}, Y = 1) = 0.5$
G. All $\boldsymbol{x}$, s.t. $\log p(\boldsymbol{x}, Y = 1) - \log p(\boldsymbol{x}, Y = 0) = 0$
H. None of the above

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H. None of the above

Piazza Poll 2

True/False: Logistic regression always produces a linear decision boundary.

A. I don’t know

B. True

C. False

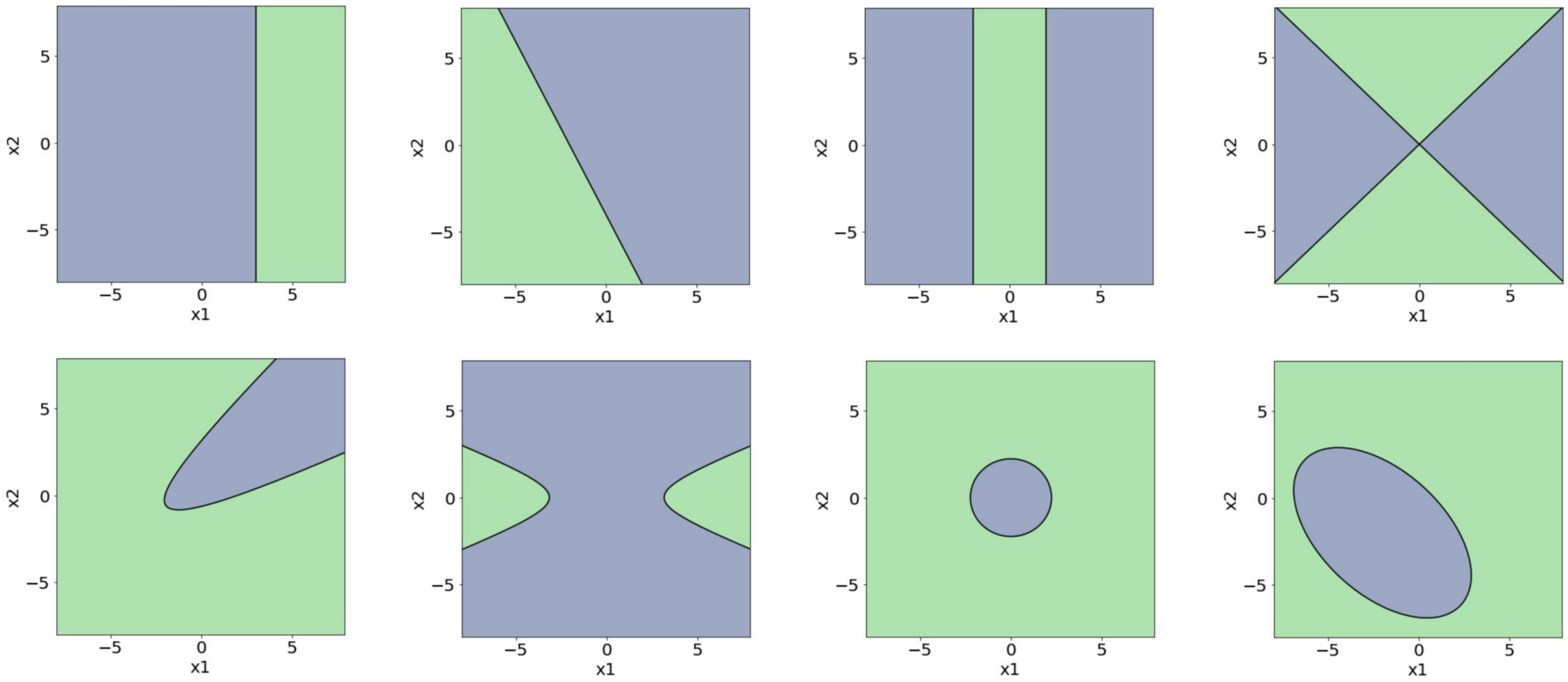
Piazza Poll 2

True/False: Logistic regression always produces a linear decision boundary.

A. I don't know

B. True

C. False



Generative Models

SPAM:

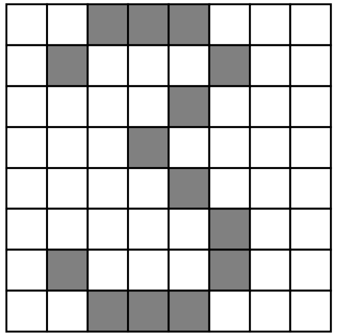
- Class distribution: $Y \sim \ Bern(\phi)$
- Class conditional distribution: $X_m \sim \ Bern(\theta_{m,y})$
- Naïve Bayes X_i conditionally independent X_j given Y for all $i \neq j$

$$p(X_i, X_j \mid Y) \ = \ p(X_i \mid Y) \mid p(X_j \mid Y)$$

Digits:

- Class distribution: $Y \sim \ Multinomial(\phi, 1)$
- Class conditional distribution: $X_m \sim \ Bern(\theta_{m,y})$
- Naïve Bayes X_i conditionally independent X_j given Y for all $i \neq j$

$$p(X_i, X_j \mid Y) \ = \ p(X_i \mid Y) \mid p(X_j \mid Y)$$



Recitation?

Fisher Iris Dataset

https://en.wikipedia.org/wiki/Iris_flower_data_set

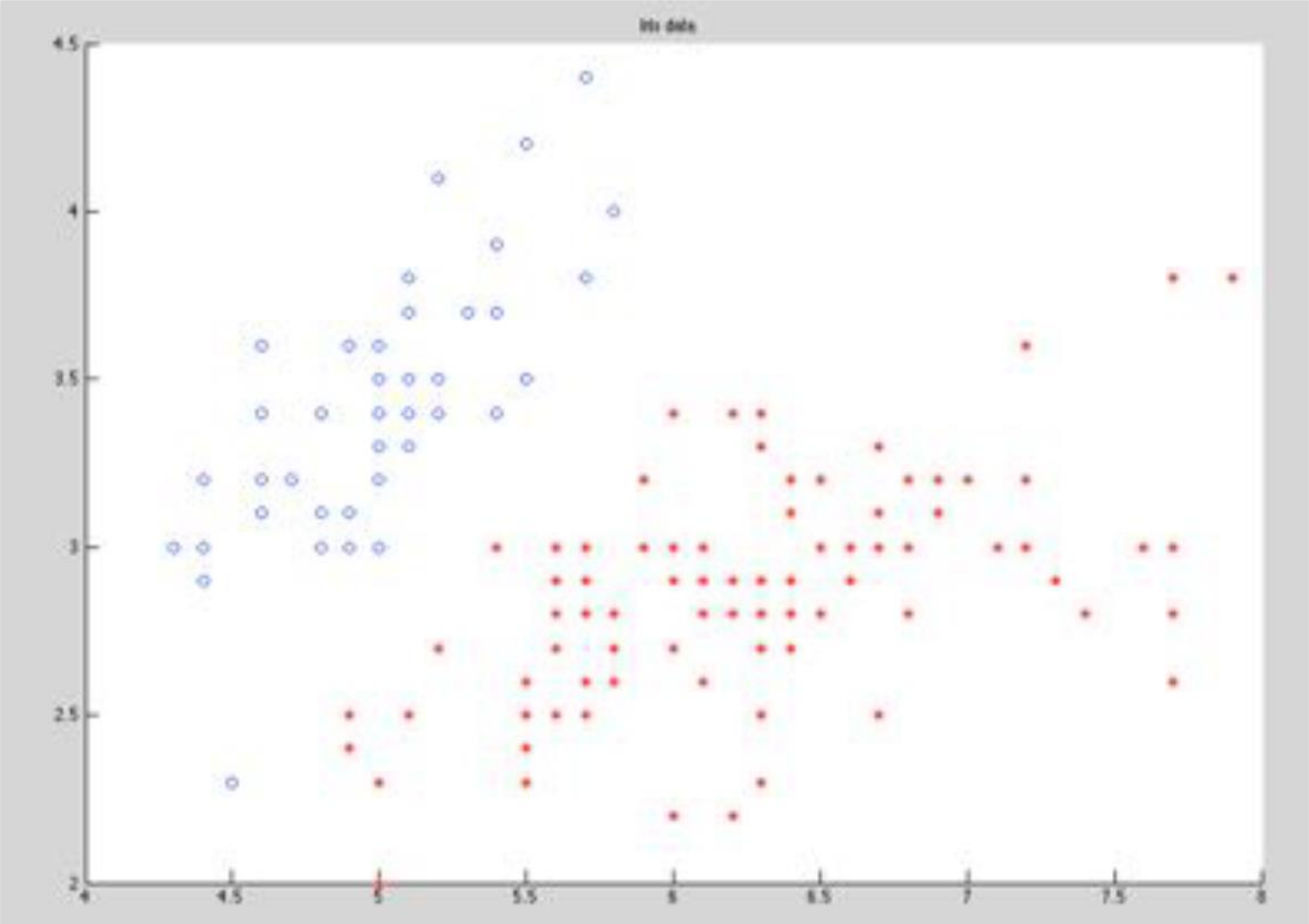
Fisher Iris Dataset

https://en.wikipedia.org/wiki/Iris_flower_data_set

Fisher (1936) used 150 measurements of flowers from 3 different species: Iris setosa (0), Iris virginica (1), Iris versicolor (2) collected by Anderson (1936)

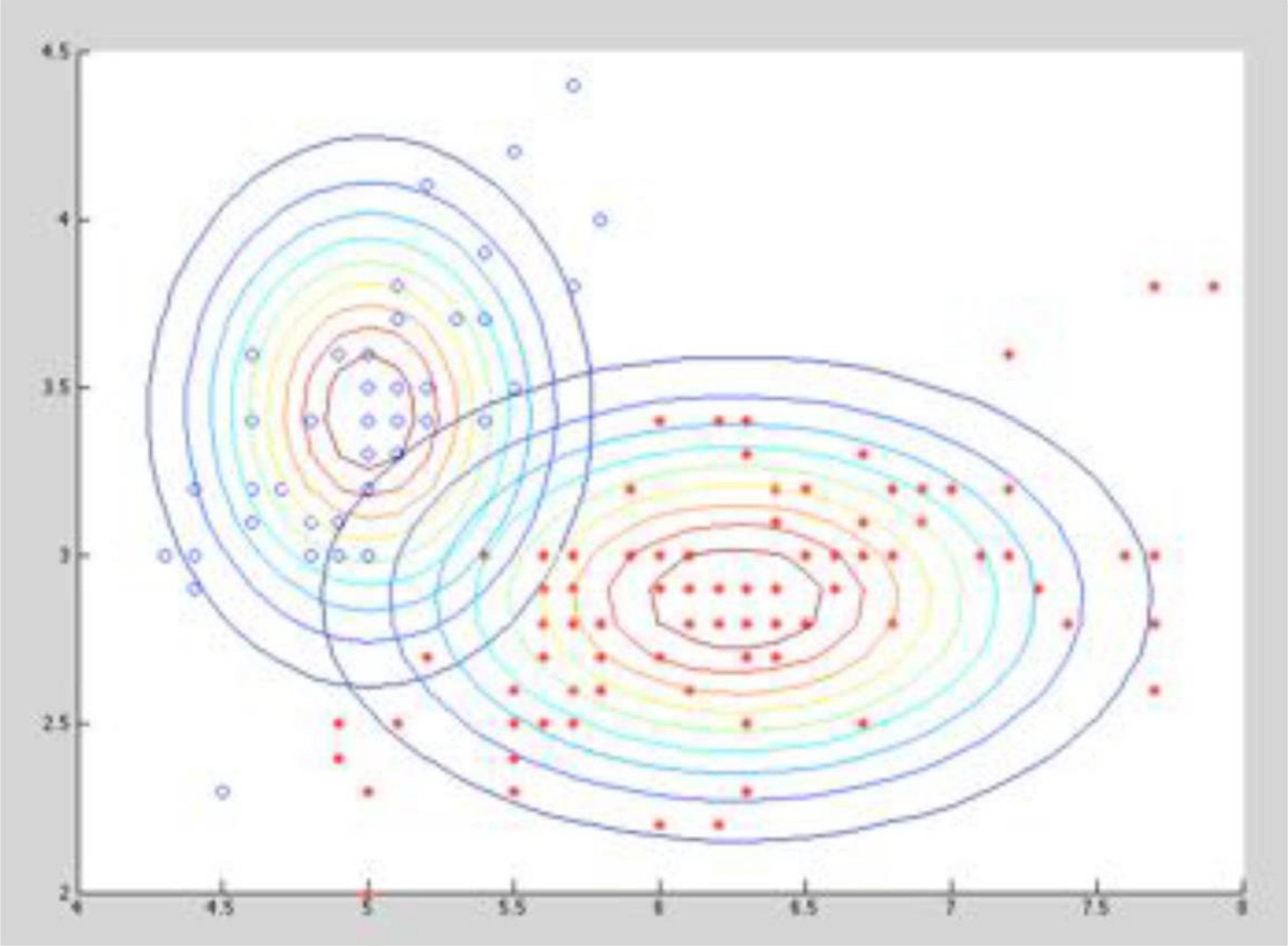
Species	Sepal Length	Sepal Width	Petal Length	Petal Width
0	4.3	3.0	1.1	0.1
0	4.9	3.6	1.4	0.1
0	5.3	3.7	1.5	0.2
1	4.9	2.4	3.3	1.0
1	5.7	2.8	4.1	1.3
1	6.3	3.3	4.7	1.6
1	6.7	3.0	5.0	1.7

Fisher Iris Dataset



Slides from William Cohen, CMU

Fisher Iris Dataset



Slides from William Cohen, CMU

Generative Models with Continuous Features

Iris dataset:

- Class distribution: $Y \sim \ Bern(\phi)$
- Class conditional distribution: $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{\mu}_y, \boldsymbol{\Sigma}_y)$
- Naïve Bayes assumption?

Piazza Poll 3

Iris dataset:

- Class distribution: $Y \sim \; Bern(\phi)$
- Class conditional distribution: $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{\mu}_y, \boldsymbol{\Sigma}_y)$
- Naïve Bayes assumption?

Which of the following pairs of Gaussian class conditional distributions satisfy the Naïve Bayes assumptions? Select ALL that apply.

A. $\boldsymbol{\mu}_{y=0} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{y=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \boldsymbol{\mu}_{y=1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{y=1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B. $\boldsymbol{\mu}_{y=0} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{y=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \boldsymbol{\mu}_{y=1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{y=1} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

C. $\boldsymbol{\mu}_{y=0} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{y=0} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \qquad \boldsymbol{\mu}_{y=1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{y=1} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

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Piazza Poll 3

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Piazza Poll 3

Iris dataset:

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- Naïve Bayes assumption?

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Decision Boundaries

Iris dataset:

- Class distribution: $Y \sim \ Bern(\phi)$
- Class conditional distribution: $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{\mu}_y, \boldsymbol{\Sigma}_y)$

- Naïve Bayes assumption:

- Linear Decision Boundary:

- Quadradic Decision Boundary:

Introduction to Machine Learning

Intro to Neural Networks

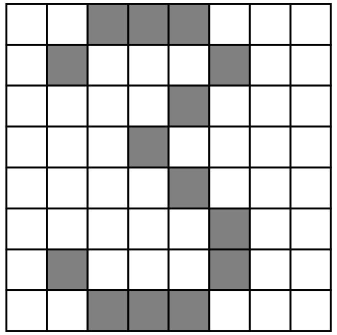
Instructor: Pat Virtue

Neural Networks from HW2

1-D Regression

Neural Networks from HW2

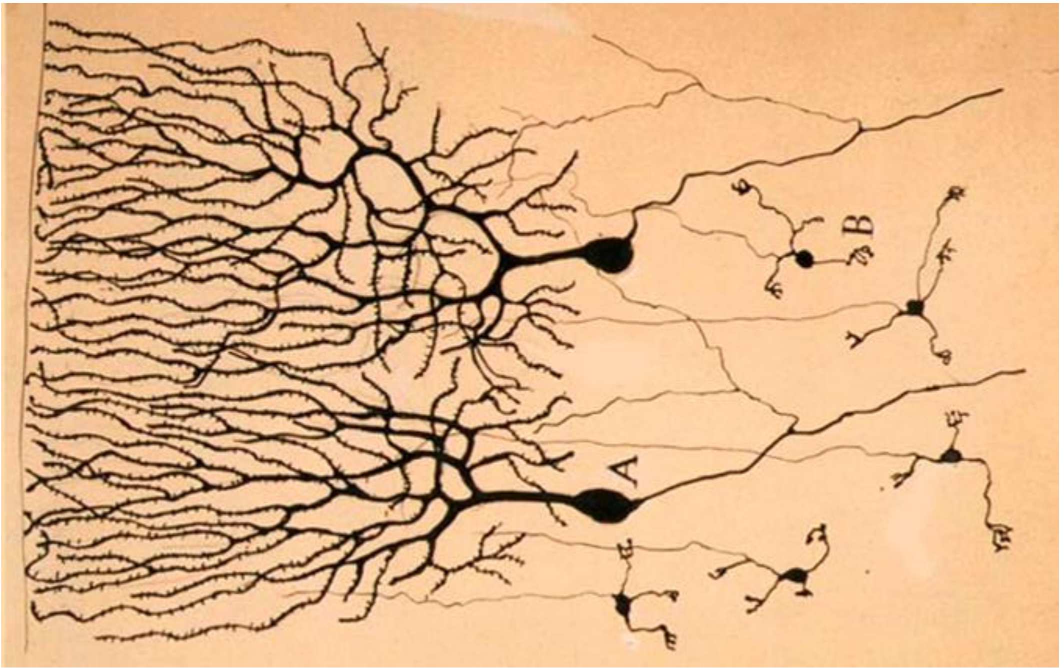
Digit Classification



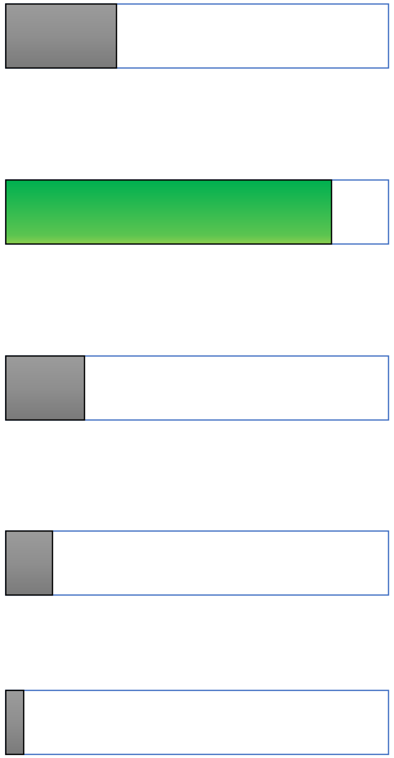
Neural Networks

Inspired by actual human brain

Input Signal



Output Signal



DOG

CAT

TREE

CAR

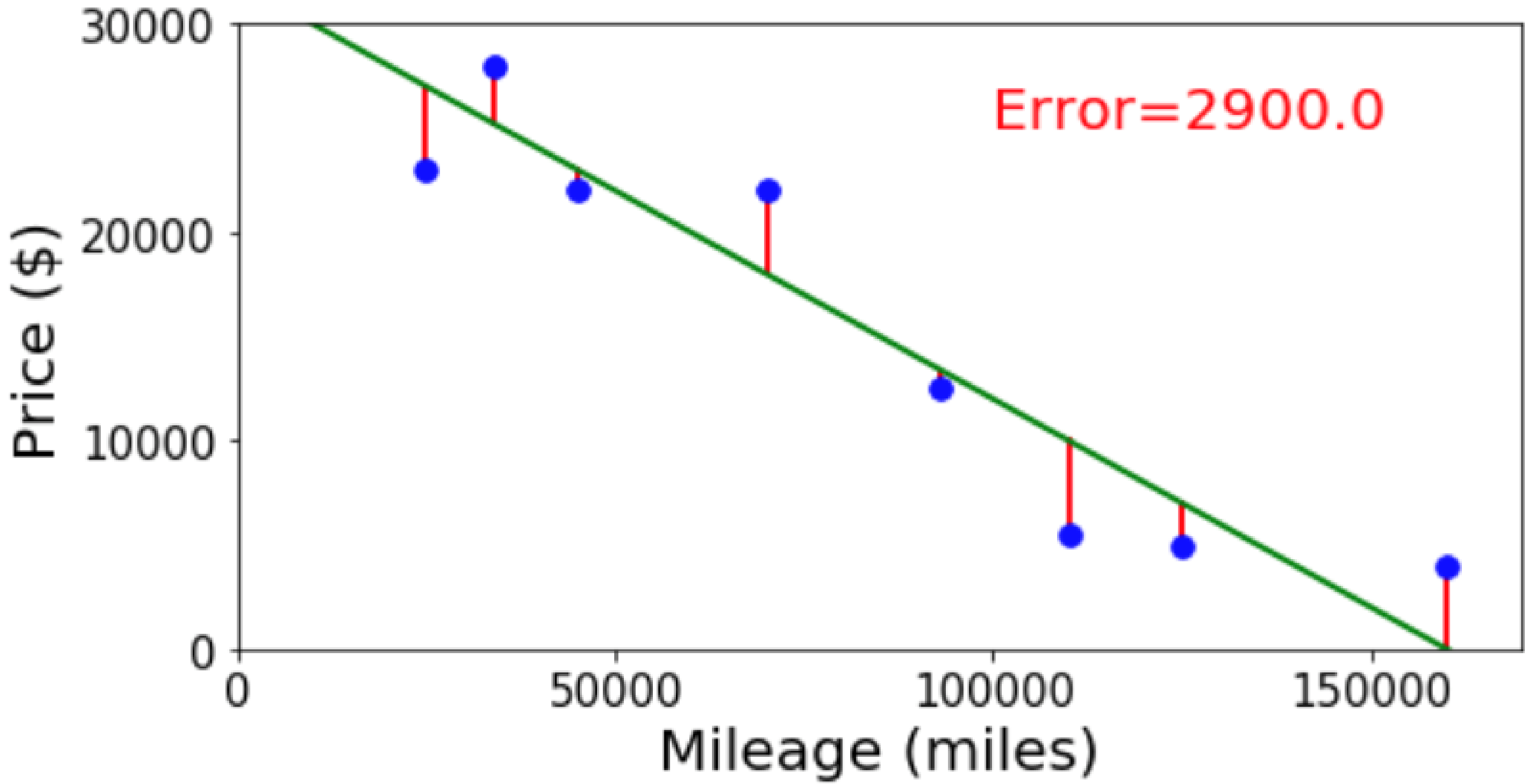
SKY

Image: https://en.wikipedia.org/wiki/Neuron

Neural Networks

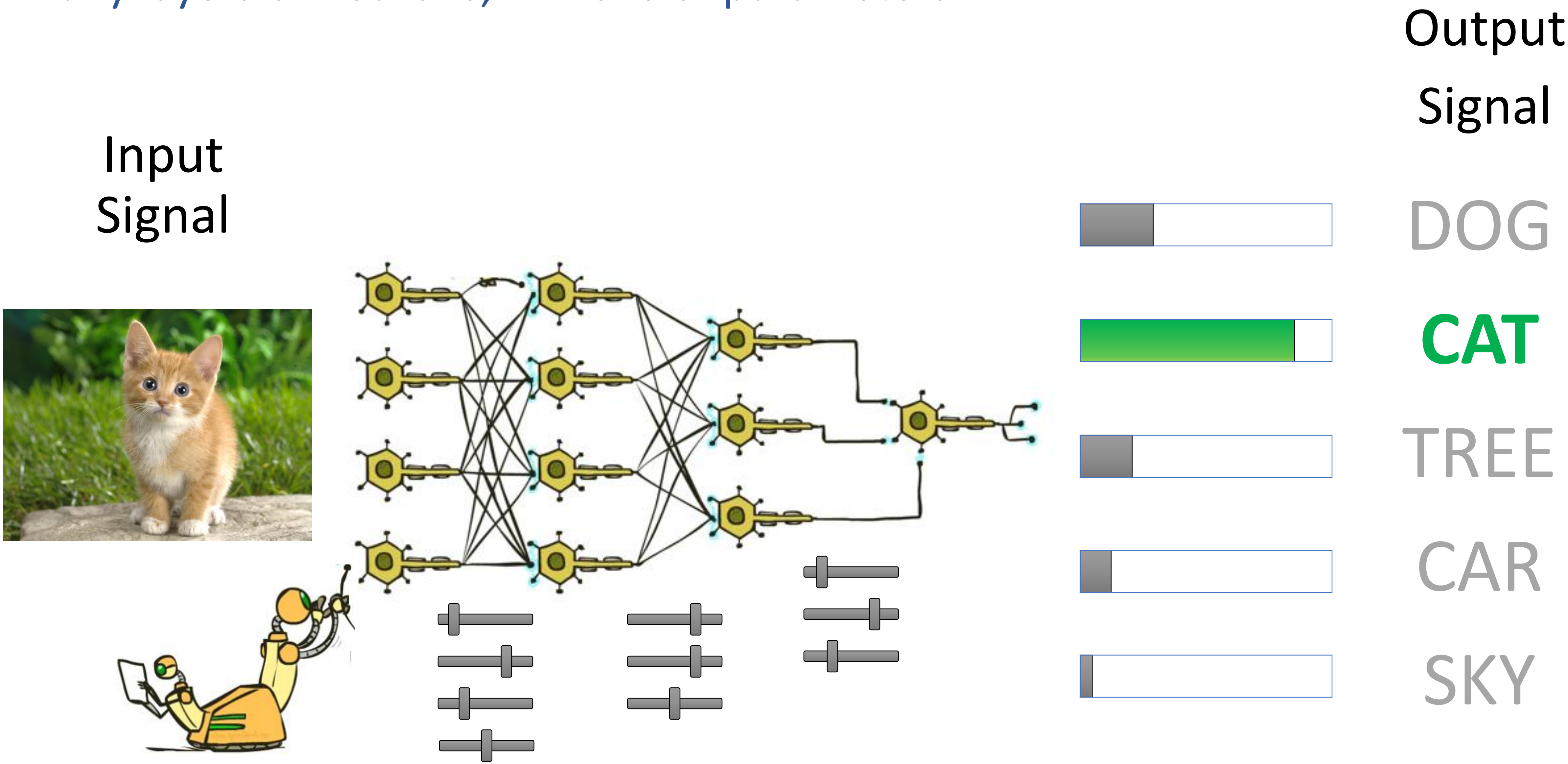
Simple single neuron example:

- Selling my car



Neural Networks

Many layers of neurons, millions of parameters



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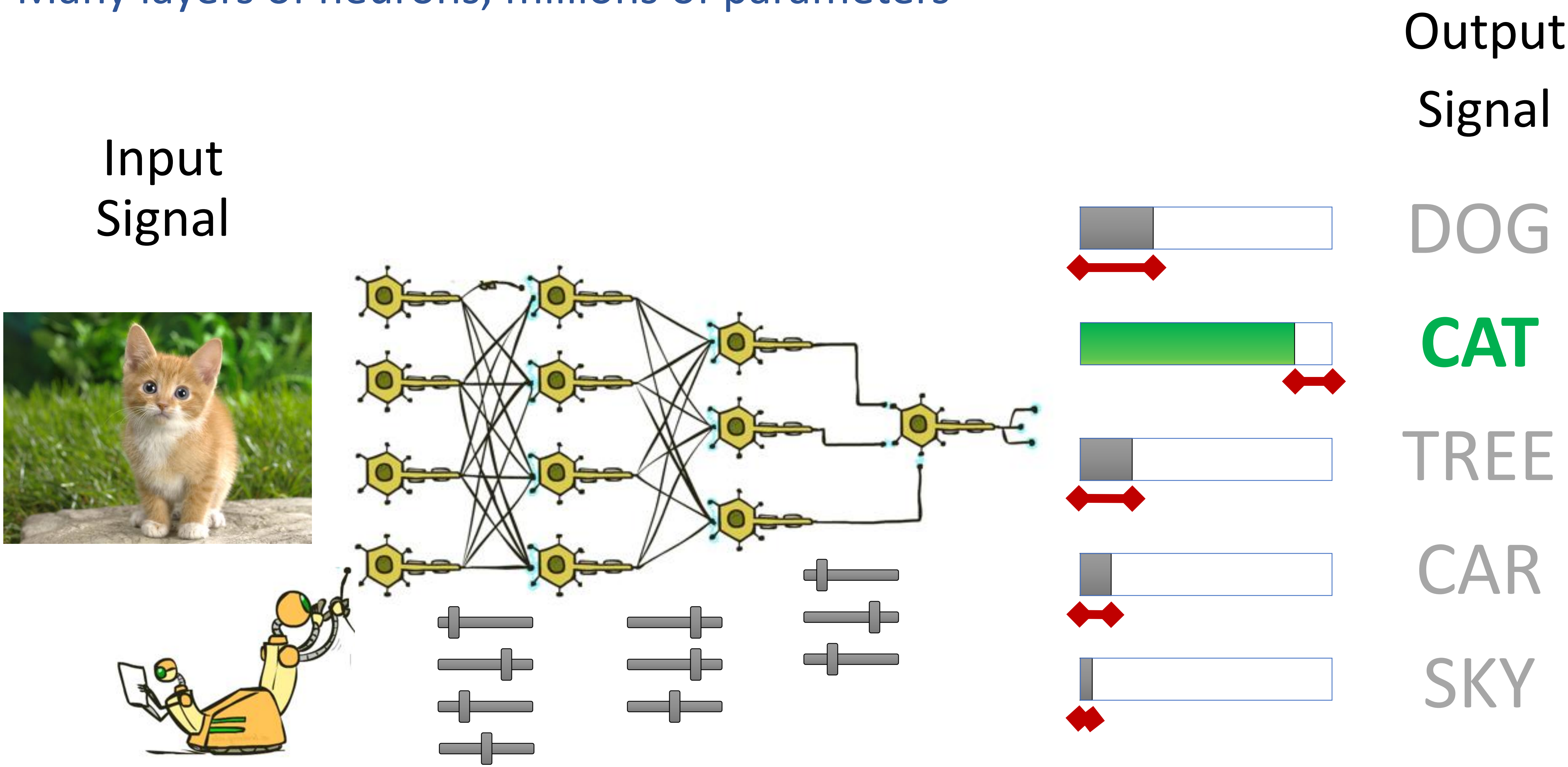
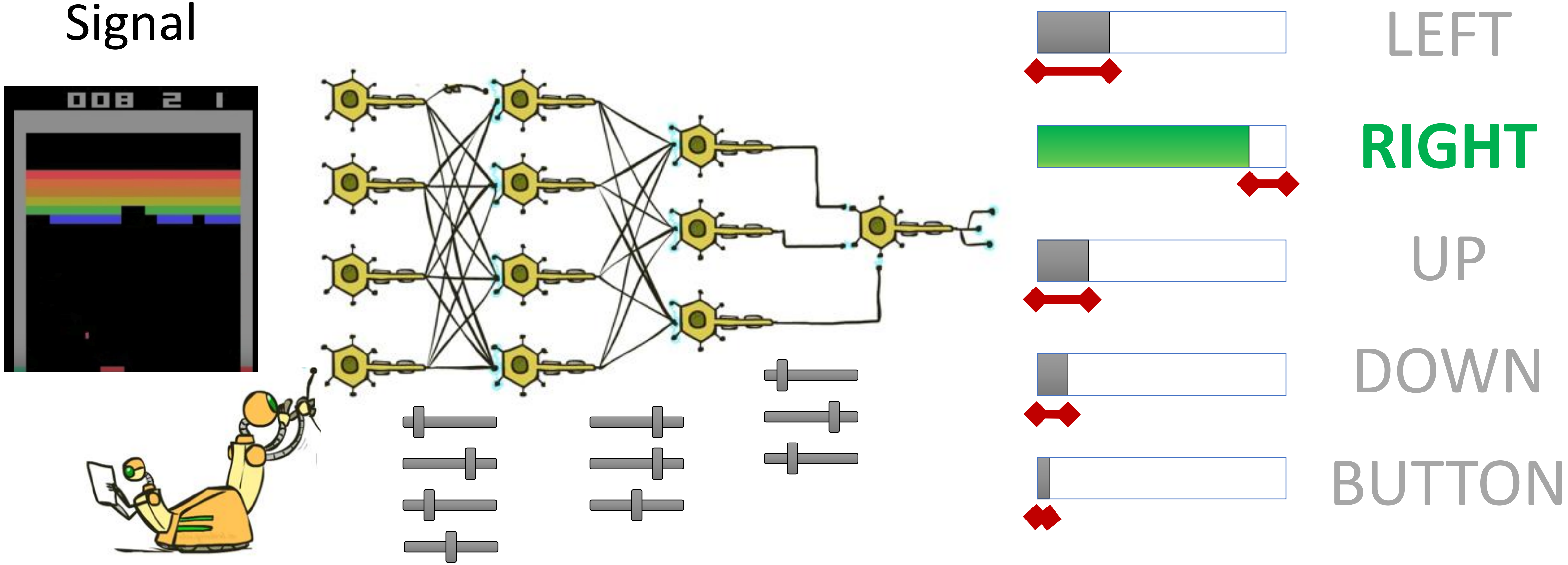


Neural Networks

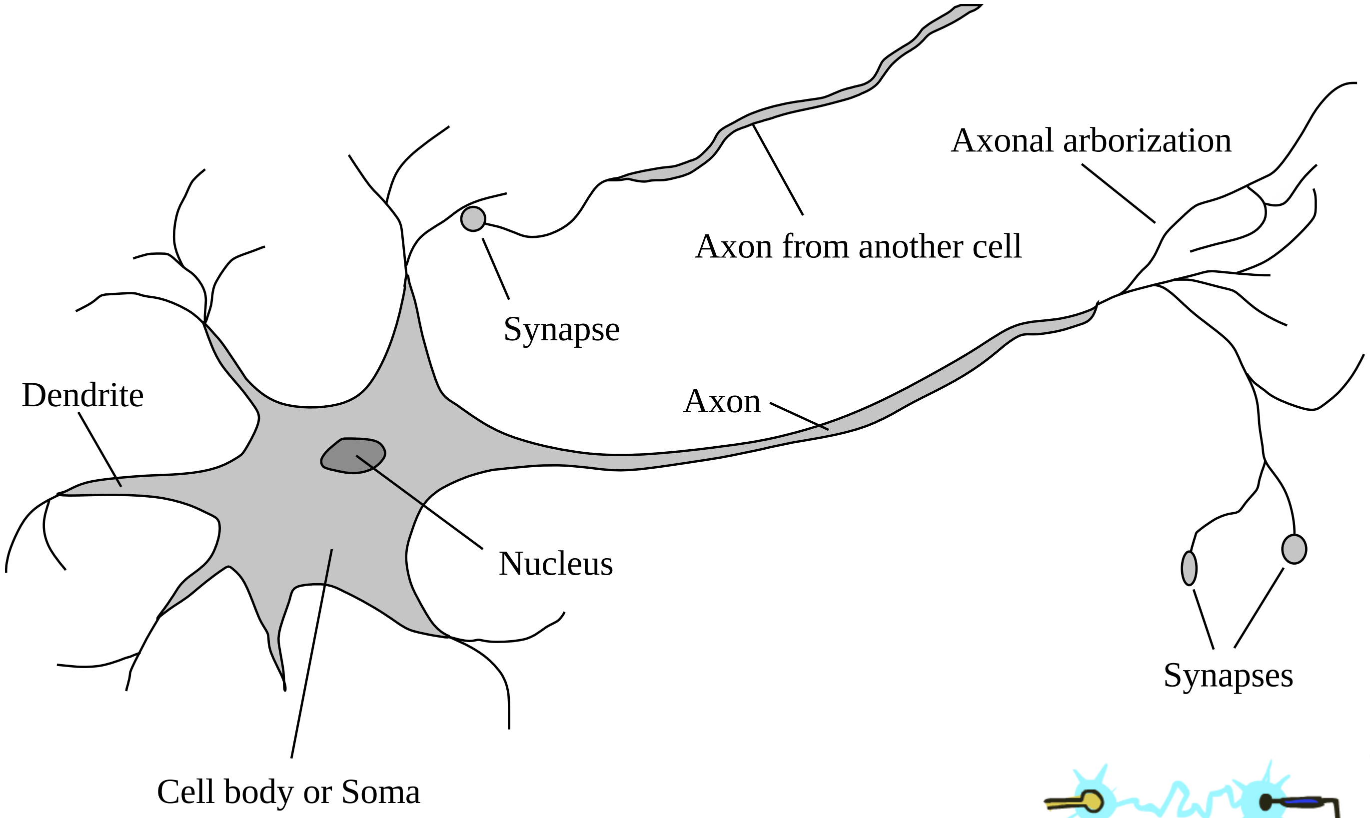
Many layers of neurons, millions of parameters

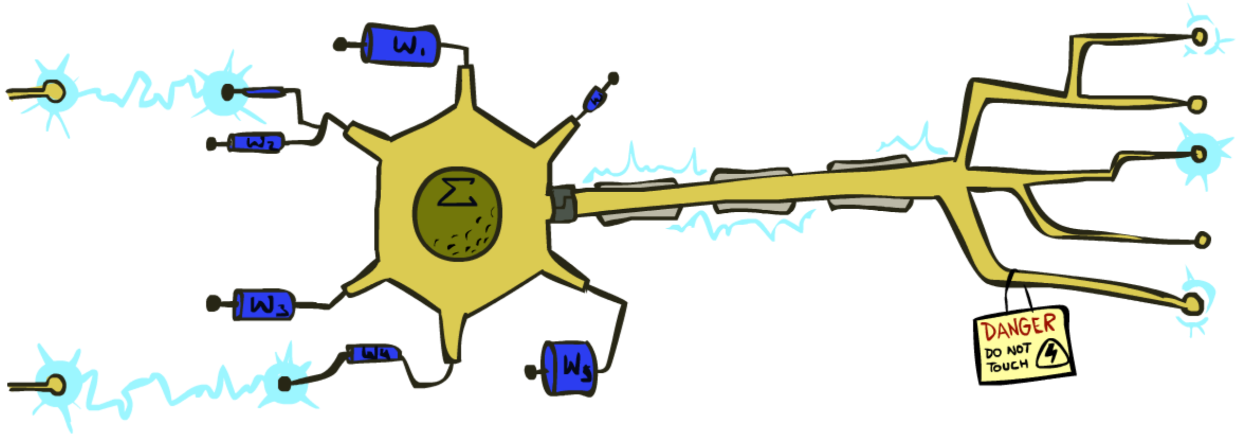
Output Signal

Input Signal



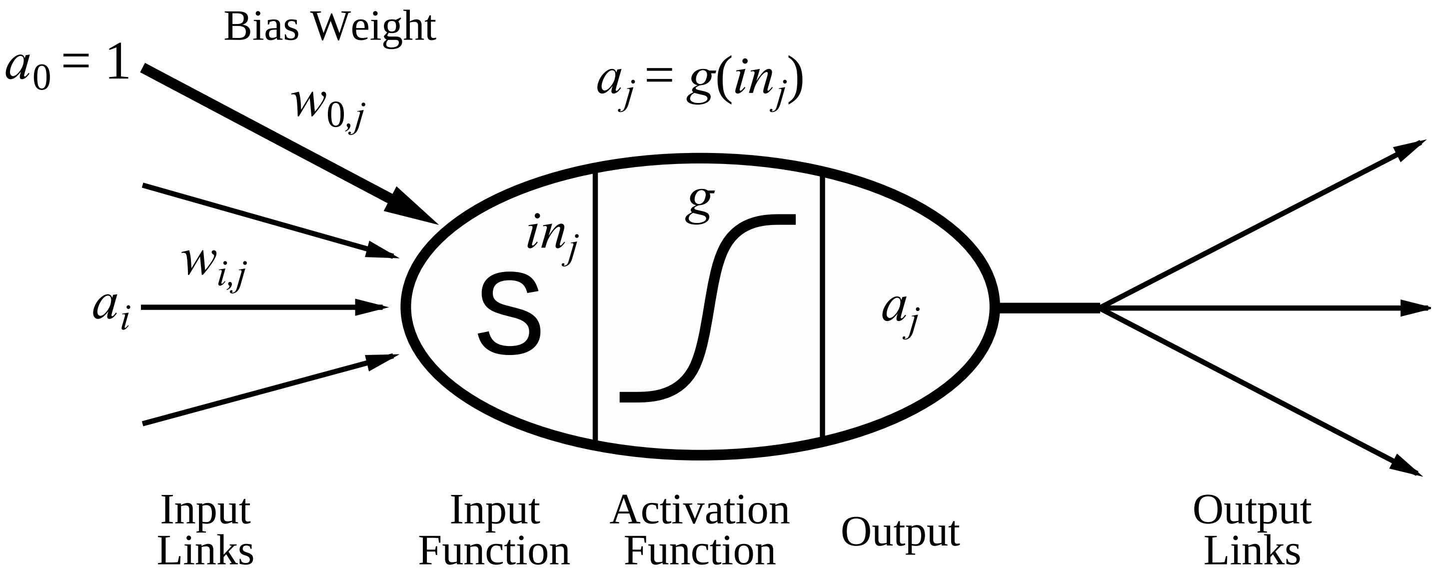
Very Loose Inspiration: Human Neurons





Slide from UC Berkeley AI

Simple Model of a Neuron (McCulloch & Pitts, 1943)



Inputs a_i come from the output of node i to this node j (or from "outside")

Each input link has a ***weight*** $w_{i,j}$

There is an additional fixed input a_0 with ***bias*** weight $w_{0,j}$

The total input is $in_j = \Sigma_i\, w_{i,j}\, a_i$

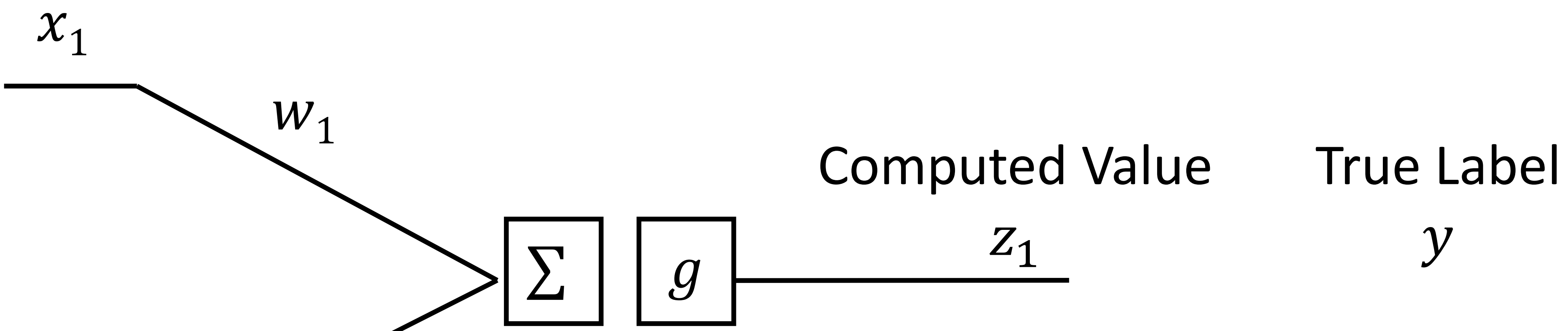
The output is $a_j = g(in_j) = g(\Sigma_i\, w_{i,j}\, a_i) = g(\mathbf{w.a})$

Slide from UC Berkeley AI

Single Neuron

Single neuron system

- Perceptron (if g is step function)
- Logistic regression (if g is sigmoid)



$$h_{\boldsymbol{w}}(\boldsymbol{x}) = z_1$$

$$h_{\boldsymbol{w}}(\boldsymbol{x}) = g\left(\sum_i w_i x_i\right)$$

Optimizing

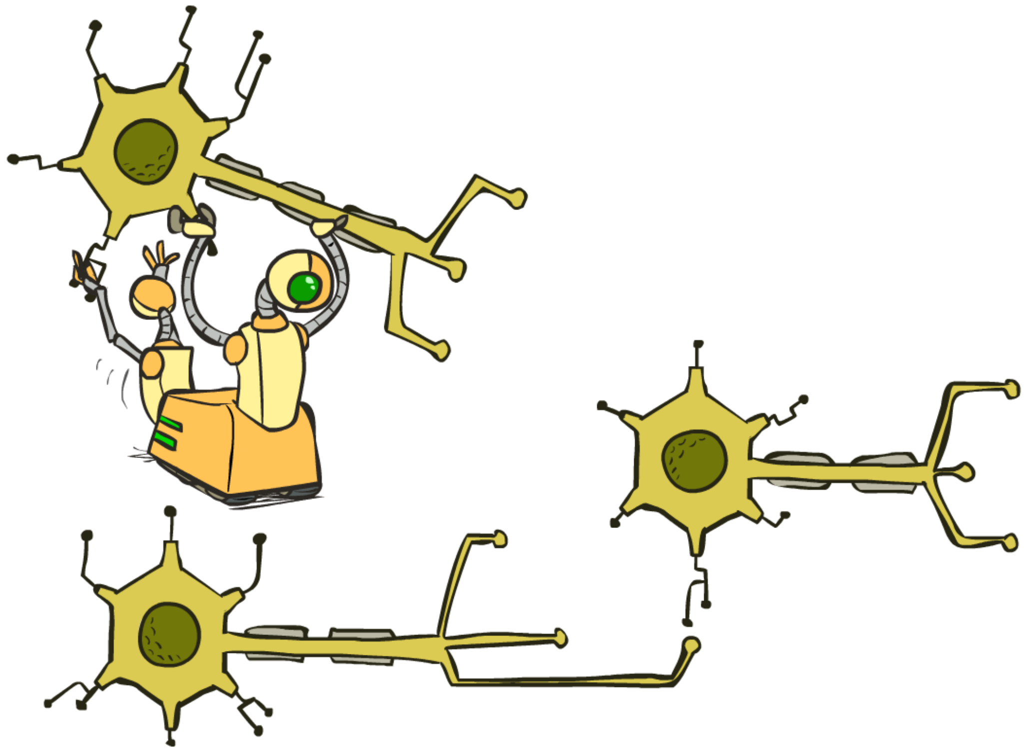
How do we find the "best" set of weights?

$$h_{\boldsymbol{w}}(\boldsymbol{x}) = g\left(\sum_i w_i x_i\right)$$

Multilayer Perceptrons

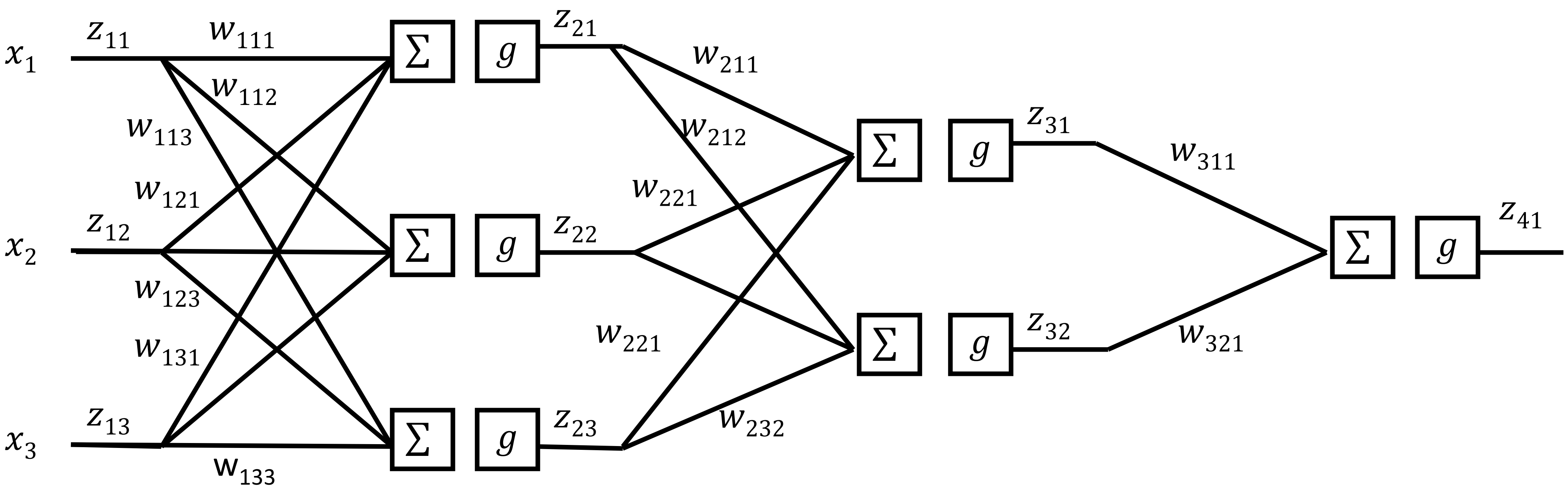
A ***multilayer perceptron*** is a feedforward neural network with at least one ***hidden layer*** (nodes that are neither inputs nor outputs)

MLPs with enough hidden nodes can represent any function



Slide from UC Berkeley AI

Neural Network Equations



$$h_w(\boldsymbol{x}) = z_{4,1}$$

$$z_{1,1} = x_1$$

$$z_{4,1} = g\left(\textstyle\sum_i w_{3,i,1}\, z_{3,i}\right)$$

$$z_{3,1} = g\left(\textstyle\sum_i w_{2,i,1}\, z_{2,i}\right)$$

$$z_{d,1} = g\left(\textstyle\sum_i w_{d-1,i,1}\, z_{d-1,i}\right)$$

$$h_w(x) = g\left(\sum_k w_{3,k,1}\; g\left(\sum_j w_{2,j,k}\; g\left(\sum_i w_{1,i,j}\; x_i\right)\right)\right)$$

Optimizing

How do we find the "best" set of weights?

$$h_w(x) = g\left(\sum_k w_{3,k,1}\ g\left(\sum_j w_{2,j,k}\ g\left(\sum_i w_{1,i,j}\ x_i\right)\right)\right)$$

Neural Networks Properties

Practical considerations

- Large number of neurons
  - Danger for overfitting
- Modelling assumptions vs data assumptions trade-off
- Gradient descent can get stuck in bad local optima

What if there are not non-linear activations?

- A deep neural network with only linear layers can be reduced to an exactly equivalent single linear layer

Universal Approximation Theorem:

- A two-layer neural network with a sufficient number of neurons can approximate any continuous function to any desired accuracy.