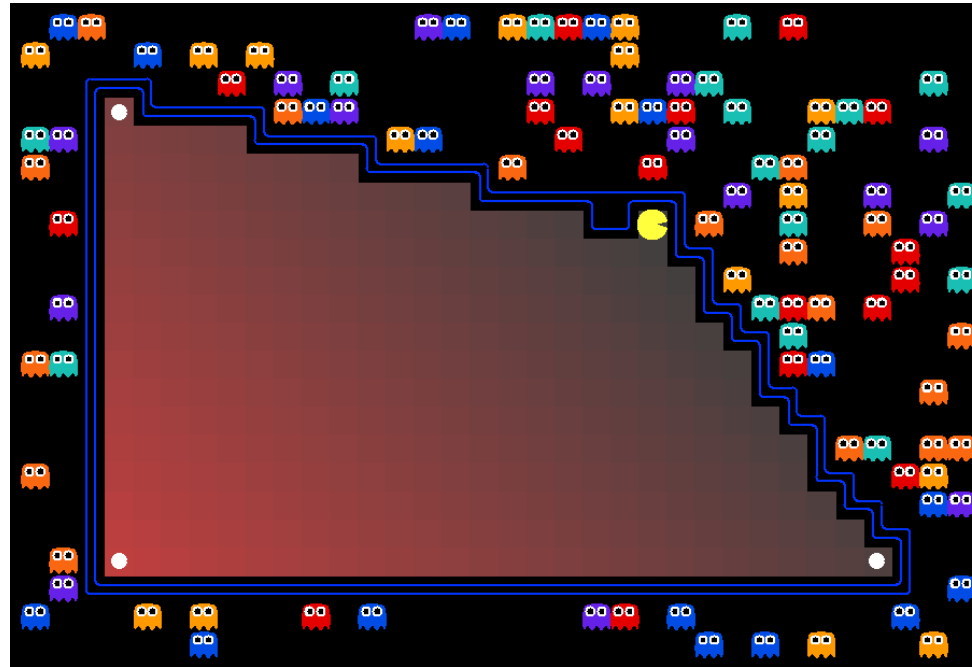


07-380 AI & ML II

Linear Programming



Instructors: Pat Virtue & Mohammad Salameh

Slide credits: CMU AI with drawings from <http://ai.berkeley.edu>

Warm-up: What to eat?

We are trying healthy by finding the optimal amount of food to purchase.

We can choose the amount of **stir-fry** (ounce) and **boba** (fluid ounces).

Healthy Squad Goals

- $2000 \leq \text{Calories} \leq 2500$
- $\text{Sugar} \leq 100 \text{ g}$
- $\text{Calcium} \geq 700 \text{ mg}$

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

What is the cheapest way to stay “healthy” with this menu?

How much **stir-fry** (ounce) and **boba** (fluid ounces) should we buy?

Linear Programming

Notation Alert!
Notation Alert!

Linear objective with linear constraints

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \preceq \mathbf{b} \end{array} \quad \leq \text{ vs. } \preceq$$

"such that"

Key point:

Problem description $\rightarrow$ LP formulation $\rightarrow$ LP Solver

Assume vectors are column vectors

Linear Programming Applications

Many logistics / operations research problems can be modeled as an LP

Gemini summary of the size of the global logistics market:

Market Value Estimates

- **Core Logistics Market:** Valued at roughly **\$4.1 trillion** as of 2025 by [Grand View Research](#), and projected to grow to over \$8.4 trillion by 2033.  Grand View Research
- **Extended Logistics & Infrastructure:** Broad global market estimates reach up to **\$11.2 trillion**, with expectations to climb past \$23 trillion over the next decade according to Precedence Research data.  Yahoo Finance
- **Operations Research (OR):** While standalone market figures for operations research specifically applied to logistics are rarely isolated, broader supply chain analytics, software, and AI integration represent a fast-growing subset driven by e-commerce efficiency needs and automated demand forecasting.

Linear Programming Applications

Many logistics / operations research problems can be modeled as an LP

Gemini summary of the size of the global logistics market:

Market Value Estimates

- **Core Logistics Market:** Valued at roughly **\$4.1 trillion** as of 2025 by [Grand View Research](#), and projected to grow to over \$8.4 trillion by 2033.  Grand View Research

Key Growth Drivers

- **Extended** trillion, with Preceden
- **E-commerce:** Rising consumer demand for fast shipping drives continuous tech and infrastructure upgrades.
- **Operation** specifically software, efficiency
- **Technology:** Companies increasingly adopt AI and advanced analytics for routing, inventory, and demand prediction.
- **Labor Demand:** Employment for trained professionals like [BLS Logisticians](#) is projected to grow 18% from 2025 to 2035.  Bureau of Labor Statisti... +4

Optimization: Constraint Satisfaction Problems

Map coloring

Any $\mathbf{x}$

s.t. $\mathbf{x}$ satisfies constraints



Linear Programming

Linear objective with linear constraints

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

Linear regression optimization?

As opposed to general optimization

Neural net optimization?

$$\begin{array}{ll}\min_{\mathbf{x}} & f_0(\mathbf{x}) \\ \text{s.t.} & f_i(\mathbf{x}) \leq 0, \quad i = 1 \dots M \\ & \mathbf{a}_i^\top \mathbf{x} = \mathbf{b}_i, \quad i = 1 \dots P\end{array}$$

Linear Programming

Different formulations

Inequality form

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

General form

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} + \mathbf{d} \\ \text{s.t.} & G\mathbf{x} \leq \mathbf{h} \\ & A\mathbf{x} = \mathbf{b}\end{array}$$

Standard form

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq 0\end{array}$$

Important to pay attention to form!

We'll always use inequality form unless noted otherwise

Can switch between formulations!

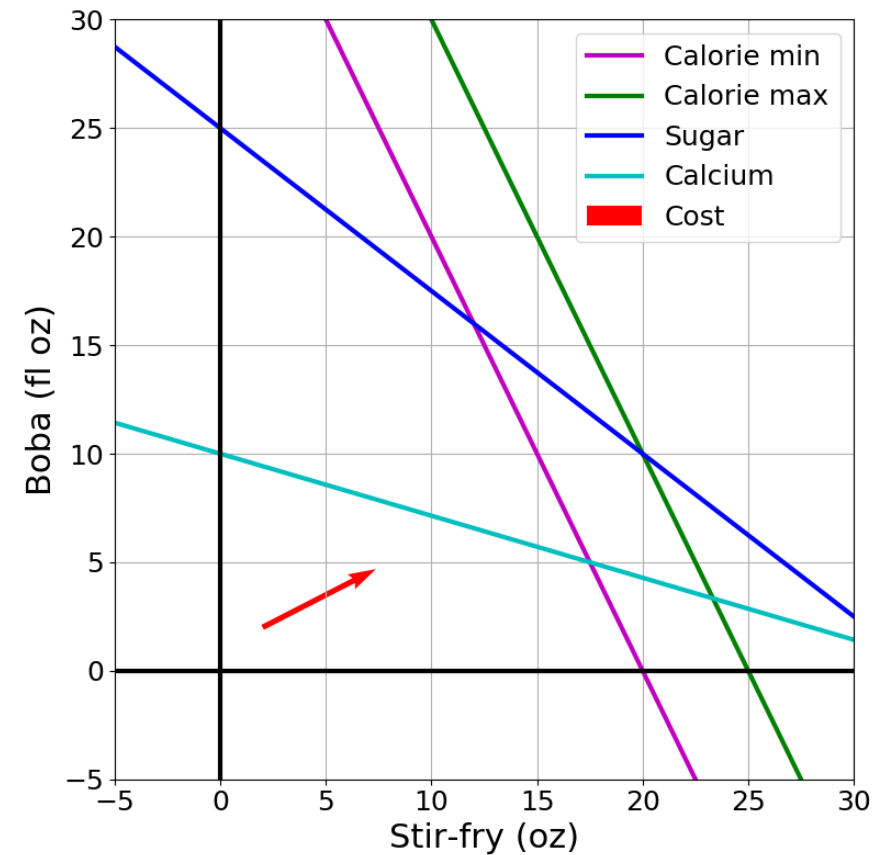
Optimization

Problem
Description

Optimization
Representation

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

Graphical Representation



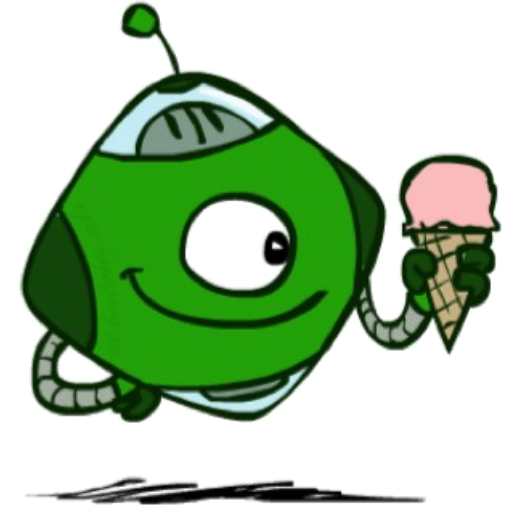
Pre-reading

Optimization formulation

Poll 1

What has to increase to add more nutrition constraints?

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$



Select all that apply

- A) length $\mathbf{x}$
- B) length $\mathbf{c}$
- C) height A
- D) width A
- E) length $\mathbf{b}$

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

Poll 1

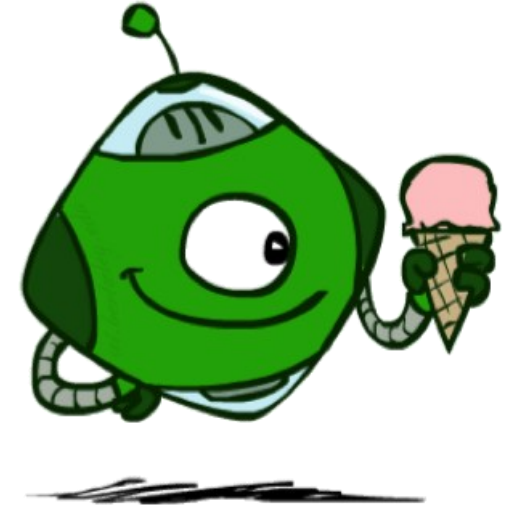
What has to increase to add more nutrition constraints?

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$A = \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix}$$

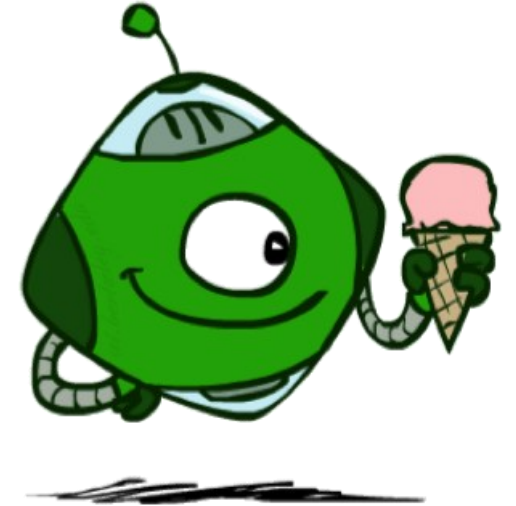
$$\mathbf{b} = \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix}$$



Poll 2

What has to increase to add more menu items?

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$



Select all that apply

- A) length $\mathbf{x}$
- B) length $\mathbf{c}$
- C) height A
- D) width A
- E) length $\mathbf{b}$

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

Poll 2

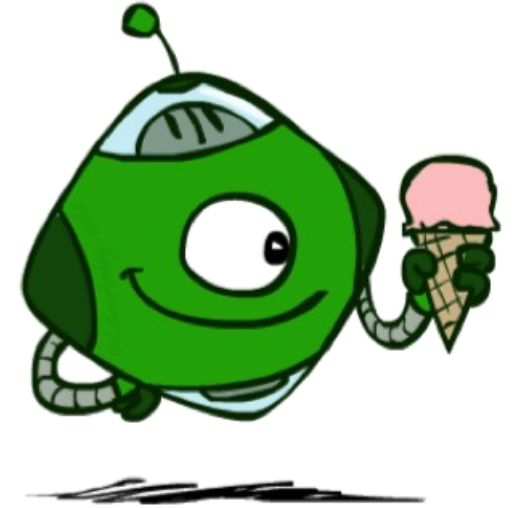
What has to increase to add more menu items?

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$$A = \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix}$$

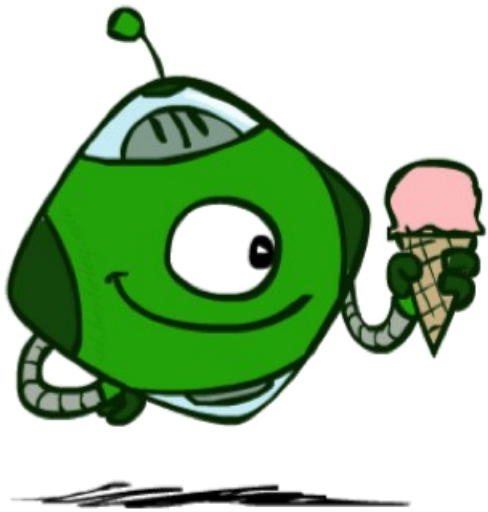
$$\mathbf{b} = \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix}$$



Optimization Formulation

Diet Problem

Any $\mathbf{x}$
s.t. $\mathbf{x}$ satisfies constraints



Healthy Squad Goals

- $2000 \leq \text{Calories} \leq 2500$
- $\text{Sugar} \leq 100 \text{ g}$
- $\text{Calcium} \geq 700 \text{ mg}$

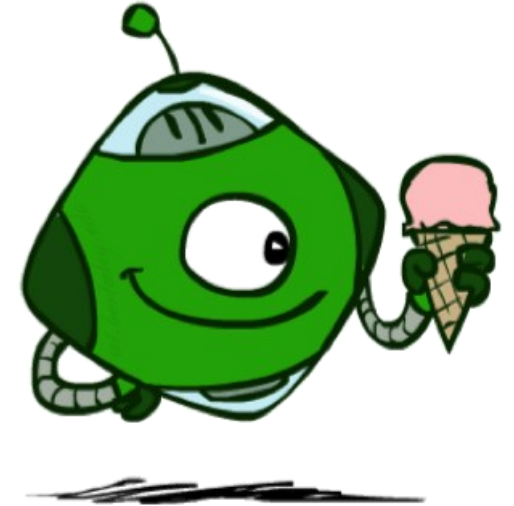
Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

Optimization Formulation

Diet Problem

$\min_{\mathbf{x}} \quad cost(\mathbf{x})$ **Objective function**

s.t. $\mathbf{x}$ satisfies constraints



Healthy Squad Goals

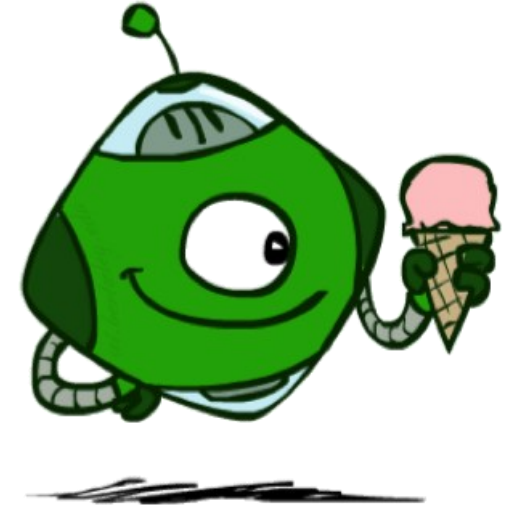
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Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

Optimization Formulation

Diet Problem

$$\begin{array}{ll}\min_{\mathbf{x}} & cost(\mathbf{x}) \\ \text{s.t.} & calories(\mathbf{x}) \text{ contained} \\ & sugar(\mathbf{x}) \leq limit \\ & calcium(\mathbf{x}) \geq limit\end{array}$$



Healthy Squad Goals

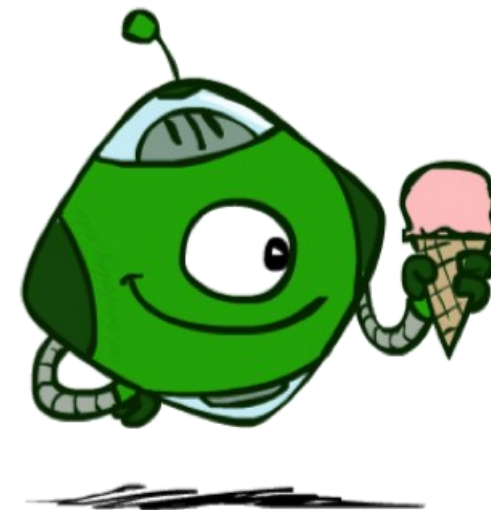
- $2000 \leq \text{Calories} \leq 2500$
- $\text{Sugar} \leq 100 \text{ g}$
- $\text{Calcium} \geq 700 \text{ mg}$

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

Optimization Formulation

Diet Problem

$$\begin{array}{ll}\min_{x_1, x_2} & 1 x_1 + 0.5 x_2 \\ \text{s.t.} & 100 x_1 + 50 x_2 \geq 2000 \\ & 100 x_1 + 50 x_2 \leq 2500 \\ & 3 x_1 + 4 x_2 \leq 100 \\ & 20 x_1 + 70 x_2 \geq 700\end{array}$$



Healthy Squad Goals

- $2000 \leq \text{Calories} \leq 2500$
- $\text{Sugar} \leq 100 \text{ g}$
- $\text{Calcium} \geq 700 \text{ mg}$

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
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Optimization Formulation

Diet Problem

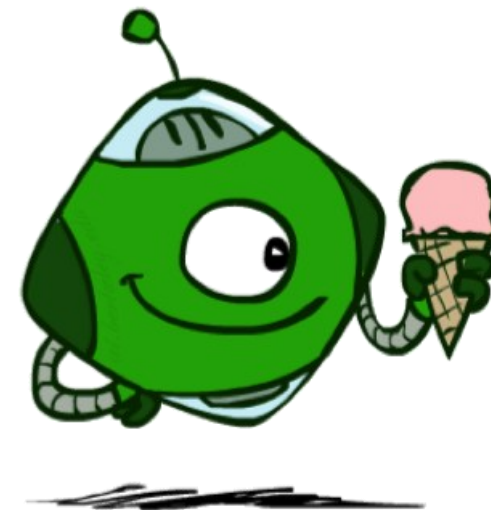
$$\min_{x_1, x_2} \quad c_1 x_1 + c_2 x_2$$

$$\begin{aligned} \text{s.t.} \quad & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4 \end{aligned}$$

$$A = \begin{array}{cc} & \begin{array}{c} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$\mathbf{b} = \begin{array}{cc} & \begin{array}{c} \text{Limit} \end{array} \\ \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} & \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array} \end{array}$$



Optimization Formulation

Diet Problem

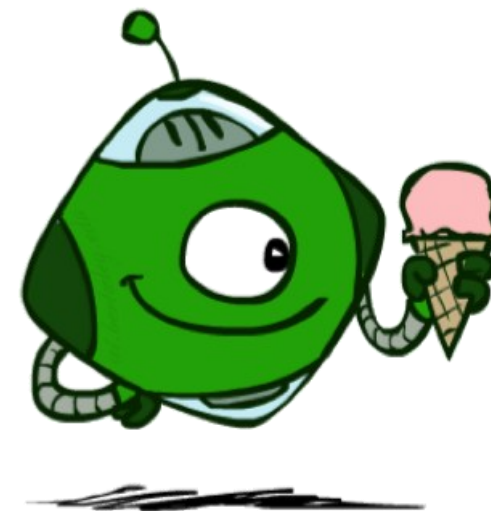
$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

$$\begin{aligned} \text{s.t.} \quad & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4 \end{aligned}$$

$$A = \begin{matrix} & \text{Stir-fry} & \text{Boba} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{matrix}$$

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} \begin{matrix} \text{Limit} \\ \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{matrix}$$



Optimization Formulation

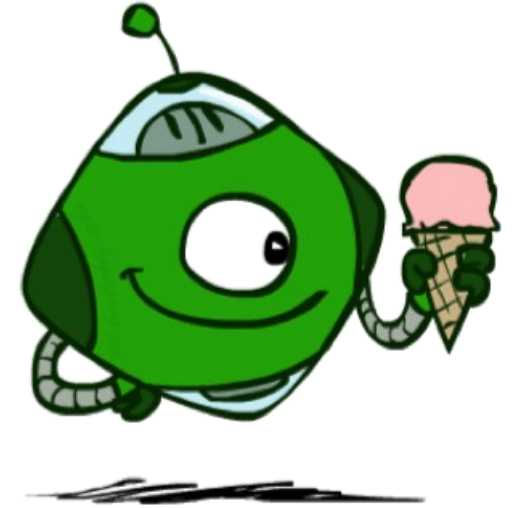
Diet Problem

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

$$\begin{aligned} \text{s.t.} \quad & -a_{1,1} x_1 - a_{1,2} x_2 \leq -b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & -a_{4,1} x_1 - a_{4,2} x_2 \leq -b_4 \end{aligned}$$

$$A = \begin{array}{cc} & \begin{array}{c} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$

$$\mathbf{b} = \begin{array}{cc} & \text{Limit} \\ \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} & \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array} \end{array}$$



Cost

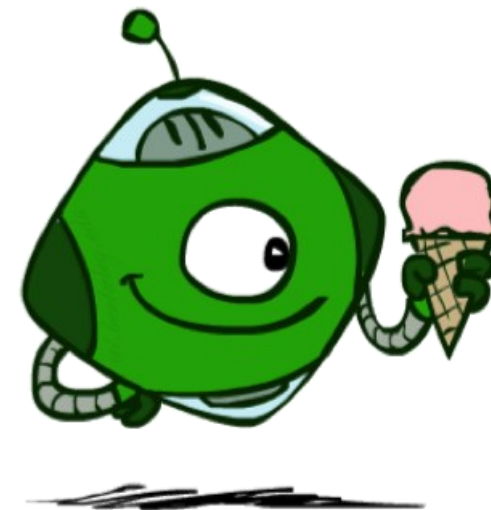
$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Optimization Formulation

Diet Problem

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

$$\begin{aligned} \text{s.t.} \quad & a_{1,1} x_1 + a_{1,2} x_2 \leq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \leq b_4 \end{aligned}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

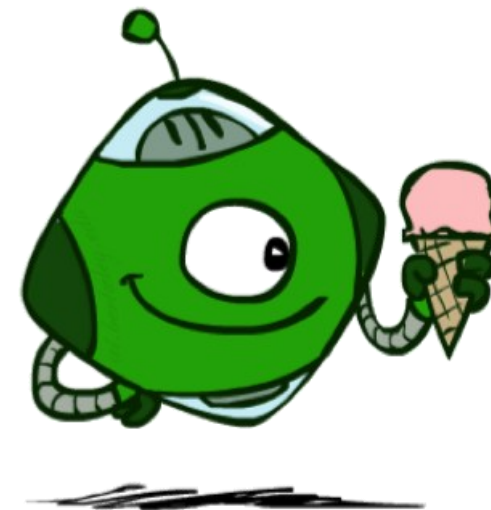
Limit

$$A = \begin{matrix} & \text{Stir-fry} & \text{Boba} \\ \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix} & \mathbf{b} = \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix} & \begin{matrix} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{matrix} \end{matrix}$$

Optimization Formulation

Diet Problem

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Limit

$$A = \begin{array}{cc} \text{Stir-fry} & \text{Boba} \\ \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix} & \mathbf{b} = \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix} \end{array}$$

Calorie min
Calorie max
Sugar
Calcium

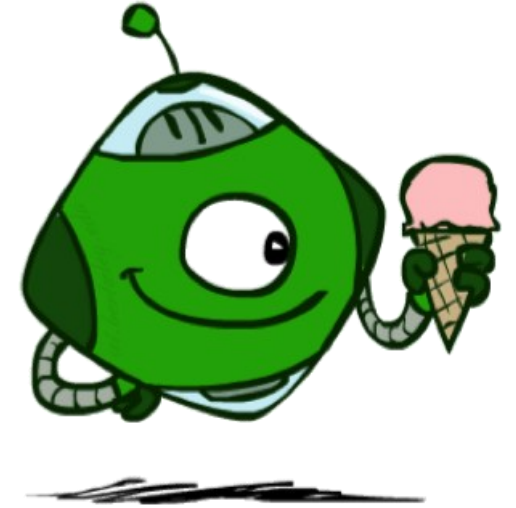
Question

If $A \in \mathbb{R}^{M \times N}$, which of the following also equals N ?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

Select all that apply

- A) length $\mathbf{x}$
- B) length $\mathbf{c}$
- C) length $\mathbf{b}$



Plan

Pre-reading (with some polls today)

Linear programming (LP) formulation

- Problem description
- Optimization representation
- Graphical representation

Today

- LP: Continue graphical representation
- Solving linear programs
- Higher dimensions than just 2

Today

- Integer programming (IP)

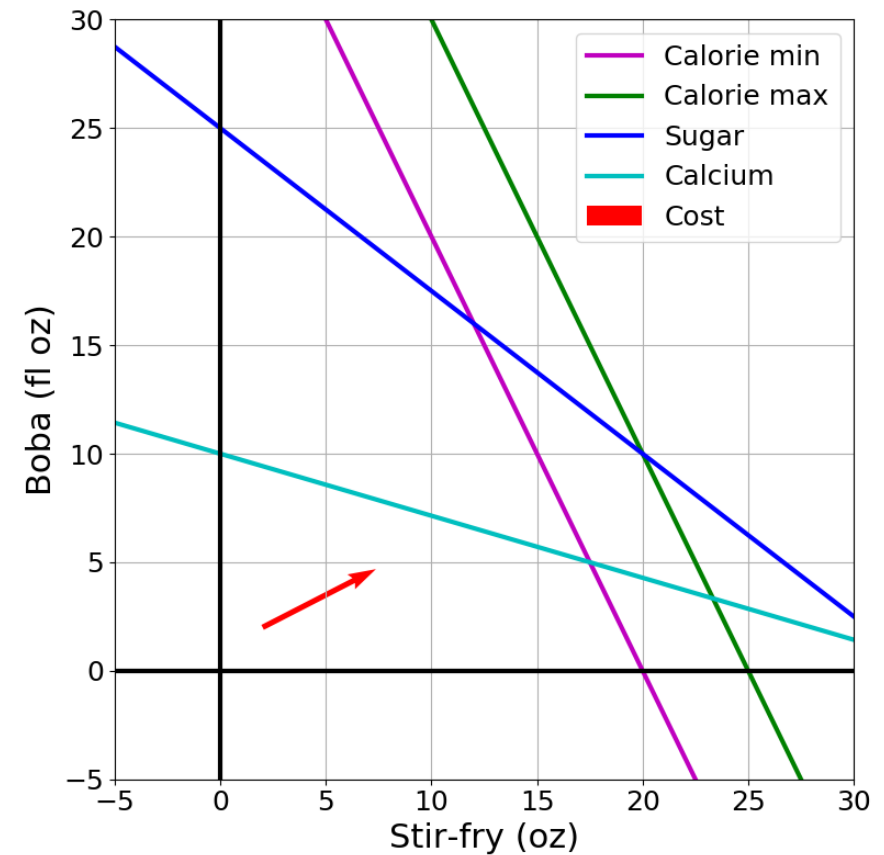
Optimization

Problem
Description

Optimization
Representation

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

Graphical Representation



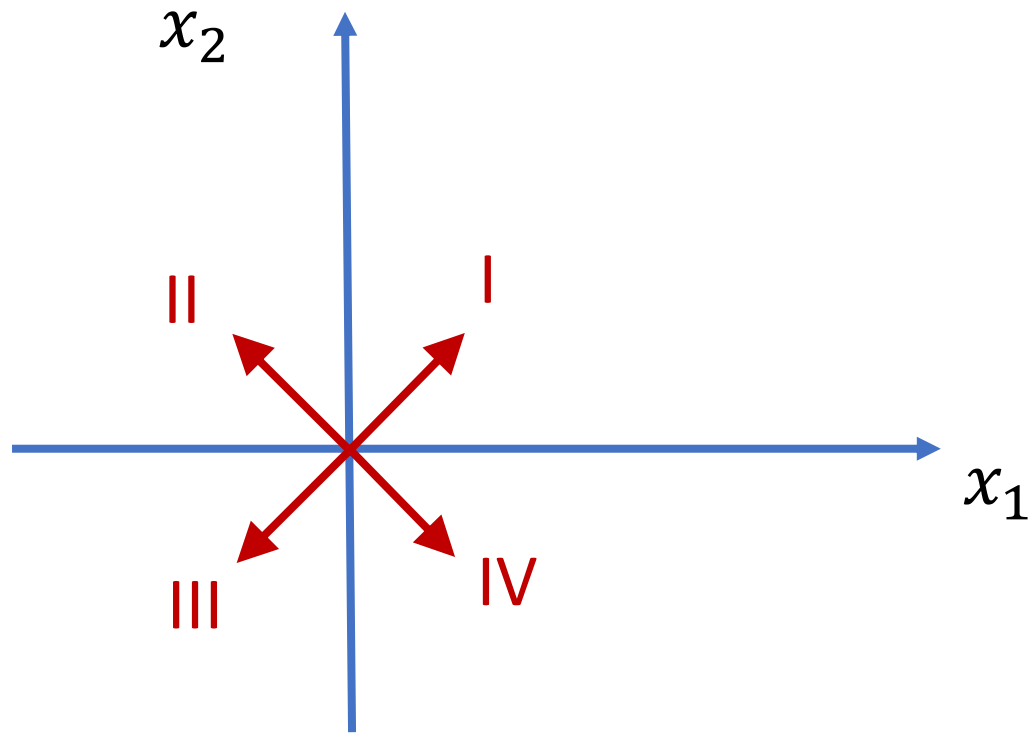
Pre-reading

Graphical Representation (2-D)

Poll 3

Which of these points have cost $\mathbf{c}^\top \mathbf{x} = 0$?

for cost vector: $\mathbf{c} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$



Question

Given the cost vector $[c_1, c_2]^T$ where will

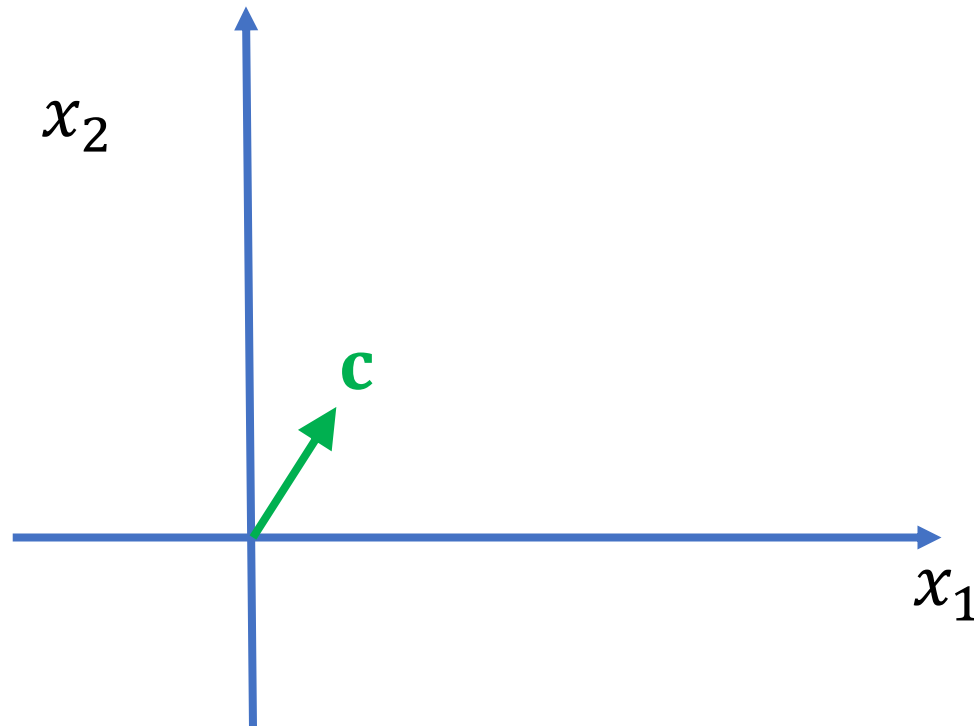
$$\mathbf{c}^T \mathbf{x} = 0 ?$$

$$\mathbf{c}^T \mathbf{x} = 1 ?$$

$$\mathbf{c}^T \mathbf{x} = 2 ?$$

$$\mathbf{c}^T \mathbf{x} = -1 ?$$

$$\mathbf{c}^T \mathbf{x} = -2 ?$$



Cost Contours

Given the cost vector $[c_1, c_2]^T$ where will

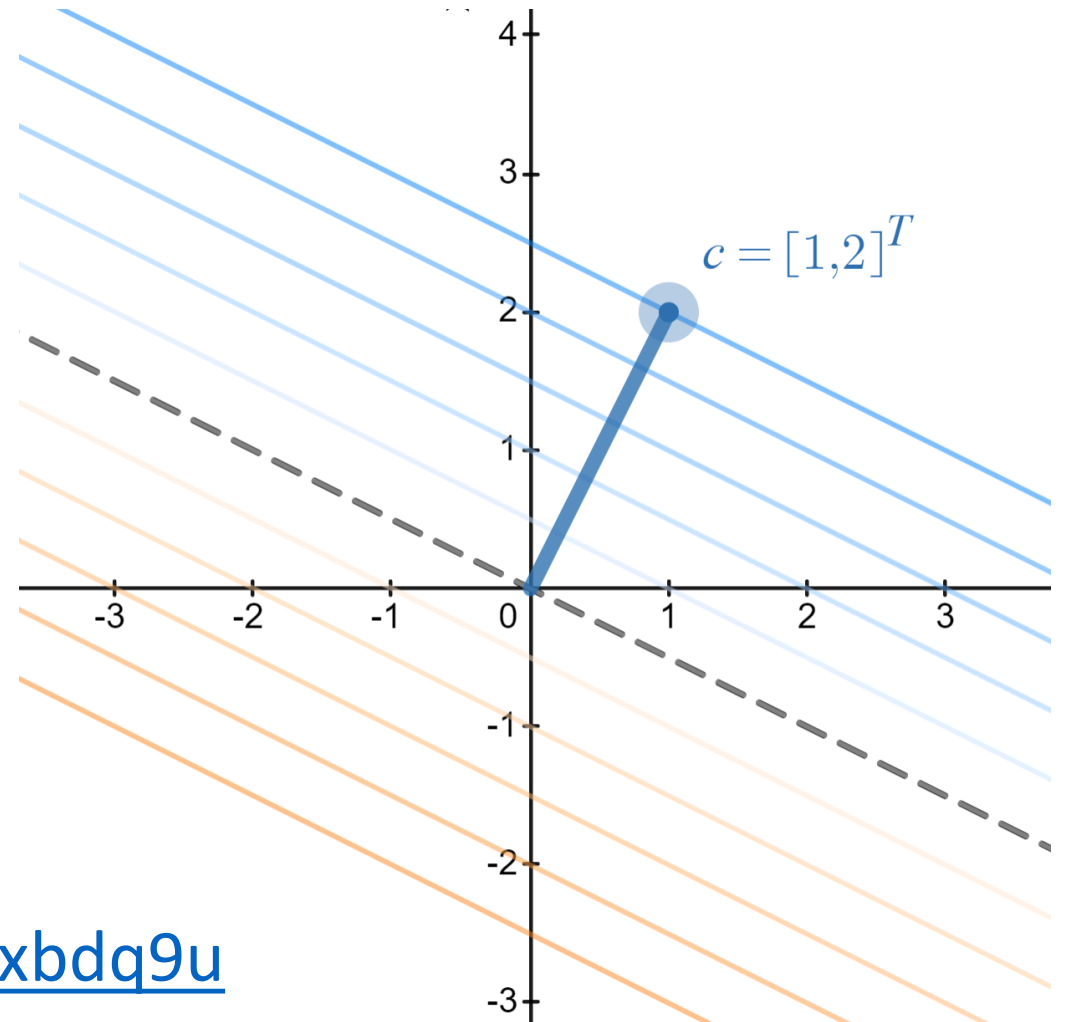
$$\mathbf{c}^T \mathbf{x} = 0 ?$$

$$\mathbf{c}^T \mathbf{x} = 1 ?$$

$$\mathbf{c}^T \mathbf{x} = 2 ?$$

$$\mathbf{c}^T \mathbf{x} = -1 ?$$

$$\mathbf{c}^T \mathbf{x} = -2 ?$$

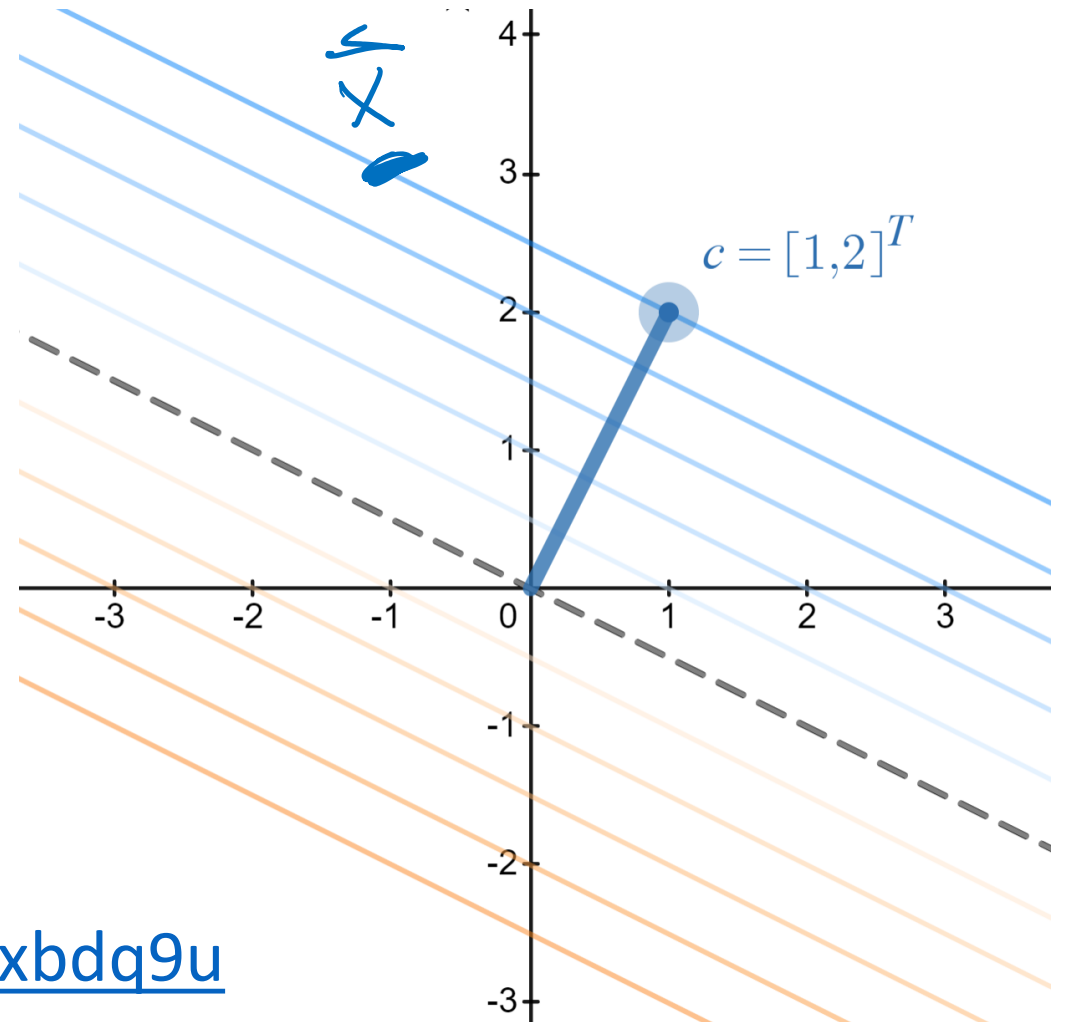


<https://www.desmos.com/calculator/8d9kxbdq9u>

Question

As the magnitude of $\mathbf{c}$ increases, the distance between the contours lines of the objective $\mathbf{c}^T \mathbf{x}$:

- A) Increases
- B) Decreases



<https://www.desmos.com/calculator/8d9kxbdq9u>

Graphics Representation

Geometry / Algebra I Quiz

What shape does this inequality represent?

$$a_1 x_1 + a_2 x_2 \leq b_1$$

Graphics Representation

Geometry / Algebra I Quiz

What shape do these represent?

1. $a_1 x_1 + a_2 x_2 = b_1$

2. $a_1 x_1 + a_2 x_2 \leq b_1$

3. $a_{1,1} x_1 + a_{1,2} x_2 \leq b_1$
 $a_{2,1} x_1 + a_{2,2} x_2 \leq b_2$
 $a_{3,1} x_1 + a_{3,2} x_2 \leq b_3$
 $a_{4,1} x_1 + a_{4,2} x_2 \leq b_4$

Feasible region:

All points x that satisfy the constraints

Graphics Representation

Geometry / Algebra I Quiz

What shape do these represent?

1. $a_1 x_1 + a_2 x_2 = b_1$

2. $a_1 x_1 + a_2 x_2 \leq b_1$

3. $a_{1,1} x_1 + a_{1,2} x_2 \leq b_1$
 $a_{2,1} x_1 + a_{2,2} x_2 \leq b_2$
 $a_{3,1} x_1 + a_{3,2} x_2 \leq b_3$
 $a_{4,1} x_1 + a_{4,2} x_2 \leq b_4$

Feasible region:

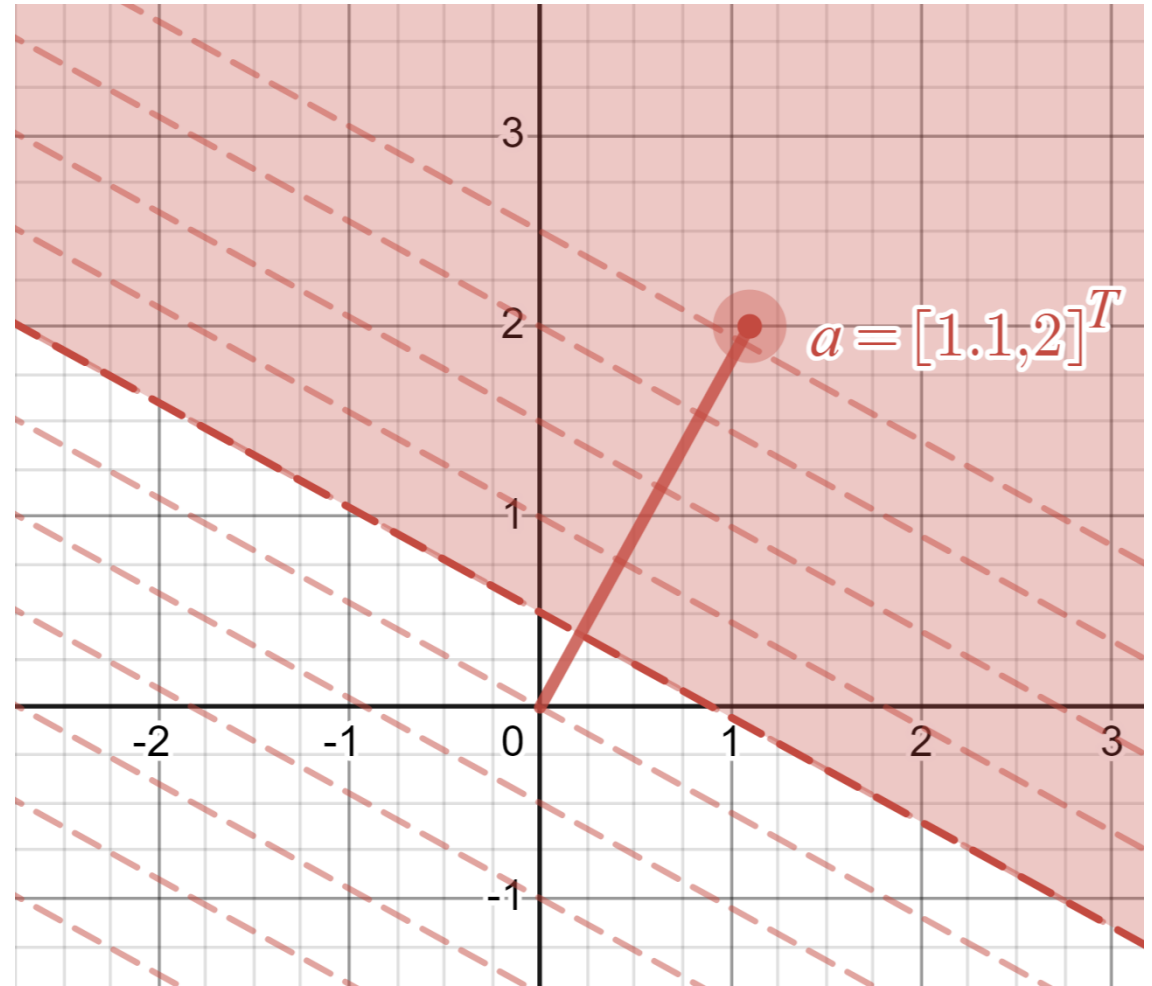
All points x that satisfy the constraints

Graphics Representation

Geometry / Algebra I Quiz

What shape do these represent?

1. $a_1 x_1 + a_2 x_2 = b_1$
2. $a_1 x_1 + a_2 x_2 \leq b_1$
- 3.



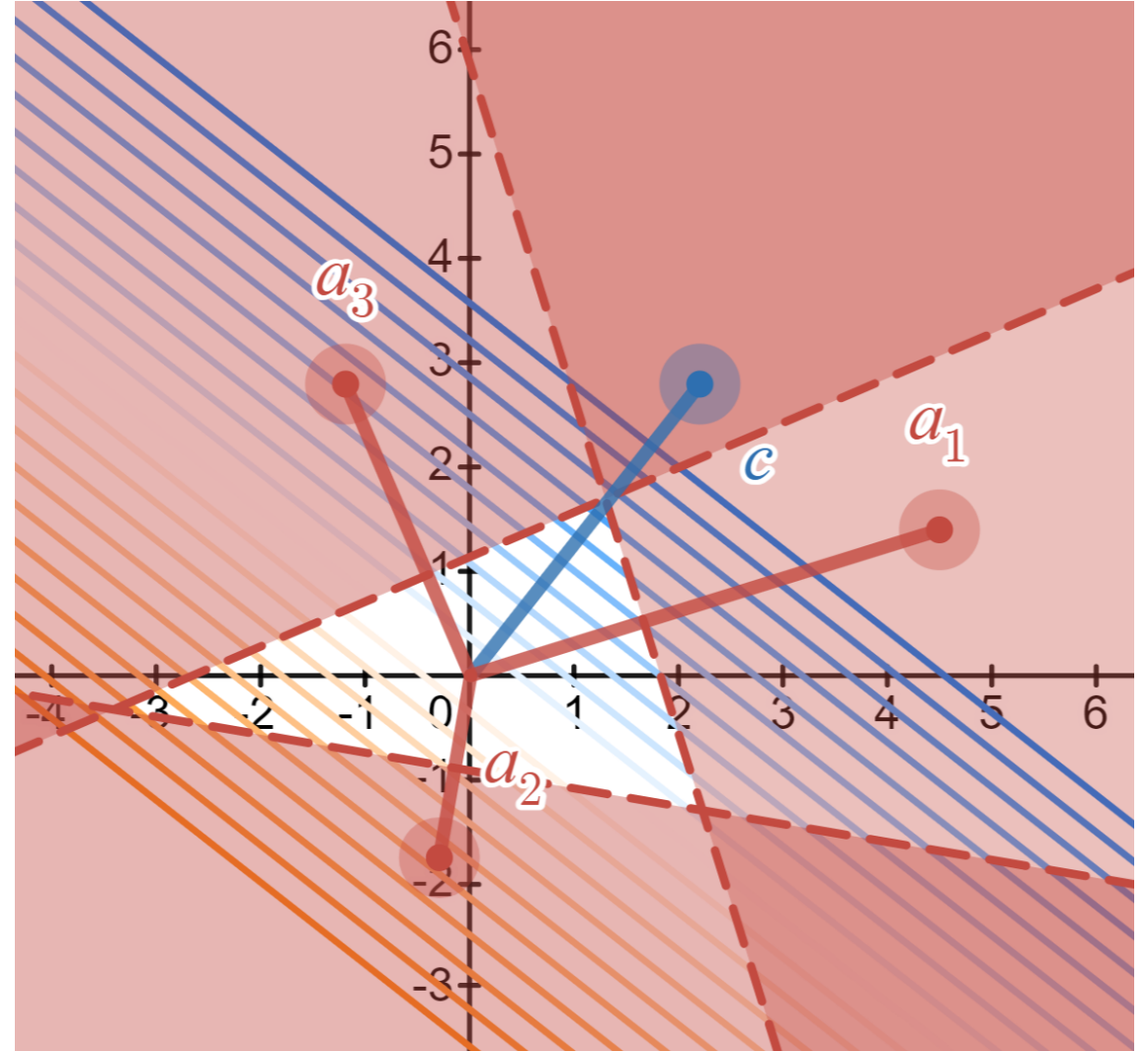
<https://www.desmos.com/calculator/lp0rqsb1w6>

Graphics Representation

Geometry / Algebra I Quiz

What shape do these represent?

1. $a_1 x_1 + a_2 x_2 = b_1$
2. $a_1 x_1 + a_2 x_2 \leq b_1$
3.
$$\begin{aligned} a_{1,1} x_1 + a_{1,2} x_2 &\leq b_1 \\ a_{2,1} x_1 + a_{2,2} x_2 &\leq b_2 \\ a_{3,1} x_1 + a_{3,2} x_2 &\leq b_3 \\ a_{4,1} x_1 + a_{4,2} x_2 &\leq b_4 \end{aligned}$$



<https://www.desmos.com/calculator/plp1thgsbh>

Reminder: Cost Contours

Given the cost vector $[c_1, c_2]^T$ where will

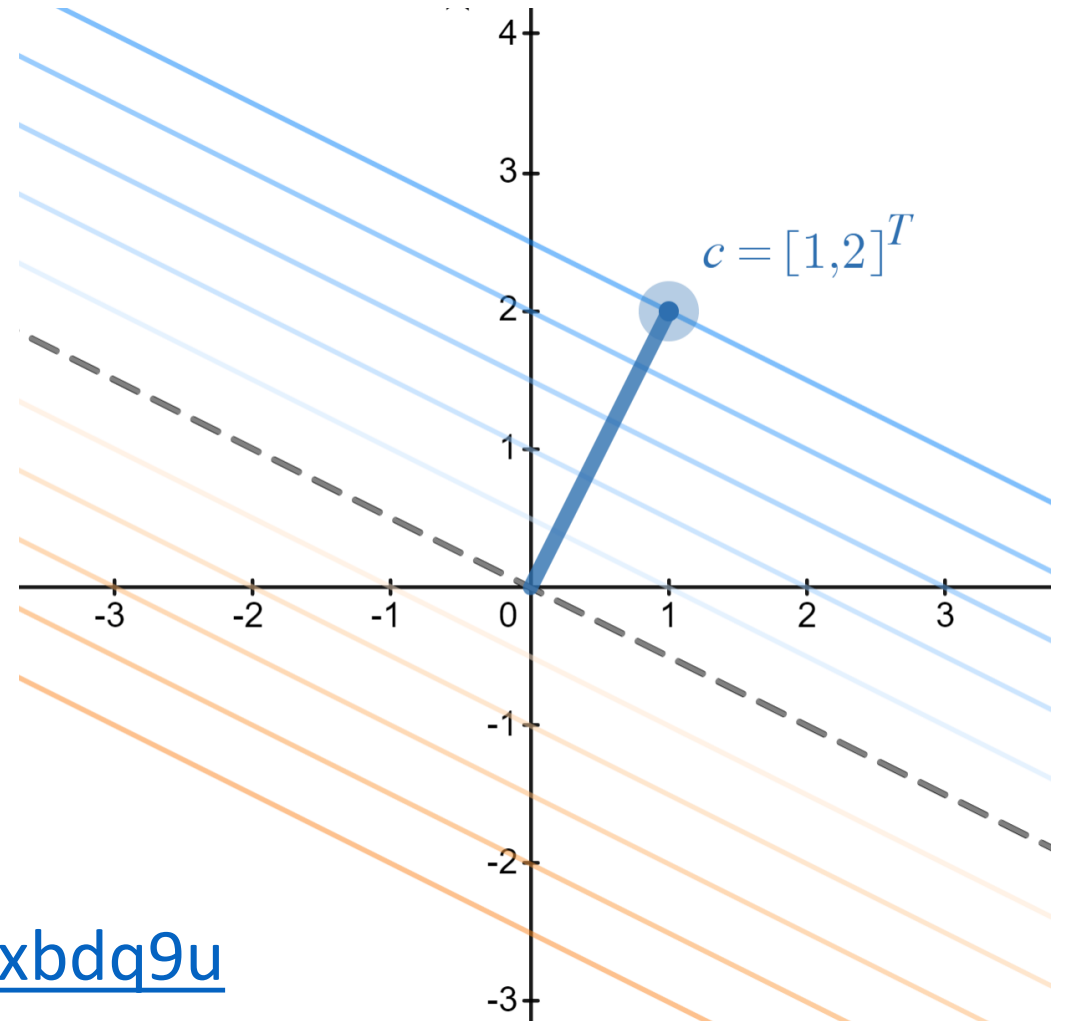
$$\mathbf{c}^T \mathbf{x} = 0 ?$$

$$\mathbf{c}^T \mathbf{x} = 1 ?$$

$$\mathbf{c}^T \mathbf{x} = 2 ?$$

$$\mathbf{c}^T \mathbf{x} = -1 ?$$

$$\mathbf{c}^T \mathbf{x} = -2 ?$$



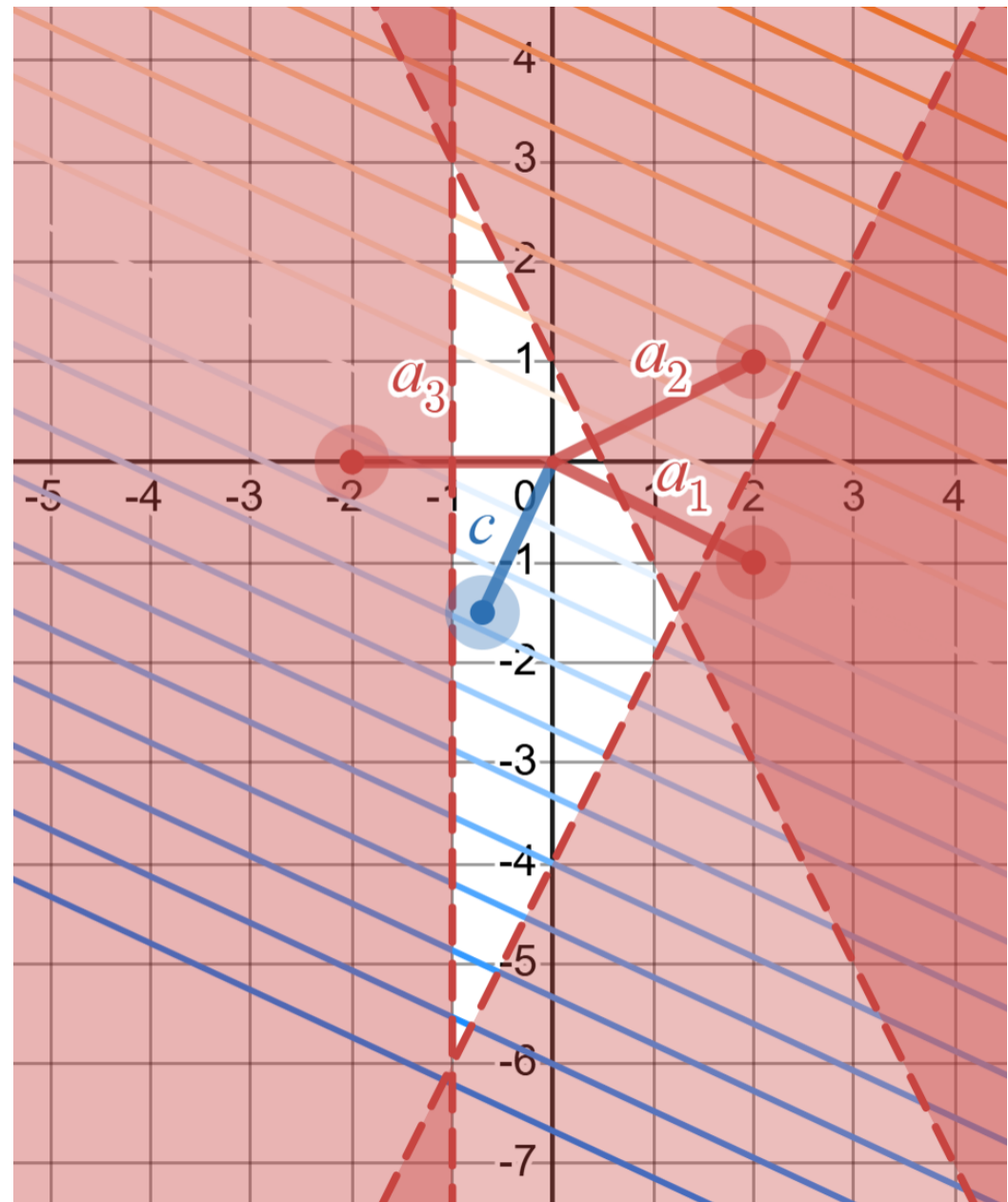
<https://www.desmos.com/calculator/8d9kxbdq9u>

Finding LP Solutions

Intuition

Poll 4

What is the solution to this LP?



<https://www.desmos.com/calculator/tnlo7p5plp>

Solving a Linear Program

Inequality form, with constraints

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

Solving a Linear Program

Inequality form, with two constraints

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

Solving a Linear Program

Inequality form, with one constraint

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & a_1 x_1 + a_2 x_2 \leq b\end{array}$$

Desmos demo: LP with one constraint

<https://www.desmos.com/calculator/ufrnzjgyvb>

Poll 5

True or False: A minimizing LP with exactly one constraint, will always have a minimum objective at $-\infty$.

$$\begin{array}{ll}\min. & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} & a_1 x_1 + a_2 x_2 \leq b\end{array}$$

Finding LP Solutions

Algorithms

Solving an LP

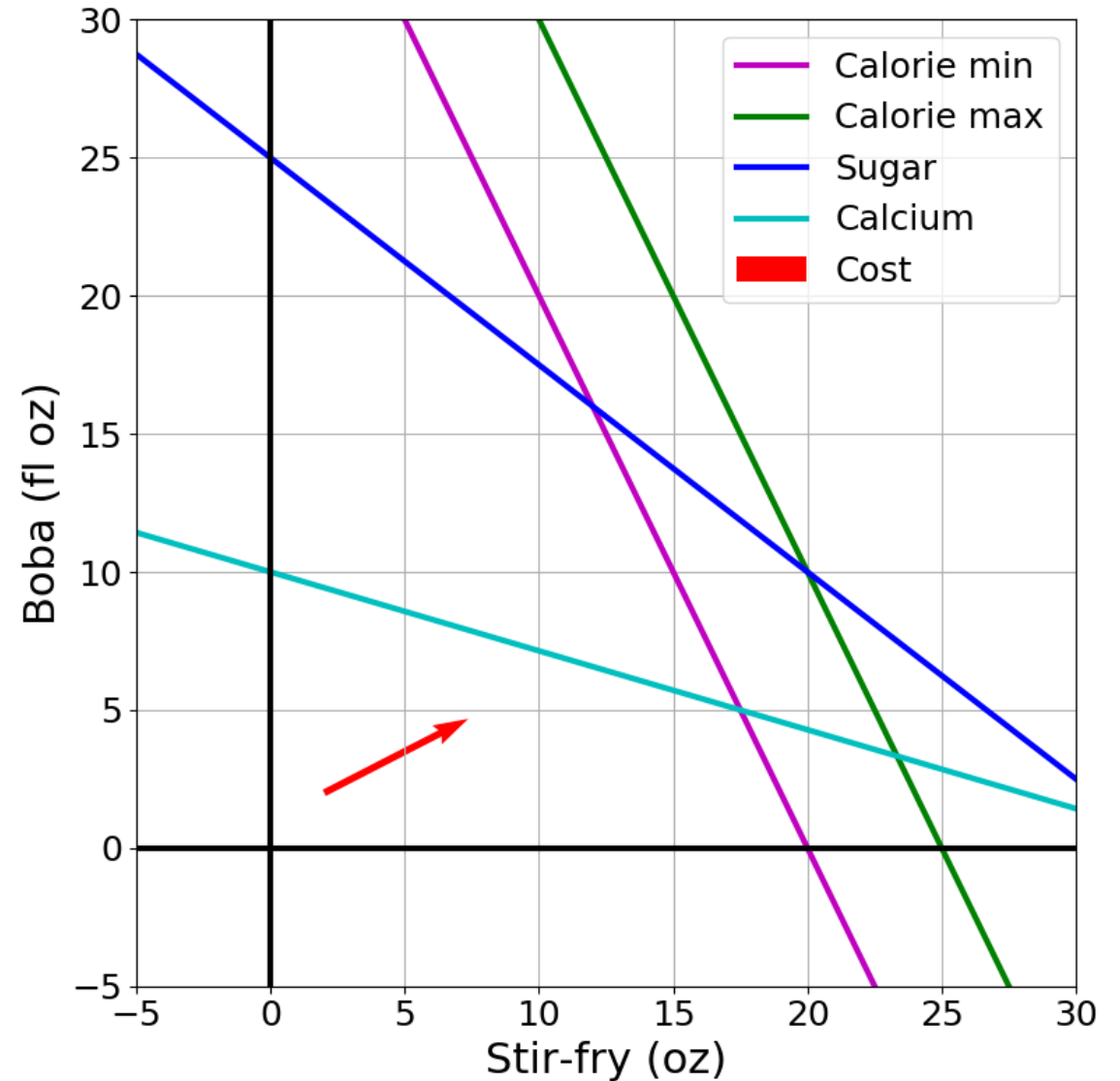
Solutions are at feasible intersections of constraint boundaries!!

Algorithms

- Check objective at all feasible intersections

In more detail:

1. Enumerate all intersections
2. Keep only those that are feasible (satisfy *all* inequalities)
3. Return feasible intersection with the lowest objective value



Solving an LP

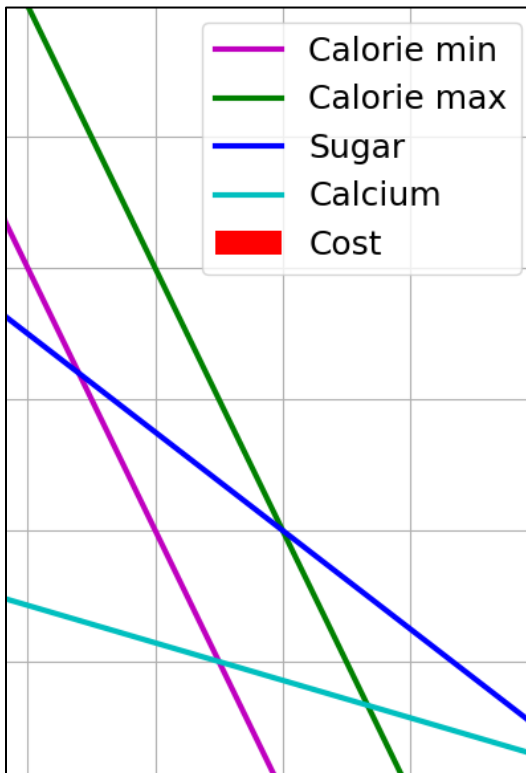
But, how do we find the intersection between boundaries?

$$\begin{array}{ll}\min & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

$$A = \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix}$$

Calorie min
Calorie max
Sugar
Calcium

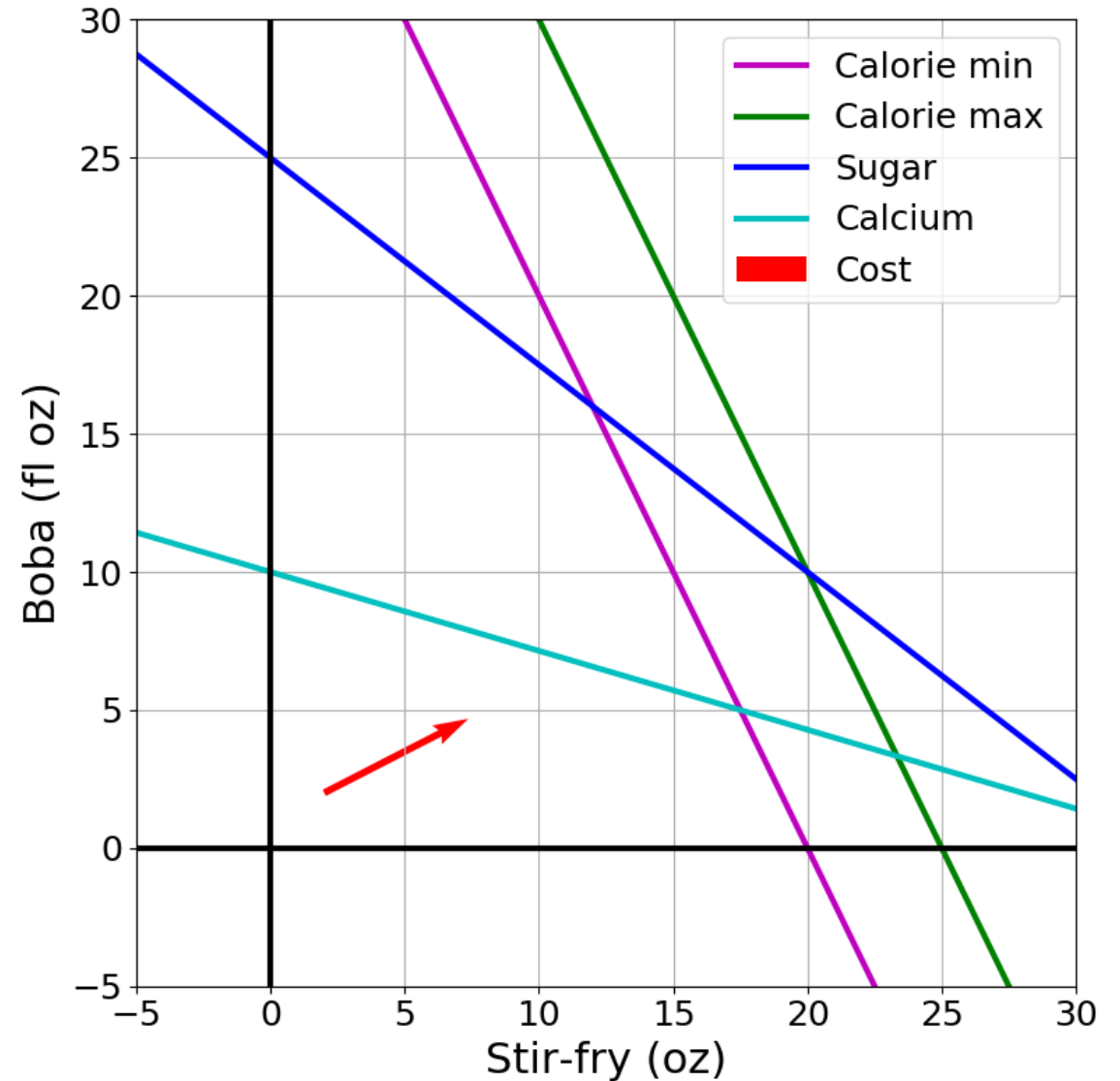


Solving an LP

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Algorithms

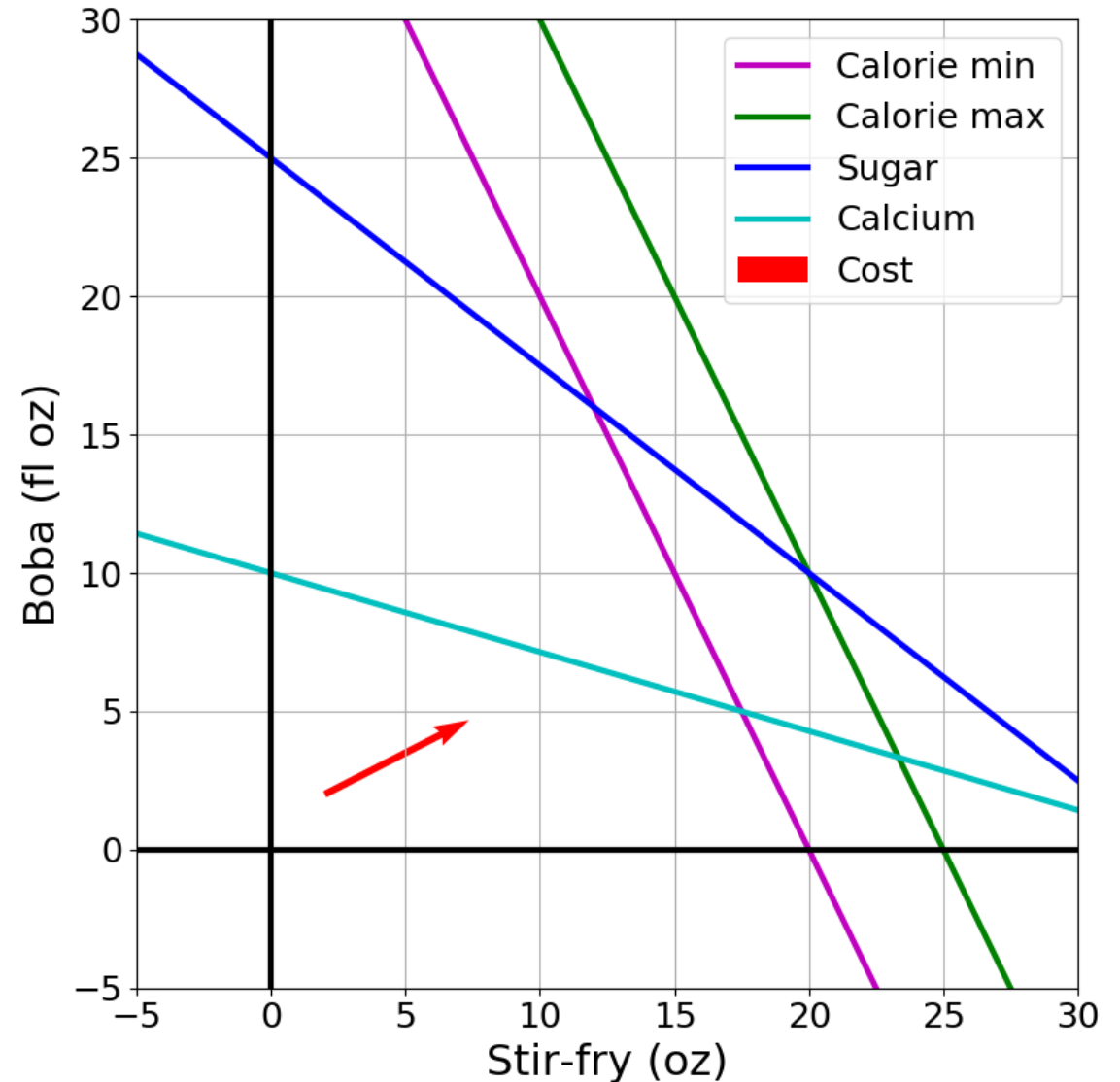
- Check objective at all feasible intersections
- Simplex



Solving an LP

Simplex algorithm

- Start at a feasible intersection (if not trivial, can solve another LP to find one)
- Define successors as “neighbors” of current intersection
 - i.e., remove one row from our square subset of A , and add another row not in the subset; then check feasibility
- Move to any successor with lower objective than current intersection
 - If no such successors, we are done

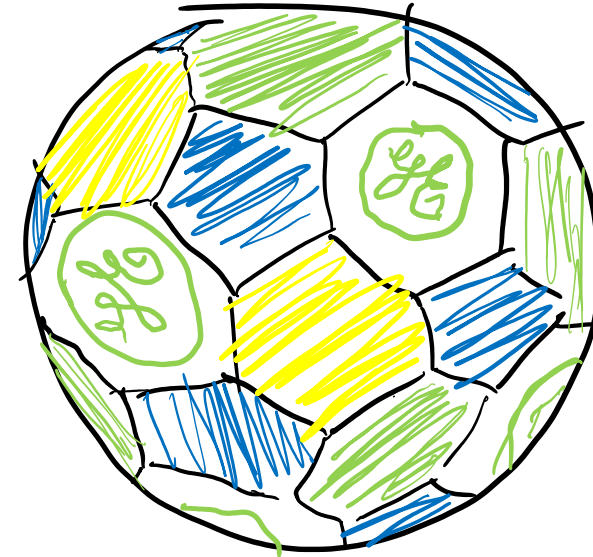


Greedy local hill-climbing search! ... but always finds *optimal* solution

Solving an LP

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Greedy local hill-climbing search! ... but always finds *optimal* solution

Solving an LP

Solutions are at feasible intersections
of constraint boundaries!!

Algorithms

- Check objective at all feasible intersections
- Simplex (intuition)
- Interior Point (out of scope)

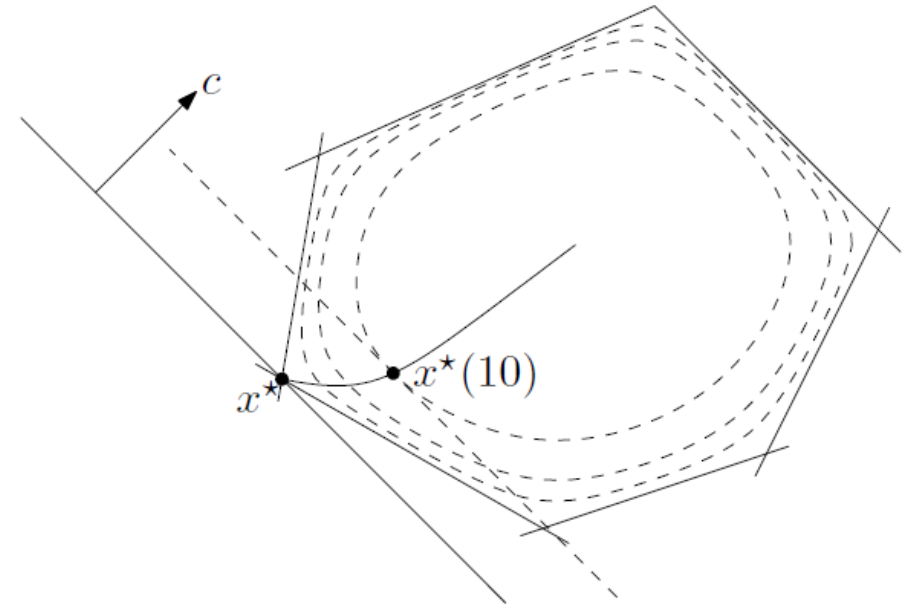


Figure 11.2 from Boyd and Vandenberghe, *Convex Optimization*

LPs in Higher Dimensions

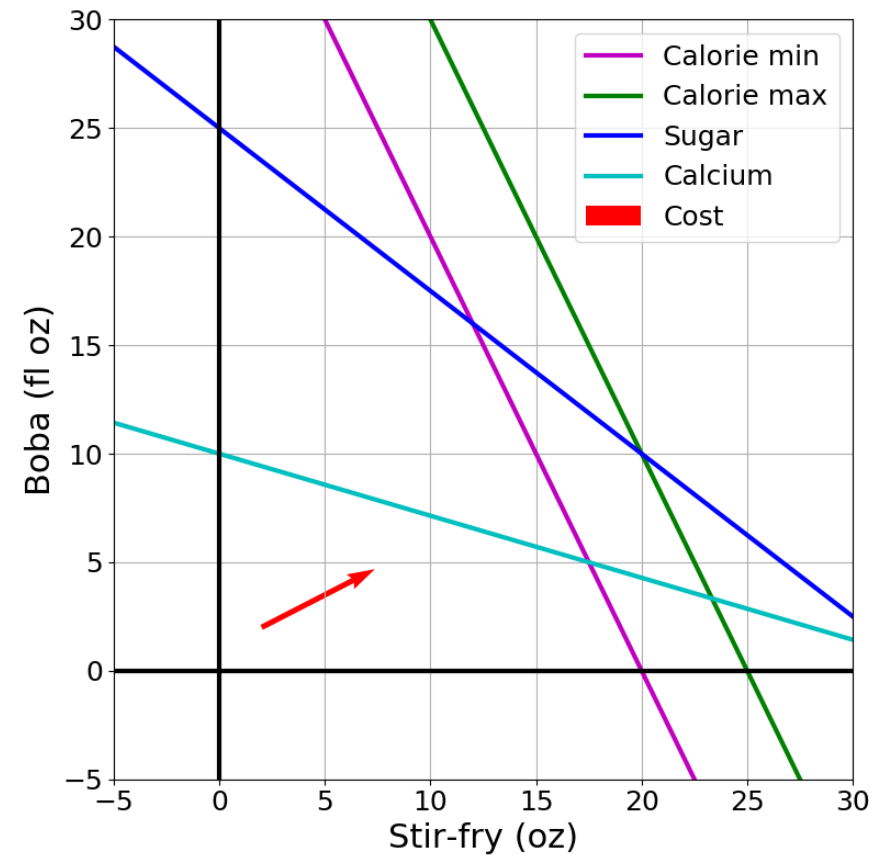
What about higher dimensions?

Problem Description

Optimization Representation

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

Graphical Representation



“Marty, you’re not thinking fourth-dimensionally”



<https://youtu.be/pTxW4n58eao?si=b5CdpwsJrpyVVNHv&t=39>

Shapes in higher dimensions

How do these linear shapes extend to 3-D, N-D?

$$a_1 x_1 + a_2 x_2 = b_1$$

$$a_1 x_1 + a_2 x_2 \leq b_1$$

$$a_{1,1} x_1 + a_{1,2} x_2 \leq b_1$$

$$a_{2,1} x_1 + a_{2,2} x_2 \leq b_2$$

$$a_{3,1} x_1 + a_{3,2} x_2 \leq b_3$$

$$a_{4,1} x_1 + a_{4,2} x_2 \leq b_4$$

What are intersections in higher dimensions?

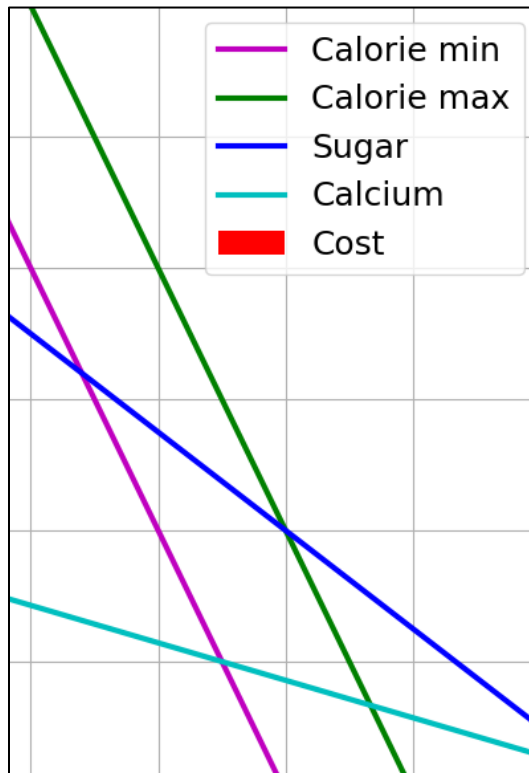
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How do we find intersections in higher dimensions?

Still looking at subsets of A matrix

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