Correlates of homicide: new space/time interaction tests for spatiotemporal point processes

Seth Flaxman Carnegie Mellon University

Spatial Statistics Conference June 7, 2013

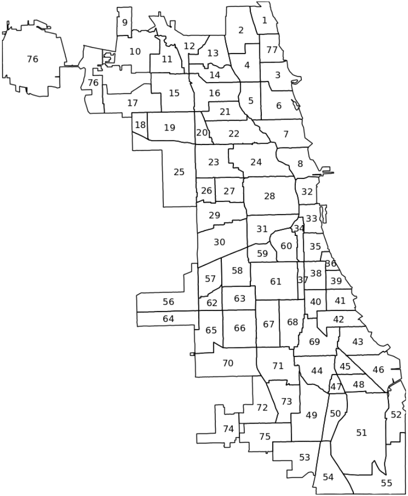
Joint work with....

Daniel Neill[1], Wilpen Gorr[1], Alex Smola[2]
1. H. J. Heinz III College, Carnegie Mellon University
2. School of Computer Science, Carnegie Mellon University

This work was partially supported by the National Science Foundation, grants IIS-0916345, IIS-0911032, and IIS-0953330.

- **Research question:** what predicts homicides?
- **Background:** space-time interaction tests
- **Methods:** kernel-based measures of independence
- **Applications:** 911 call data, crime offense reports from Chicago

Chicago



- Population: 2.7 million
- Area: 234 square miles
- **Research question:** Which types of calls to 911 are predictive of homicides and shootings nearby?

Space-time interaction

- Residual space-time dependence, after controlling for purely spatial and purely temporal dependence.

Space-time interaction

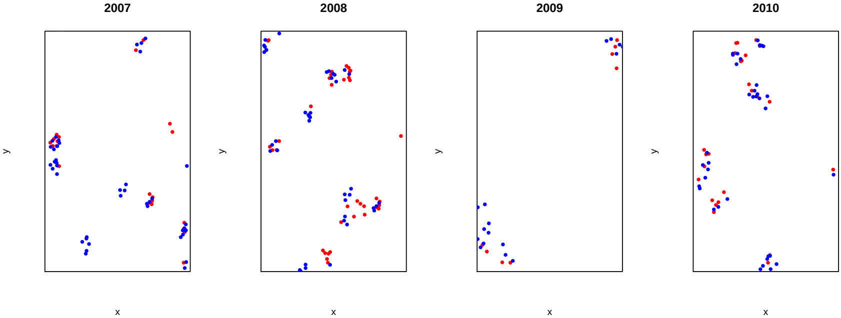
- Residual space-time dependence, after controlling for purely spatial and purely temporal dependence.
- If two events are close in space, they are likely to be close in time [Diggle, 1995].

Space-time interaction

- Residual space-time dependence, after controlling for purely spatial and purely temporal dependence.
- If two events are close in space, they are likely to be close in time [Diggle, 1995].

Data: point processes

$$\mathcal{P}_1 = \{(x_i^1, y_i^1, t_i^1), i \in 1, \ldots, n_1\}, \mathcal{P}_2 = \{(x_j^2, y_j^2, t_j^2), j \in 1, \ldots, n_2\}$$



Knox test [1964]

Put the $N = n_1 \cdot n_2$ pairs of points into a contingency table:

	close in space	far in space	
close in time	X	a	$= N_t$
far in time	b	c	
	$= N_s$		

Knox test [1964]

Put the $N = n_1 \cdot n_2$ pairs of points into a contingency table:

	close in space	far in space	
close in time	X	a	$= N_t$
far in time	b	c	
	$= N_s$		

Test statistic: $\frac{X}{N} - \frac{N_t}{N} \cdot \frac{N_s}{N}$

Knox test [1964]

Put the $N = n_1 \cdot n_2$ pairs of points into a contingency table:

	close in space	far in space	
close in time	X	a	$= N_t$
far in time	b	c	
	$= N_s$		

Test statistic: $\frac{X}{N} - \frac{N_t}{N} \cdot \frac{N_s}{N}$
(or just X)

Mantel test [1967]

Put the N pairs of points into two matrices:

Mantel test [1967]

Put the N pairs of points into two matrices:

$$\text{space:} \quad K = \begin{pmatrix} 0 & \|s_1 - s_2\| & \dots & \|s_1 - s_n\| \\ \|s_2 - s_1\| & 0 & \dots & \|s_2 - s_n\| \\ & & \ddots & \\ \|s_n - s_1\| & \|s_n - s_2\| & \dots & 0 \end{pmatrix}$$

Mantel test [1967]

Put the N pairs of points into two matrices:

$$\text{space:} \quad K = \begin{pmatrix} 0 & \|s_1 - s_2\| & \dots & \|s_1 - s_n\| \\ \|s_2 - s_1\| & 0 & \dots & \|s_2 - s_n\| \\ & & \ddots & \\ \|s_n - s_1\| & \|s_n - s_2\| & \dots & 0 \end{pmatrix}$$

$$\text{time:} \quad L = \begin{pmatrix} 0 & |t_1 - t_2| & \dots & |t_1 - t_n| \\ |t_2 - t_1| & 0 & \dots & |t_2 - t_n| \\ & & \ddots & \\ |t_n - t_1| & |t_n - t_2| & \dots & 0 \end{pmatrix}$$

Mantel test [1967]

Put the N pairs of points into two matrices:

$$\text{space:}\quad K = \begin{pmatrix} 0 & \|s_1 - s_2\| & \dots & \|s_1 - s_n\| \\ \|s_2 - s_1\| & 0 & \dots & \|s_2 - s_n\| \\ & & \ddots & \\ \|s_n - s_1\| & \|s_n - s_2\| & \dots & 0 \end{pmatrix}$$

$$\text{time:}\quad L = \begin{pmatrix} 0 & |t_1 - t_2| & \dots & |t_1 - t_n| \\ |t_2 - t_1| & 0 & \dots & |t_2 - t_n| \\ & & \ddots & \\ |t_n - t_1| & |t_n - t_2| & \dots & 0 \end{pmatrix}$$

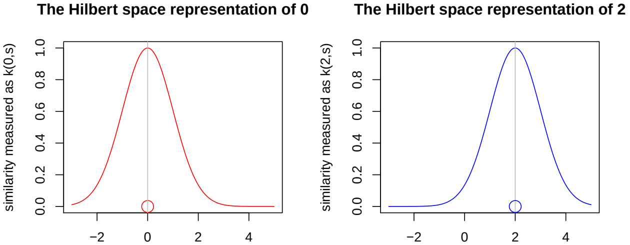
Test statistic: $\sum_{i,j} K_{i,j} L_{i,j}$

Shortcomings

- Knox: discretizes using pre-specified cutoffs
- Mantel: linear measure of independence (correlation)
- Focus is exclusively on interpoint (Euclidean) distances
- No way to include covariates, more spatial or temporal structure

Machine learning to the rescue

- A kernel is a real-valued paired similarity function: $k(x, y) \in \mathcal{R}$. Larger values $\Rightarrow$ more similar.
- You've heard of kernels: they're a generalization of covariance functions $C(x, y)$!
- Example: Gaussian $k(x, y) = e^{-\|x-y\|^2}$
- Mathematical theory: kernels turn points into infinite dimensional vectors, i.e. functions:



The “kernel trick”

- Take a standard statistical tool (e.g. clustering, PCA, SVMs), replace dot products or similarity metrics $\langle x, y \rangle$ with kernels $k(x, y)$ throughout.
- Enables the application of simple, linear methods in non-linear settings

Mantel test [1967]

Put the N pairs of points into two matrices:

$$\text{space:}\quad K = \begin{pmatrix} 0 & \|s_1 - s_2\| & \dots & \|s_1 - s_n\| \\ \|s_2 - s_1\| & 0 & \dots & \|s_2 - s_n\| \\ & & \ddots & \\ \|s_n - s_1\| & \|s_n - s_2\| & \dots & 0 \end{pmatrix}$$

$$\text{time:}\quad L = \begin{pmatrix} 0 & |t_1 - t_2| & \dots & |t_1 - t_n| \\ |t_2 - t_1| & 0 & \dots & |t_2 - t_n| \\ & & \ddots & \\ |t_n - t_1| & |t_n - t_2| & \dots & 0 \end{pmatrix}$$

Test statistic: $\sum_{i,j} K_{i,j} L_{i,j}$

Mantel test: transformed

$$k(s) := \frac{1}{s + \epsilon_1}, \ell(t) := \frac{1}{t + \epsilon_2}$$

Mantel test: transformed

$$k(s) := \frac{1}{s + \epsilon_1}, \ell(t) := \frac{1}{t + \epsilon_2}$$

$$K = \begin{pmatrix} 0 & k(\|s_1 - s_2\|) & \dots & k(\|s_1 - s_n\|) \\ k(\|s_2 - s_1\|) & 0 & \dots & k(\|s_2 - s_n\|) \\ & & \ddots & \\ k(\|s_n - s_1\|) & k(\|s_n - s_2\|) & \dots & 0 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & \ell(|t_1 - t_2|) & \dots & \ell(|t_1 - t_n|) \\ \ell(|t_2 - t_1|) & 0 & \dots & \ell(|t_2 - t_n|) \\ & & \ddots & \\ \ell(|t_n - t_1|) & \ell(|t_n - t_2|) & \dots & 0 \end{pmatrix}$$

Mantel test: transformed

$$k(s) := \frac{1}{s+\epsilon_1}, \ell(t) := \frac{1}{t+\epsilon_2}$$

$$K = \begin{pmatrix} 0 & k(\|s_1 - s_2\|) & \dots & k(\|s_1 - s_n\|) \\ k(\|s_2 - s_1\|) & 0 & \dots & k(\|s_2 - s_n\|) \\ & & \ddots & \\ k(\|s_n - s_1\|) & k(\|s_n - s_2\|) & \dots & 0 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & \ell(|t_1 - t_2|) & \dots & \ell(|t_1 - t_n|) \\ \ell(|t_2 - t_1|) & 0 & \dots & \ell(|t_2 - t_n|) \\ & & \ddots & \\ \ell(|t_n - t_1|) & \ell(|t_n - t_2|) & \dots & 0 \end{pmatrix}$$

Test statistic: $\sum_{i,j} K_{i,j} L_{i,j}$

"Kernelized" Mantel test

Choose kernels k and ℓ:

$$K = \begin{pmatrix} 0 & k(s_1, s_2) & \dots & k(s_1, s_n) \\ k(s_2, s_1) & 0 & \dots & k(s_2, s_n) \\ & & \ddots & \\ k(s_n, s_1) & k(s_n, s_2) & \dots & 0 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & \ell(t_1, t_2) & \dots & \ell(t_1, t_n) \\ \ell(t_2, t_1) & 0 & \dots & \ell(t_2, t_n) \\ & & \ddots & \\ \ell(t_n, t_1) & \ell(t_n, t_2) & \dots & 0 \end{pmatrix}$$

"Kernelized" Mantel test

Choose kernels k and ℓ:

$$K = \begin{pmatrix} 0 & k(s_1, s_2) & \dots & k(s_1, s_n) \\ k(s_2, s_1) & 0 & \dots & k(s_2, s_n) \\ & & \ddots & \\ k(s_n, s_1) & k(s_n, s_2) & \dots & 0 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & \ell(t_1, t_2) & \dots & \ell(t_1, t_n) \\ \ell(t_2, t_1) & 0 & \dots & \ell(t_2, t_n) \\ & & \ddots & \\ \ell(t_n, t_1) & \ell(t_n, t_2) & \dots & 0 \end{pmatrix}$$

Test statistic: $\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j)$

Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:

$$k(p_i, p_j) := k(s_i, s_j) \ \text{(similarity in space)}$$

$$\ell(p_i, p_j) := \ell(t_i, t_j) \ \text{(similarity in time)}$$

Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:

$$k(p_i, p_j) := k(s_i, s_j) \quad \text{(similarity in space)}$$

$$\ell(p_i, p_j) := \ell(t_i, t_j) \quad \text{(similarity in time)}$$

- Are these two notions of similarity independent?

Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:

$$k(p_i, p_j) := k(s_i, s_j) \text{ (similarity in space)}$$

$$\ell(p_i, p_j) := \ell(t_i, t_j) \text{ (similarity in time)}$$

- Are these two notions of similarity independent?
- Equivalently: given a random point $p \sim \mathcal{P}$, are the Hilbert space representations $k(p, \cdot)$ and $\ell(p, \cdot)$ independent?

Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:

$$k(p_i, p_j) := k(s_i, s_j) \text{ (similarity in space)}$$

$$\ell(p_i, p_j) := \ell(t_i, t_j) \text{ (similarity in time)}$$

- Are these two notions of similarity independent?
- Equivalently: given a random point $p \sim \mathcal{P}$, are the Hilbert space representations $k(p, \cdot)$ and $\ell(p, \cdot)$ independent?
- Mantel test only picks up linear dependence (measured as correlation):

$$\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j)$$

Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:
$$k(p_i, p_j) := k(s_i, s_j) \ \ \text{(similarity in space)}$$
$$\ell(p_i, p_j) := \ell(t_i, t_j) \ \ \text{(similarity in time)}$$
- Are these two notions of similarity independent?
- Equivalently: given a random point $p \sim \mathcal{P}$, are the Hilbert space representations $k(p, \cdot)$ and $\ell(p, \cdot)$ independent?
- Mantel test only picks up linear dependence (measured as correlation):
$$\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j)$$
- Statistical machine learning says there's a better way...

Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)

Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)
- Given observations $(s_i, t_i) \sim S \times T$, is $P(S, T) = P(S)P(T)$?

Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)
- Given observations $(s_i, t_i) \sim S \times T$, is $P(S, T) = P(S)P(T)$?
- Looks for a function f:

$$\text{HSIC} = \sup_f \left(\mathop{\mathrm{E}}_{(s,t)\sim S\times T} f(s,t) - \mathop{\mathrm{E}}_{s\sim S, t\sim T} f(s,t) \right)^2$$

Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)
- Given observations $(s_i, t_i) \sim S \times T$, is $P(S, T) = P(S)P(T)$?
- Looks for a function f:

$$\text{HSIC} = \sup_f \left(\mathop{\mathrm{E}}_{(s,t)\sim S\times T} f(s,t) - \mathop{\mathrm{E}}_{s\sim S, t\sim T} f(s,t) \right)^2$$

- Intuition: f is a “witness” function, meant to find discrepancies between $P(S, T)$ and $P(S)P(T)$.

Kernel measures of independence

- For f sufficiently complex, can do much more than distinguish linear dependence:
  Theorem. $\mathrm{HSIC} = 0$ if and only if $P(S, T) = P(S)P(T)$

Kernel measures of independence

- For f sufficiently complex, can do much more than distinguish linear dependence:
  Theorem. HSIC $= 0$ if and only if $P(S, T) = P(S)P(T)$
- For f in a Hilbert space and bounded, f^* can be found in closed form:

Kernel measures of independence

- For f sufficiently complex, can do much more than distinguish linear dependence:
  Theorem. HSIC $= 0$ if and only if $P(S, T) = P(S)P(T)$
- For f in a Hilbert space and bounded, f^* can be found in closed form:

$$\mathrm{HSIC} = \frac{1}{n^2} \sum_{i,j} k(s_i, s_j)\ell(t_i, t_j) - \frac{2}{n^3} \sum_{i,j,r} k(s_i, s_j)\ell(t_i, t_r) + \frac{1}{n^4} \sum_{i,j,q,r} k(s_i, s_j)\ell(t_q, t_r)$$

Kernel measures of independence

- For f sufficiently complex, can do much more than distinguish linear dependence:
  Theorem. HSIC $= 0$ if and only if $P(S, T) = P(S)P(T)$
- For f in a Hilbert space and bounded, f^* can be found in closed form:

$$\mathrm{HSIC} = \frac{1}{n^2}\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j) - \frac{2}{n^3}\sum_{i,j,r} k(s_i, s_j)\ell(t_i, t_r) + \frac{1}{n^4}\sum_{i,j,q,r} k(s_i, s_j)\ell(t_q, t_r)$$

- We show that these results hold for testing whether $k(p, \cdot)$ and $\ell(p, \cdot)$ are independent $\Rightarrow$ new test for space-time interaction

Space-time interaction statistics

HSIC:

$$\frac{1}{n^2}\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j) - \frac{2}{n^3}\sum_{i,j,r} k(s_i, s_j)\ell(t_i, t_r) + \frac{1}{n^4}\sum_{i,j,q,r} k(s_i, s_j)\ell(t_q, t_r)$$

Kernelized Mantel:

$$\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j)$$

- Notice: missing terms! Mantel is like HSIC, but with some non-optimal choice of f ($\Rightarrow$ less power).

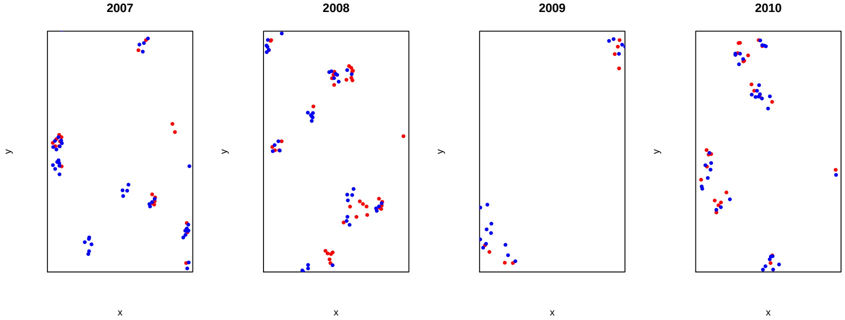
Our Contributions

- New way of thinking about space-time interaction in terms of kernels $\Rightarrow$ new interpretation of HSIC $\Rightarrow$ new test for space-time interaction
- Extensions to bivariate, forward in time cases
- Interesting connections with Mantel test, showing its shortcomings and a possible fix
- More flexible test: kernels can encode more than just distance between points. HSIC can test for non-linear dependencies.

Experimental Setup

Synthetic data: draw $n = 40$ or $n = 100$ random cluster centers, draw $k = 5$ or 1 children with locations displaced $N(0, \sigma)$ from parent in every direction.

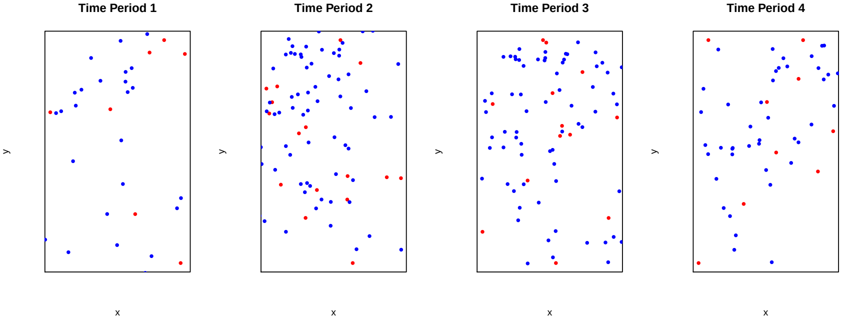
Easy example: $\sigma = .025$



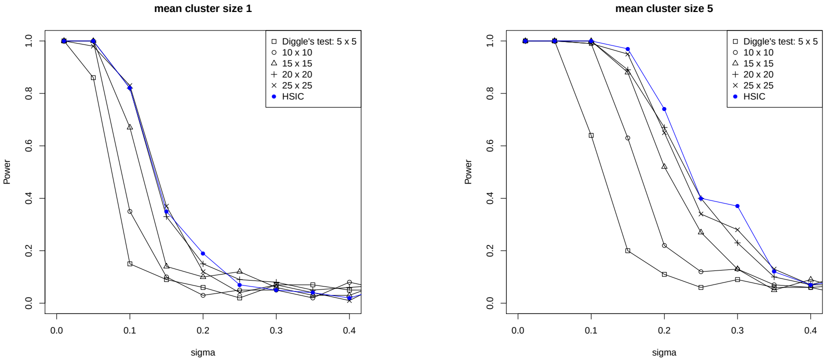
Experimental Setup

Synthetic data: draw $n = 40$ or $n = 100$ random cluster centers, draw $k = 5$ or 1 children with locations displaced $N(0, \sigma)$ from parent in every direction.

Hard example: $\sigma = .2$



Synthetic Data: Results



Experimental Setup: Crime Data

Question: which types of calls to 911 predict homicides and aggravated battery with a handgun (“shootings”)?

Experimental Setup: Crime Data

Question: which types of calls to 911 predict homicides and aggravated battery with a handgun ("shootings")?

Data:

- Dispatcher calls from January 2007-May 2010, coded by one of 271 types ($\approx$ 9 million):

  ```
  "01-01-2010","12:25:00","ARSON",1172456,1834562
  "01-02-2010","19:55:00","THEFT",1173123,1831123
  ```

- All shootings / homicides from January 2007-May 2010 (9,087 total):

  ```
  "01-01-2010","19:00:37","HOMICIDE",1172001,1834023
  "01-07-2010","19:55:00","HOMICIDE",1173934,1831384
  ```

Experimental Setup: Algorithms

- Calculate p-values for directional, bivariate space-time interaction between each 911 call type and shootings
- Compare Knox and HSIC
- Knox: cutoff for close: 500 feet, 14 days
- HSIC: Gaussian RBF kernels, with equivalent bandwidth
- Permutation testing 1000 times to calculate p-values

Results

HSIC < .01	HSIC and Knox < .01	Knox < .01
auto accident pd	10-1	arson report
battery jo	death removal	auto theft ip
battery victim inj.	evidence technician (pri. 1)	criminal tres. (ov)
beat team meeting [ov]	evidence technician (pri. 2)	evidence technician (pri. 3)
crim dam. to prop rpt	gambling	found property
mental unauth absence	gang disturbance	k9 request
mission	outdoor roll call	kidnapping report
person with a gun	person shot	notify
robbery victim injured	plan 1-5	person stabbed
theft ip	shots fired	pick up car
	shots fired (ov)	polling place check
		suspicious person (ov)
		transport

Results

HSIC < .01	**HSIC and Knox < .01**	**Knox < .01**
auto accident pd	10-1	arson report
battery jo	death removal	auto theft ip
battery victim inj.	evidence technician (pri. 1)	criminal tres. (ov)
beat team meeting [ov]	evidence technician (pri. 2)	evidence technician (pri. 3)
crim dam. to prop rpt	gambling	found property
mental unauth absence	gang disturbance	k9 request
mission	outdoor roll call	kidnapping report
person with a gun	person shot	notify
robbery victim injured	plan 1-5	person stabbed
theft ip	shots fired	pick up car
	shots fired (ov)	polling place check
		suspicious person (ov)
		transport

Expert rating: 5 (most reasonable)

Results

HSIC < .01	HSIC and Knox < .01	Knox < .01
auto accident pd	10-1	arson report
battery jo	death removal	auto theft ip
battery victim inj.	evidence technician (pri. 1)	criminal tres. (ov)
beat team meeting [ov]	evidence technician (pri. 2)	evidence technician (pri. 3)
crim dam. to prop rpt	gambling	found property
mental unauth absence	gang disturbance	k9 request
mission	outdoor roll call	kidnapping report
person with a gun	person shot	notify
robbery victim injured	plan 1-5	person stabbed
theft ip	shots fired	pick up car
	shots fired (ov)	polling place check
		suspicious person (ov)
		transport

Expert rating: 5 (most reasonable) , 4

Results

HSIC < .01	HSIC and Knox < .01	Knox < .01
auto accident pd	10-1	arson report
battery jo	death removal	auto theft ip
battery victim inj.	evidence technician (pri. 1)	criminal tres. (ov)
beat team meeting [ov]	evidence technician (pri. 2)	evidence technician (pri. 3)
crim dam. to prop rpt	gambling	found property
mental unauth absence	gang disturbance	k9 request
mission	outdoor roll call	kidnapping report
person with a gun	person shot	notify
robbery victim injured	plan 1-5	person stabbed
theft ip	shots fired	pick up car
	shots fired (ov)	polling place check
		suspicious person (ov)
		transport

Expert rating: 5 (most reasonable) , 4 , 3

Results

HSIC < .01	HSIC and Knox < .01	Knox < .01
auto accident pd	10-1	arson report
battery jo	death removal	auto theft ip
battery victim inj.	evidence technician (pri. 1)	criminal tres. (ov)
beat team meeting [ov]	evidence technician (pri. 2)	evidence technician (pri. 3)
crim dam. to prop rpt	gambling	found property
mental unauth absence	gang disturbance	k9 request
mission	outdoor roll call	kidnapping report
person with a gun	person shot	notify
robbery victim injured	plan 1-5	person stabbed
theft ip	shots fired	pick up car
	shots fired (ov)	polling place check
		suspicious person (ov)
		transport

Expert rating: 5 (most reasonable) , 4 , 3 , 2

Results

HSIC < .01	HSIC and Knox < .01	Knox < .01
auto accident pd	10-1	arson report
battery jo	death removal	auto theft ip
battery victim inj.	evidence technician (pri. 1)	criminal tres. (ov)
beat team meeting [ov]	evidence technician (pri. 2)	evidence technician (pri. 3)
crim dam. to prop rpt	gambling	found property
mental unauth absence	gang disturbance	k9 request
mission	outdoor roll call	kidnapping report
person with a gun	person shot	notify
robbery victim injured	plan 1-5	person stabbed
theft ip	shots fired	pick up car
	shots fired (ov)	polling place check
		suspicious person (ov)
		transport

Expert rating: 5 (most reasonable) , 4 , 3 , 2 , 1 (least reasonable)

Conclusions

- New data-driven formulation of "leading indicators" question as space-time interaction between pairs of point processes
- Defined a new kernel-based space-time interaction test
- HSIC performance was comparable to classical tests, parameter choices less critical
- Applied to large, real, and important dataset: shootings in Chicago

Thank you! Questions?[1]

Seth Flaxman - flaxman@cmu.edu

[1]Thank you to the Chicago Police Department for sharing data. Points of view or opinions contained within this presentation are those of the author and do not necessarily represent the official position or policies of the Chicago Police Department. Title page photo by Palsson on Flickr.

Extensions

- Bivariate case: for test statistic, restrict sums to pairs of points of different types:

Extensions

- Bivariate case: for test statistic, restrict sums to pairs of points of different types:

$$\frac{1}{n^2}\sum_{i,j} k(s_i^1, s_j^2)\ell(t_i^1, t_j^2) - \frac{2}{n^3}\sum_{i,j,r} k(s_i^1, s_j^2)\ell(t_i^1, t_r^2) + \frac{1}{n^4}\sum_{i,j,q,r} k(s_i^1, s_j^2)\ell(t_q^1, t_r^2)$$

- Interesting interpretation in Hilbert space

Extensions

- Bivariate case: for test statistic, restrict sums to pairs of points of different types:

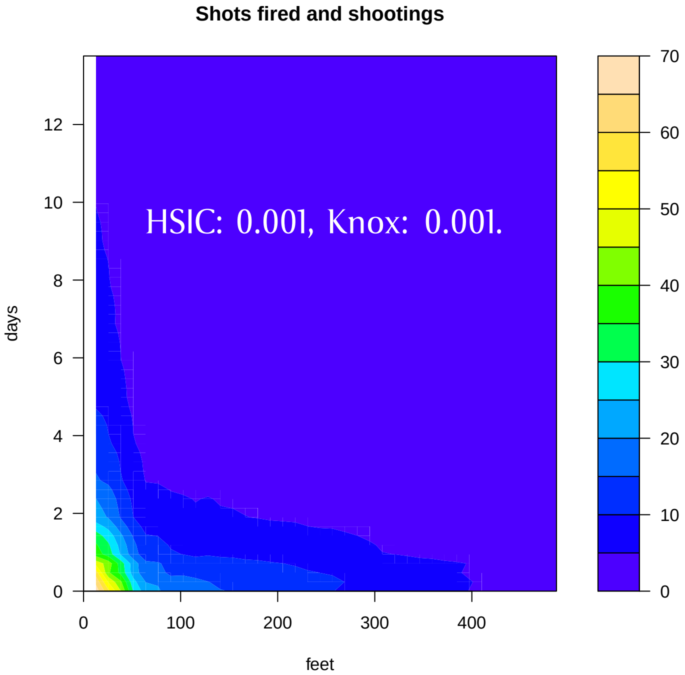
$$\frac{1}{n^2}\sum_{i,j} k(s_i^1, s_j^2)\ell(t_i^1, t_j^2) - \frac{2}{n^3}\sum_{i,j,r} k(s_i^1, s_j^2)\ell(t_i^1, t_r^2) + \frac{1}{n^4}\sum_{i,j,q,r} k(s_i^1, s_j^2)\ell(t_q^1, t_r^2)$$

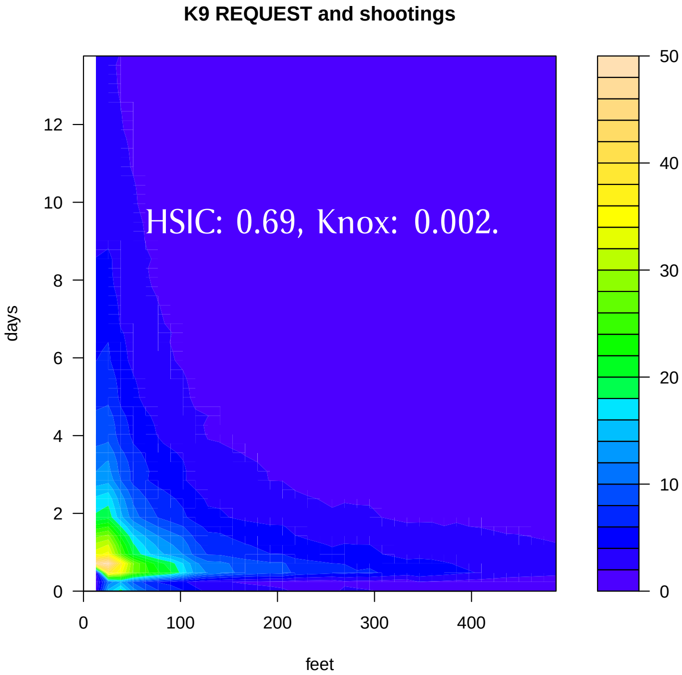
- Interesting interpretation in Hilbert space
- Only predict forward in time: restrict sums to pairs of points where $t_i < t_j$.

Excess risk attributable to space-time interaction

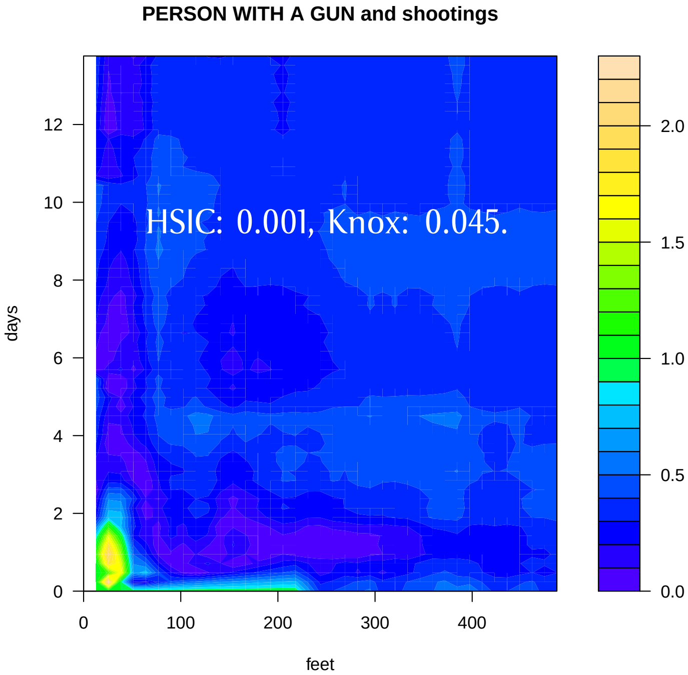
$$D(s, t) = \frac{F_{S,T}(s, t) - F_S(s)F_T(t)}{F_S(s)F_T(t)}$$

Given that we see an event of type 1, proportional increase (excess risk) of seeing an event of type 2.


Shots fired and shootings
HSIC: 0.001, Knox: 0.001.
days
feet
0
100
200
300
400
0
2
4
6
8
10
12
0
10
20
30
40
50
60
70



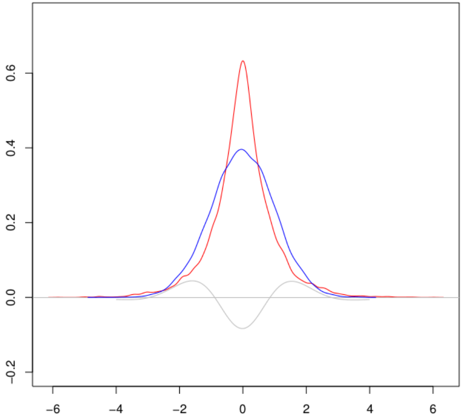
K9 REQUEST and shootings
HSIC: 0.69, Knox: 0.002.
days
feet
0
2
4
6
8
10
12
0
100
200
300
400
0
10
20
30
40
50


PERSON WITH A GUN and shootings
HSIC: 0.001, Knox: 0.045.
days
feet
0
100
200
300
400
2
4
6
8
10
12
0.0
0.5
1.0
1.5
2.0

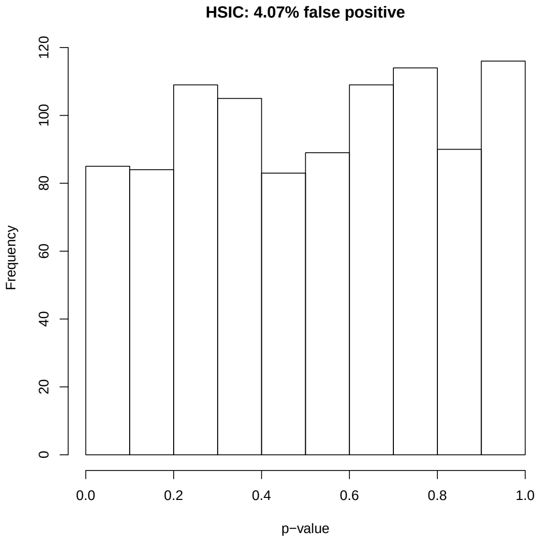
Maximum Mean Discrepancy (Gretton et al. 2012)

"Witness" $\hat{f}^*$:

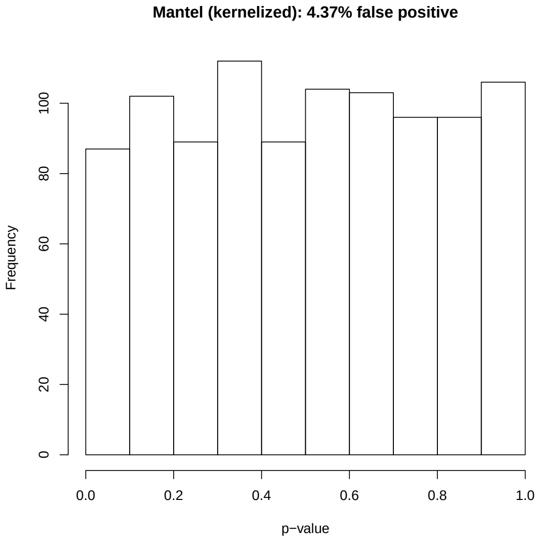
$$\hat{f}^*(x, y) \propto \sum_i k(x, x_i) - \sum_j k(y, y_j)$$



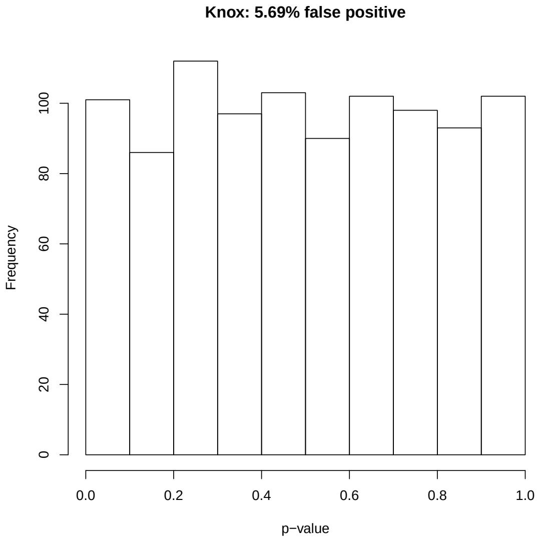
False positive rate

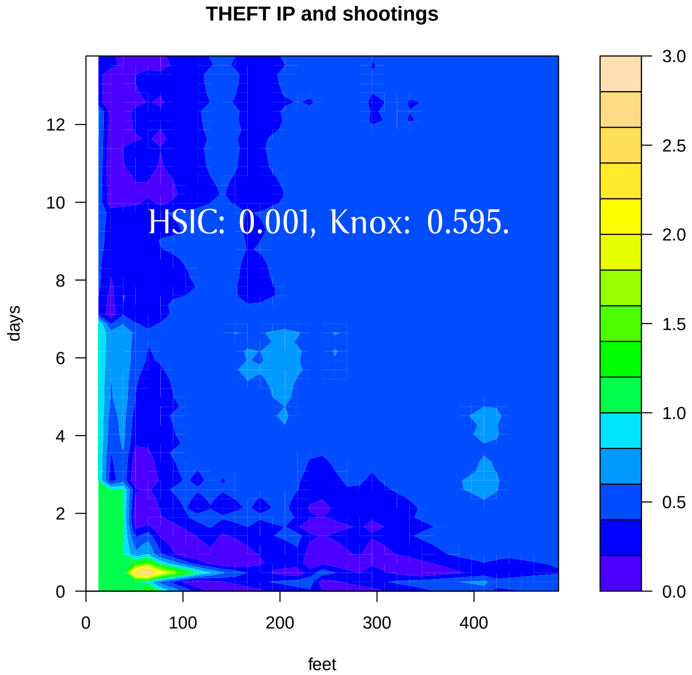


False positive rate



False positive rate




THEFT IP and shootings
HSIC: 0.001, Knox: 0.595.
days
feet
0
2
4
6
8
10
12
0
100
200
300
400
0.0
0.5
1.0
1.5
2.0
2.5
3.0