Internally Deterministic Parallel Algorithms

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Also: Jeremy Fineman, Phil Gibbons (Intel), Julian Shun, Harsha Vardham Simhadri, …

Partial Motivation

WoDet 2010: Seattle

Debate: can deterministic algorithms be as fast as nondeterministic ones?

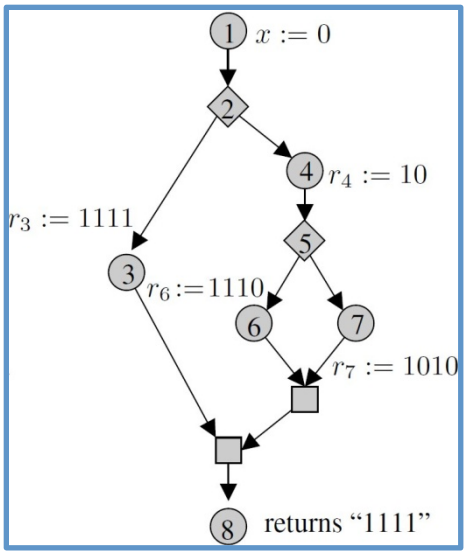
External vs. Internal Determinism [Emrath+Padua '88, Netzer+Miller '90]

- External: same input → same result
- Internal: same input → same "intermediate states" and same result

Internal Determinism

- Trace: a computation's final state, intermediate states, along with its control-flow DAG
- Internally deterministic: for any fixed input, all possible executions result in equivalent traces (with respect to some level of abstraction)
  - External determinism
  - Sequential semantics

Trace



Internally deterministic?

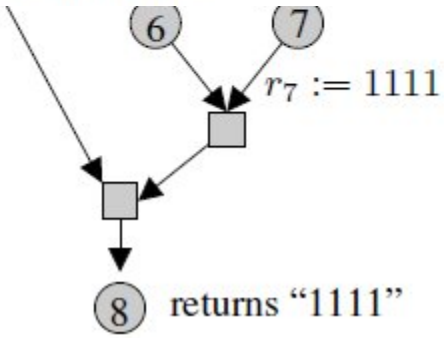
```
1.  x := 0
2.  in parallel do
3.      {   r3 := AtomicAdd(x, 1)     }
4.      {   r4 := AtomicAdd(x, 10)
5.          in parallel do
6.              {   r6 := AtomicAdd(x, 100)    }
7.              {   r7 := AtomicAdd(x, 1000)    }
        }
8.  return x
```

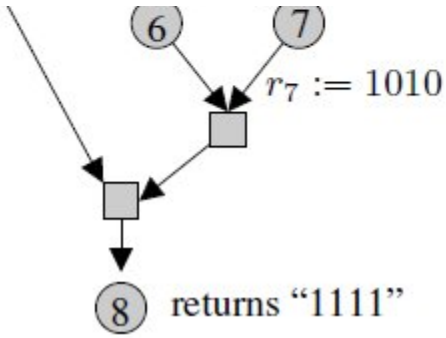

1. $x := 0$
2. **in parallel do**
3. $\{ \quad r_3 := \texttt{AtomicAdd}(x, 1) \quad \}$
4. $\{ \quad r_4 := \texttt{AtomicAdd}(x, 10)$
5. **in parallel do**
6. $\{ \quad r_6 := \texttt{AtomicAdd}(x, 100) \quad \}$
7. $\{ \quad r_7 := \texttt{AtomicAdd}(x, 1000) \quad \}$

$r_3 :=$

$\}$

8. `return` x





But!!

What does it mean for traces to be equivalent?

Emrath+Padua and Netzer+Miller: equal bit representation. Nice and simple.

But very restrictive: what about pointers from memory allocation?

Instead: abstract operations on data structures

- e.g. a dictionary
- equality is subtle

Encapsulation

Using abstract operations gives a technique to "encapsulate" non-determinism while still being internally deterministic.

- non-deterministic subroutines
- non-deterministic internal representations of data structures
- **more subtle**: non-deterministic implementations of linearizable but commutative data structures

Pros and Cons of Internal Determinism

?

- Debugging
- Composability
- Verification
- Performance analysis
- Sequential semantics

- Complicated code
- Performance penalty

Can Internally Deterministic Parallel Algorithms be fast and simple to code?

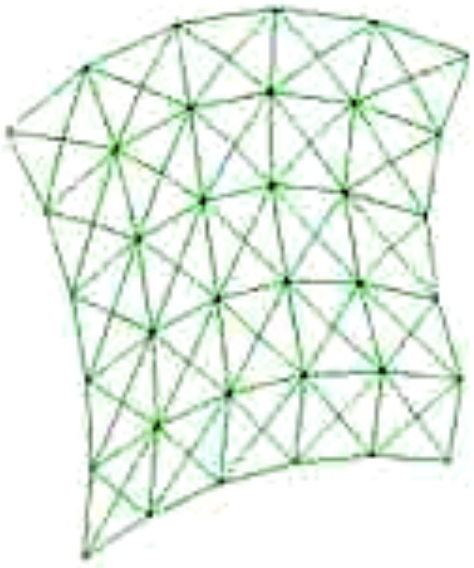
Internal Determinism in debugging

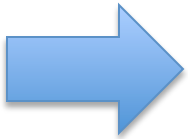
Need to be careful that measurement does not "perturb" the system.

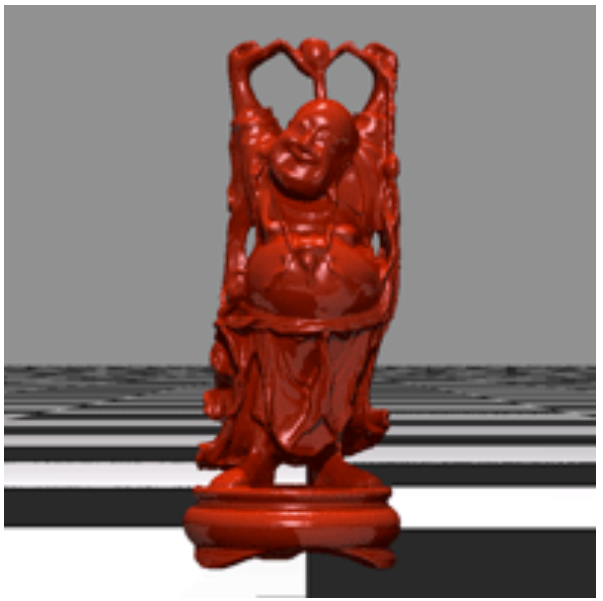
- Break points/Single stepping – can only make queries that are deterministic. Based on a particular sequential order.
- Adding observations ("print" statements) - need to collect them deterministically.

Problem Based Benchmarks

- Define a set of benchmarks in terms of **Input/ Output** behavior on specific inputs, and use them to compare solutions.







Input

Output

Many Existing Benchmarks

But none we know of match the spec

- **Code Based** : SPEC, Da Capo, PassMark, Splash-2, PARSEC, fluidMark
- **Application Specific**: Linpack, BioBench, BioParallel, MediaBench, SATLIB, CineBench, MineBench, TCP, ALPBench, Graph 500, DIMACS challenges
- **Method Based**: Lonestar
- **Machine analysis**: HPC challenge, Java Grande, NAS, Green 500, Graph 500, P-Ray, fluidMark

Preliminary Benchmarks I

Sequences	* Comparison Sorting
	* Removing Duplicates
	* Dictionary
Graphs	* Breadth First Search
	* Graph Separators
	* Minimum Spanning Tree
	* Maximal Independent Set
Geometry/ Graphics	* Delaunay Triangulation and Refinement
	* Convex Hulls
	* Ray Triangle Intersection (Ray Casting)
	Micropolygon Rendering

* finished

Preliminary Benchmarks II

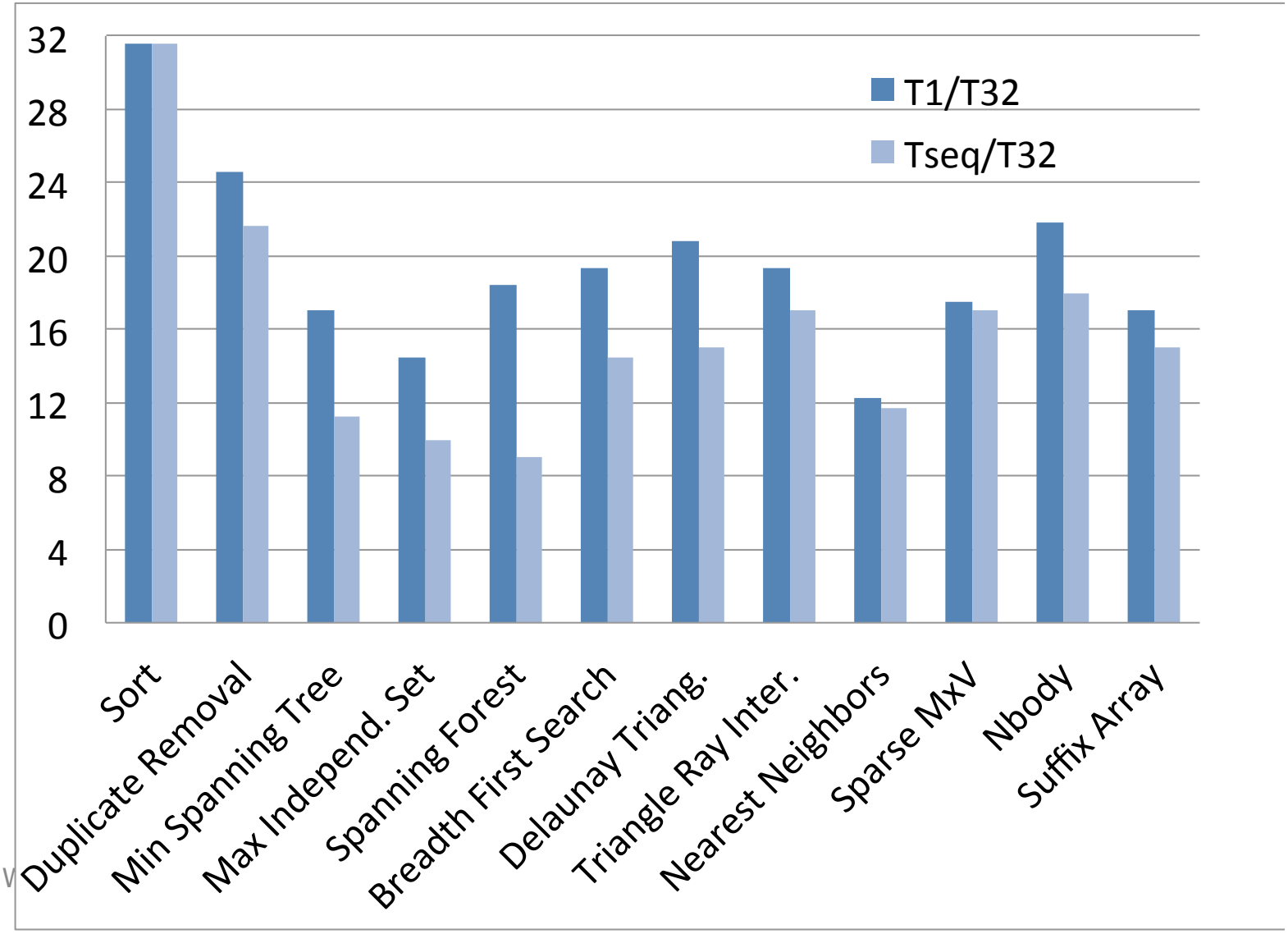
Machine Learning	* All Nearest Neighbors
	Support Vector Machines
	* K-Means
Text Processing	* Suffix Arrays
	Edit Distance
	String Search
Science	* Nbody force calculations
	Phylogenetic tree
Numerical	* Sparse Matrix Vector Multiply
	Sparse Linear Solve

* finished

Each Benchmark Consists of:

- A precise specification of the problem
- Specification of Input/Output file formats
- A set of input generators.
- Code for testing the results
- **Baseline sequential code**

How do the problems do on a modern multicore



Our Experiments

We coded up the 16 benchmarks from the PBBS with strict internal determinism trying various approaches.

Compared them with sequential code and non-deterministic code both in terms of runtime and code complexity.

Our Approach

Nested Parallelism +
Commutative/Linearizable operations (Steele '90)

Specific approaches:

- Functional programming
- History independent data structures
- Deterministic reservations

Written in g++ with just CAS, cilk_for, spawn, sync

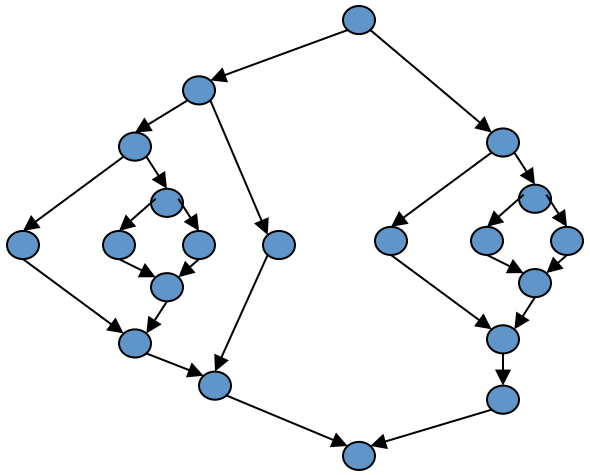
Nested Parallelism

We consider **nested parallel computations**

arbitrary nesting of fork-join and parallel loops

Has some important advantages:

- Good for caching
- Reduces scheduling overhead
- Supported by many languages
- Easy to analyze costs
- Makes it easier to verify code??



Commutativity

$f(S) \rightarrow S' \Rightarrow v$ *f converts from state S to S′ returning v*

f and g **commute** if for all S

$$f(S) \rightarrow S_f \Rightarrow v_f \quad g(S_f) \rightarrow S_{fg} \Rightarrow v_g$$

$$g(S) \rightarrow S_g \Rightarrow v'_g \quad f(S_g) \rightarrow S_{gf} \Rightarrow v'_f$$

Implies: $v_f = v'_f$ and $v_g = v'_g$ and $S_{fg} = S_{gf}$

Linearizable

Concurrent operations appear to happen atomically.

Theorem: for a nested parallel computation if all parallel operations commute and are linearizable then the computation is internally deterministic.

Commutative Operations: Examples

- write-with-min, write-with-add
- dictionary
  - inserts commute
  - deletes commute
  - searches commute
- union-find structure
  - find(x) : commute
  - link(r_1, x_1) and link(r_2, x_2) commute if $r_1 \neq r_2$

Specific Approaches

- Functional programming
- History independent data structures
- Deterministic reservations

Functional programming
Suffix array
Comparison sort
N-body
K-nearest neighbors
Triangle ray intersect

History-independent data structures
Remove duplicates
Delaunay refinement

Deterministic reservations
Spanning forest
Minimum spanning forest
Maximal independent set
Breadth first search
Delaunay triangulation
Delaunay refinement

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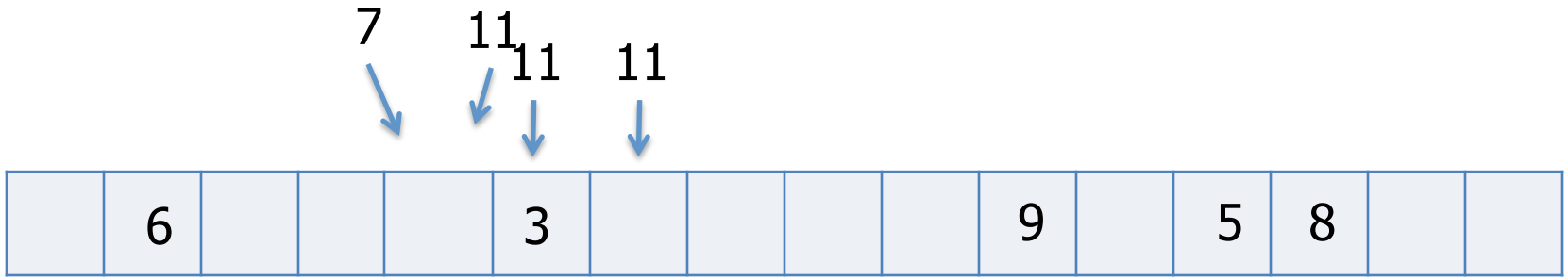
Removing Duplicates

Using hashing:

- Based on generic hash and comparison
- Problem: representation can depend on ordering. Also on which redundant element is kept.
- Solution: Use **history independent** hash table based on linear probing…once done inserting, representation is independent of order of insertion



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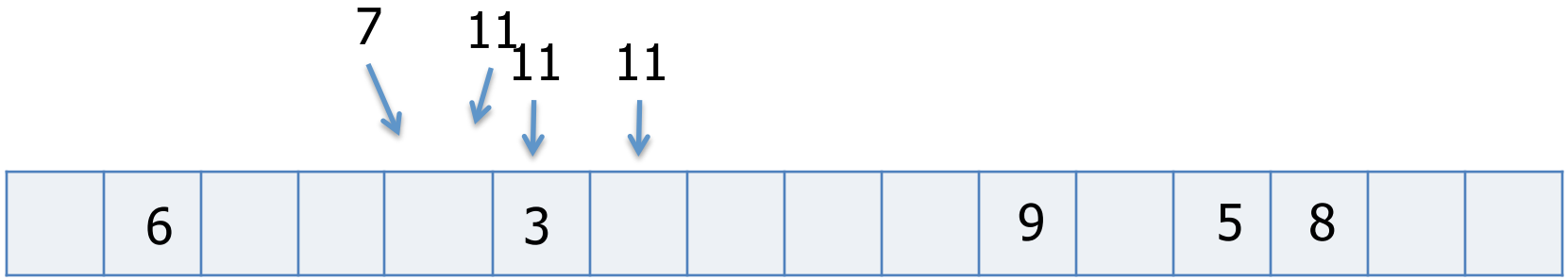

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Deterministic Reservations

Priority reserve all state that need to touch based on unique identifier. Only proceed if "won" on all reservations. Use write-with-min for reservation.

Example: speculative for

```
for (i=0; i<n; i++)
  f(i);
```

Want to simulate sequential order even with loop carried dependences which **don't commute.**

Speculative For

```
for (i=0; i<n; i++) f(i);
```

Converts to:

```
parallel_for (i=0; i<n; i++)
  reserve any shared vars with i
parallel_for (i=0; i<n; i++)
  if won all reservations, f(x)
  else mark for retry
```

Speculative For

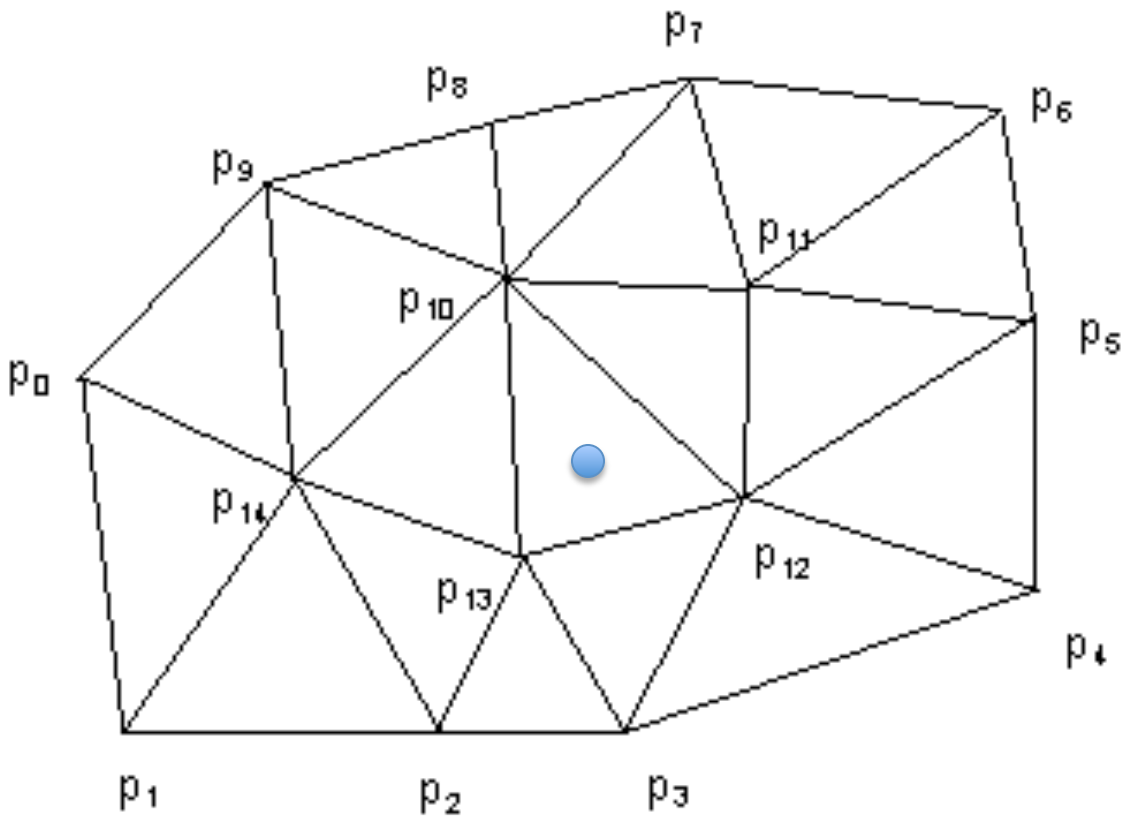
```
for (i=0; i<n; i++) f(i);
```

Or, more efficiently:

```
while (done < n)
  parallel_for (i=0; i<k; i++)
     reserve(I(i+done))
  parallel_for (i=0; i<n; i++)
     if succeed f(I(i+done))
  pack remaining indices into I
```

Delaunay Triangulation

- Adding points deterministically



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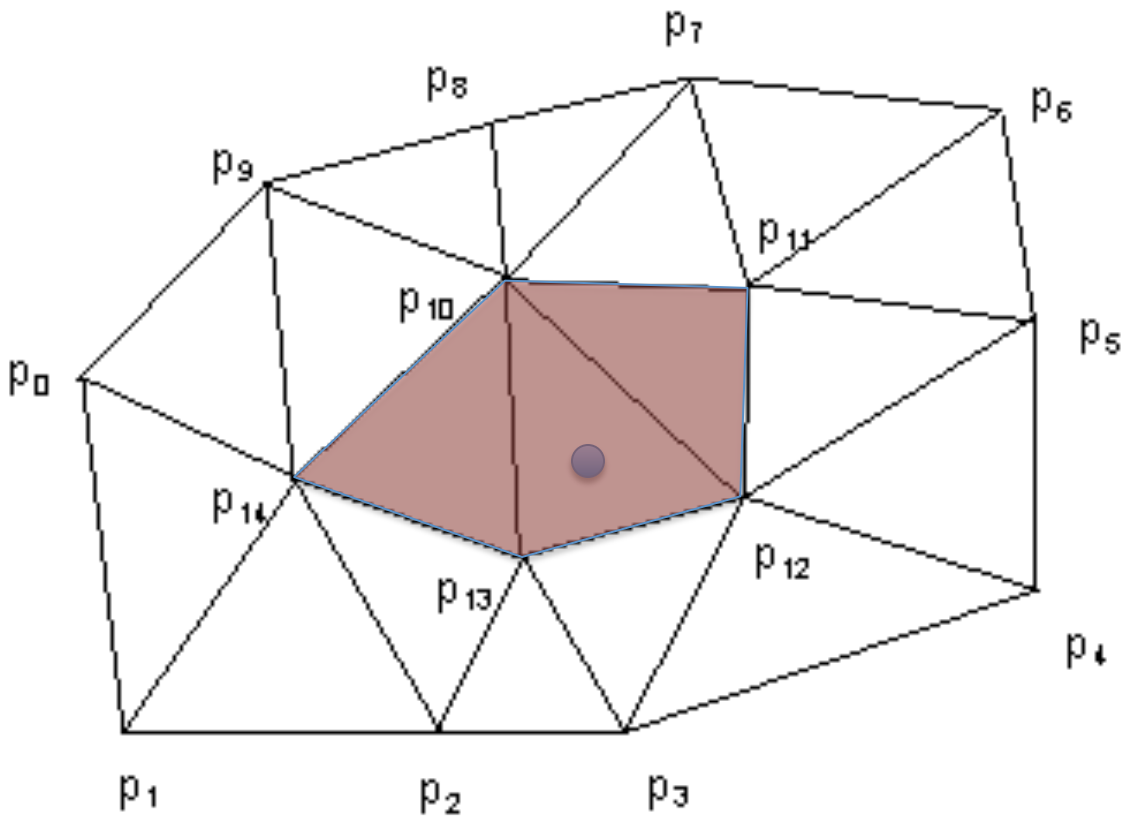

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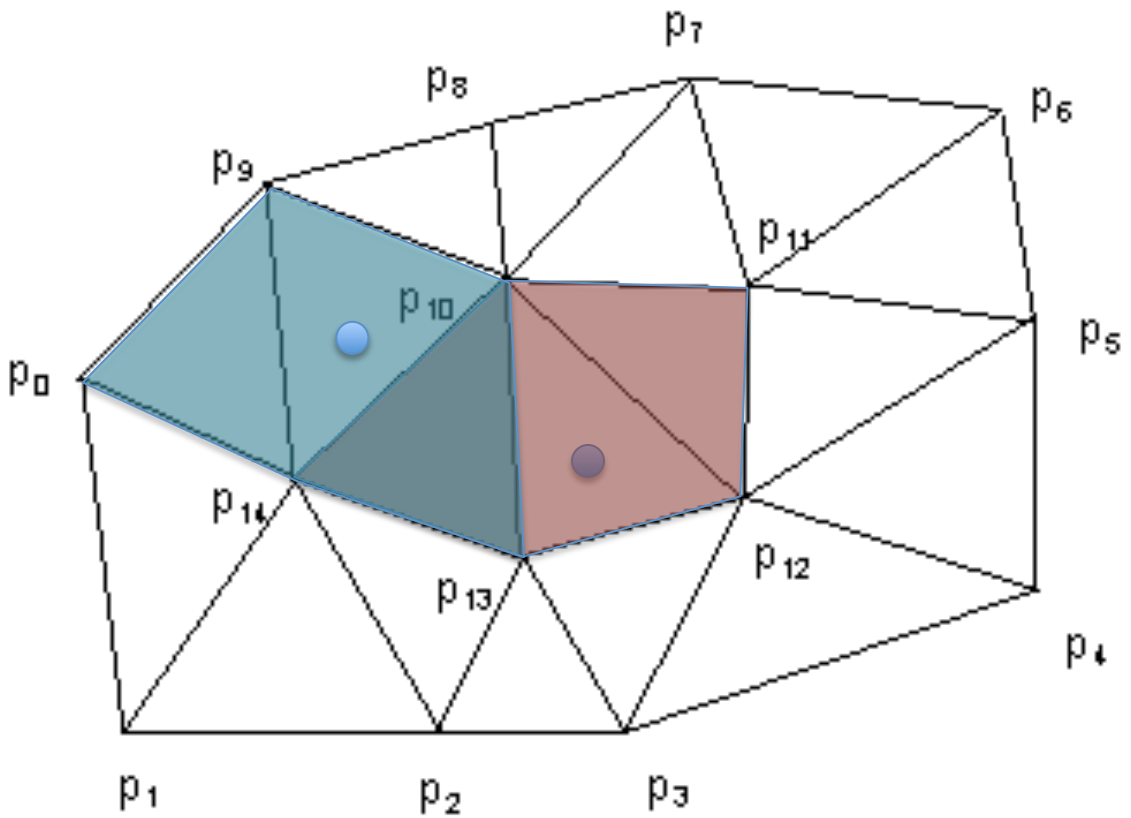

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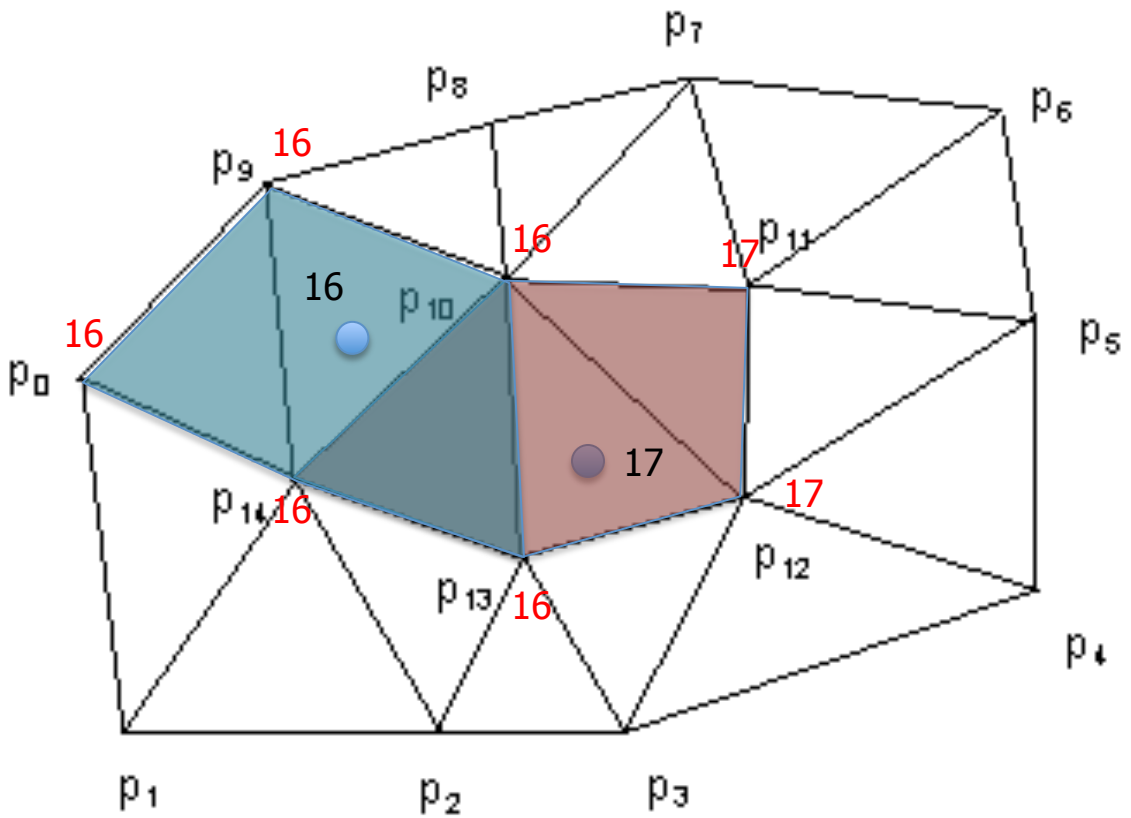
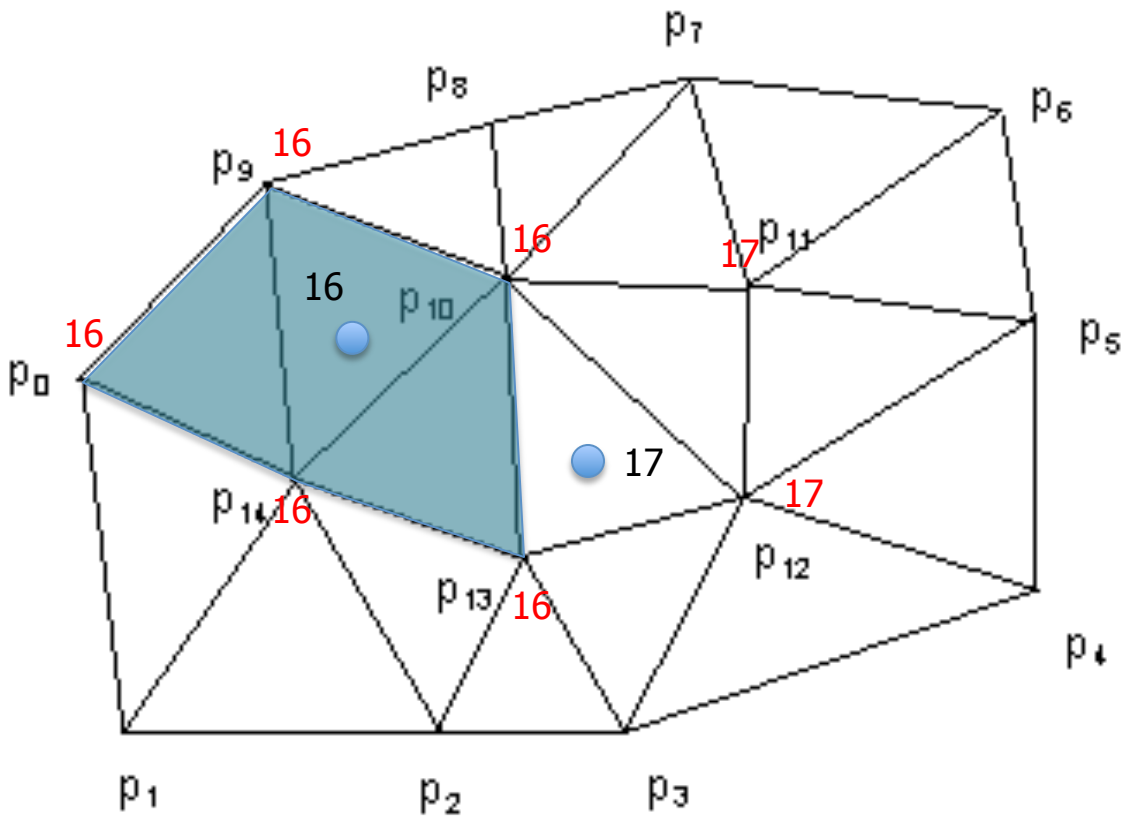


Delaunay Triangulation

- Adding points deterministically



Deterministic Reservations

Generic framework

```
iterates  = [1,…,n];
while(iterates remain){

        Phase 1: in parallel, all i in first P iterates
                        call reserve(i);

        Phase 2: in parallel, all i in first P iterates
                        call commit(i);

        Remove successfully committed i's from
            iterates;
}
```

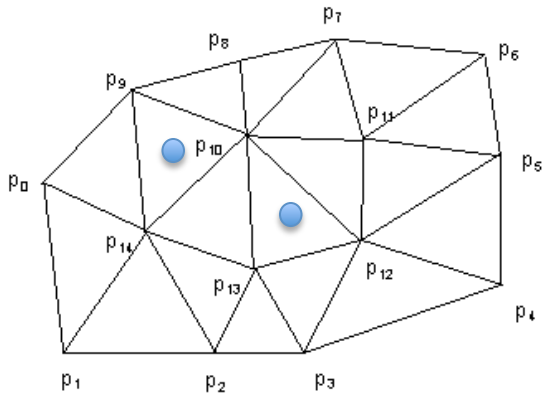
- Which iterates successfully commit is deterministic.

Delaunay triangulation

```
iterates:  points to be added

reserve(i){
      find cavity;
      reserve points in cavity;
}

commit(i){
      check reservations;
      if(all reservations successful){
            add point and triangulate;
      }
}
```



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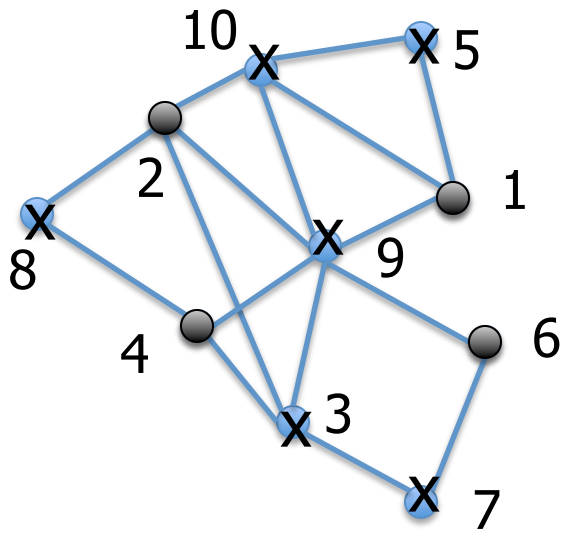
Maximal Independent Set

Important substep in Graph Coloring

Sequential algorithm:

```
for each u in V : S[u] = Remain
for each u in V
    if for all v in N(u), v < u, S[v] = Out
    then S[u] = In
    else S[u] = Out
```



Maximal Independent Set

Sequential algorithm:

```
for each u in V : S[u] = Remain
for each u in V
    if for all v in N(u), v < u, S[v] = Out
    then S[u] = In
    else S[u] = Out
```

Very efficient: most edges not even visited, simple loops

About 7x faster than sorting m edges

Maximal Independent Set

Same algorithm: with parallel speculation

```
for each u in V : S[u] = Remain
for each u in V
    if for all v in N(u), v < u, S[v] = Out
    then S[u] = In
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```
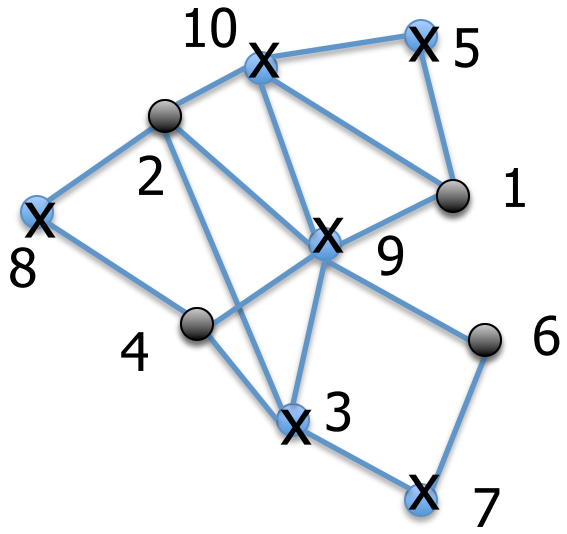
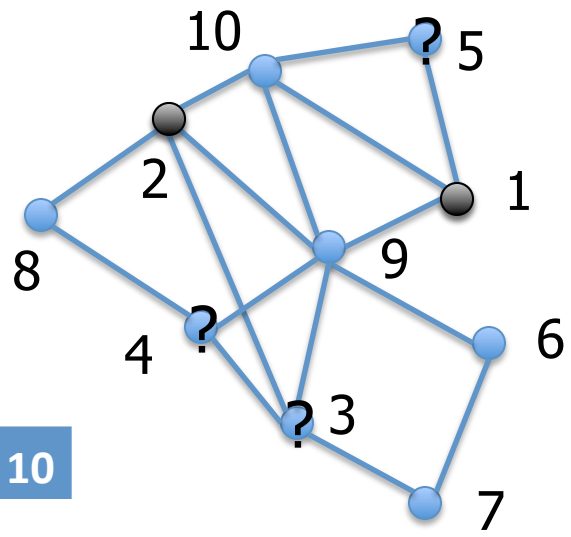



Maximal Independent Set

same algorithm: with speculation on prefix

```
for each u in V : S[u] = Remain
for each u in V
    if for all v in N(u), v < u, S[v] = Out
    then S[u] = In
    else S[u] = Out
```



1	2	3	4	5	6	7	8	9	10

MIS Parallel Code

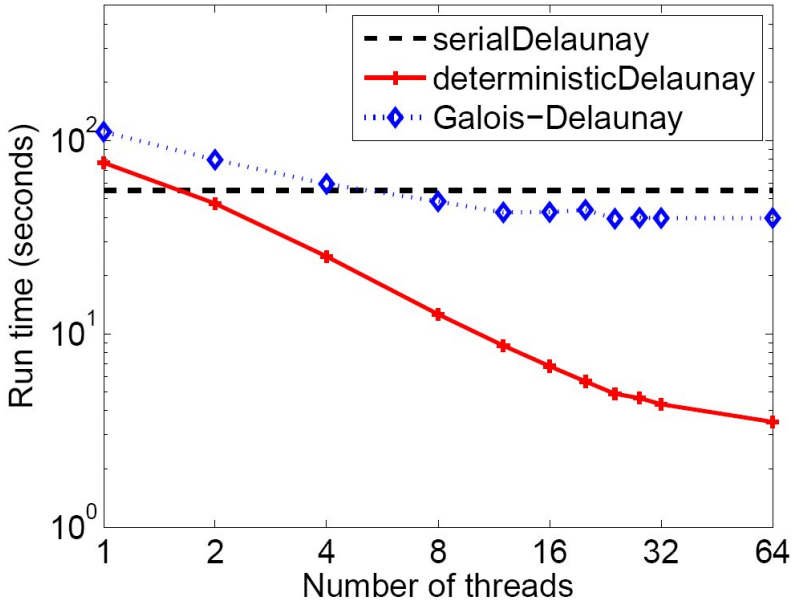
```
struct MISStep {
  bool reserve(int i) {
    int d = V[i].degree;
    flag = IN;
    for (int j = 0; j < d; j++) {
      int ngh = V[i].Neighbors[j];
      if (ngh < i) {
        if (Fl[ngh] == IN) { flag = OUT; return 1;}
        else if (Fl[ngh] == LIVE) flag = LIVE; } }
    return 1; }

  bool commit(int i) { return (Fl[i] = flag) != LIVE;}};

void MIS(FlType* Fl, vertex* V, int n, int psize)
  speculative_for(MISStep(Fl, V), 0, n, psize);}
```

Experimental Results

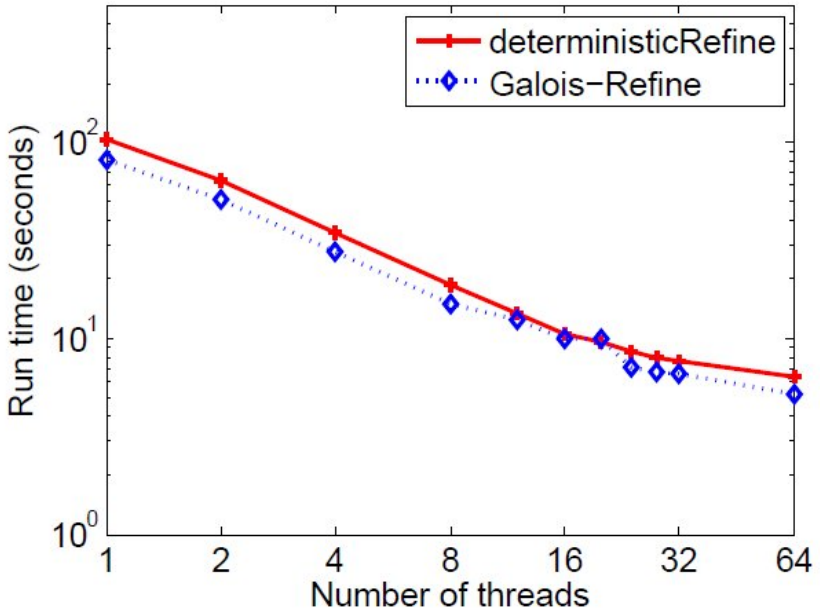
- We ran experiments comparing our deterministic implementations with serial and nondeterministic ones
- We used a 32-core (with hyper-threading) machine with 4 Intel X7560 Nehalem processors
- Used inputs drawn from a variety of distributions
- All codes are between 20 and 500 lines

Experimental Results

Delaunay Triangulation



Delaunay Refinement



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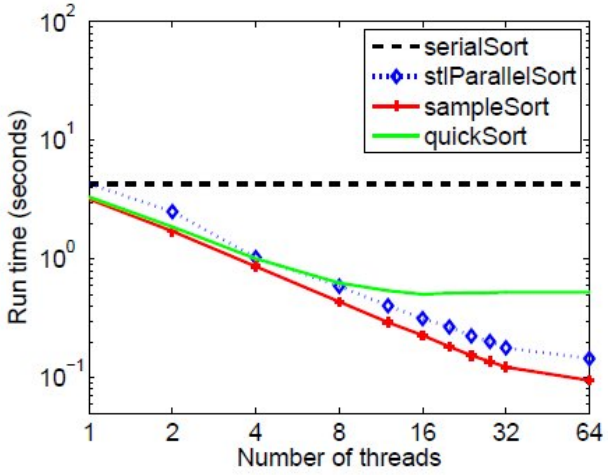
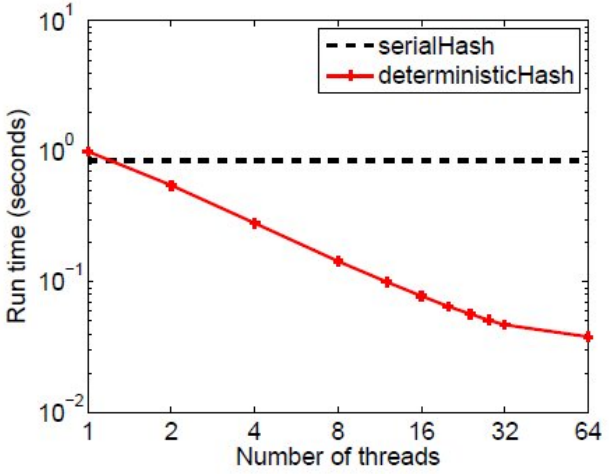
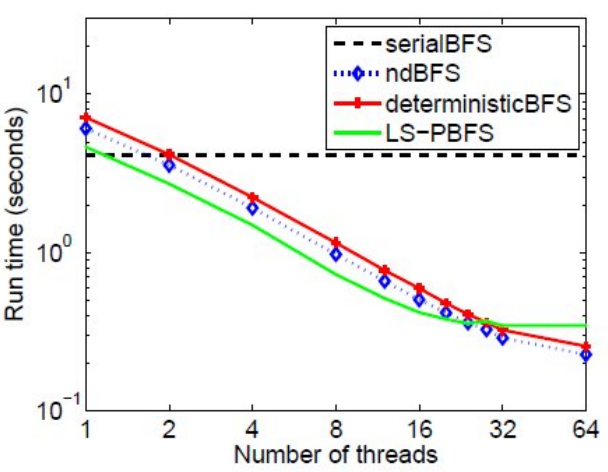


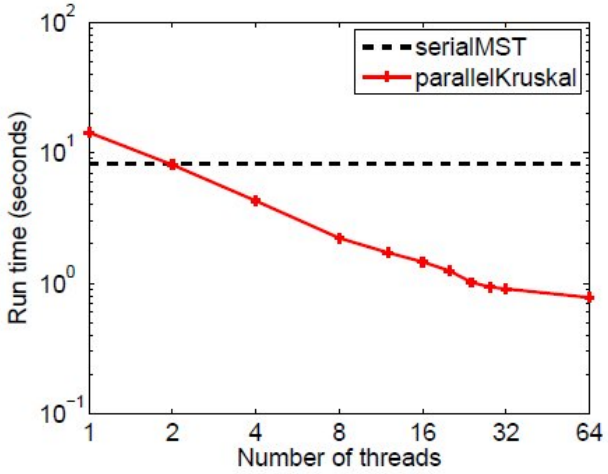
(a) comparison sorting algorithms with a **trigram string of length** 10^7



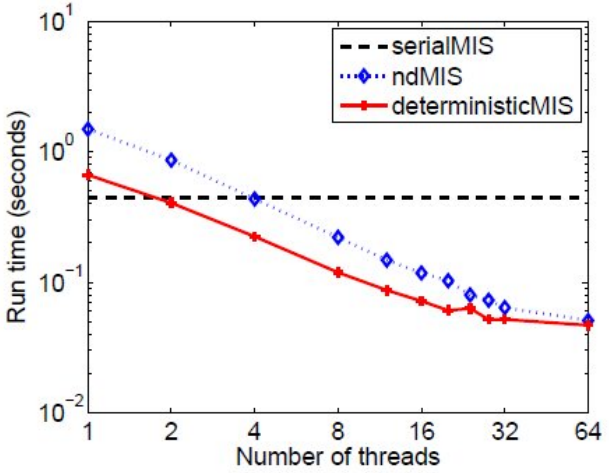
(b) remove duplicates algorithms with a **trigram string of length** 10^7



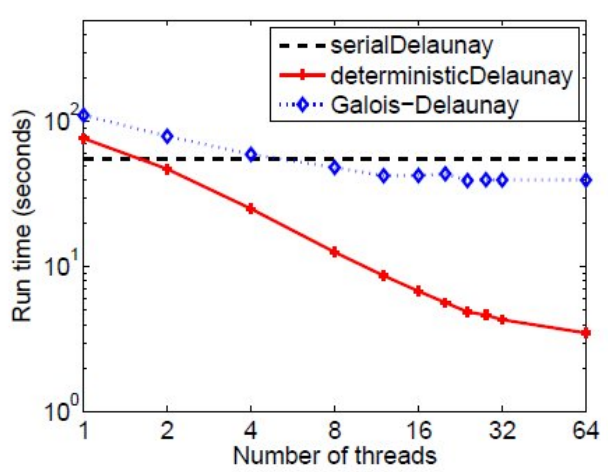
(c) BFS algorithms with a **random local graph** $(n = 10^7, m = 5 \times 10^7)$



(d) MST algorithms with a **weighted random local graph** $(n = 10^7, m = 5 \times 10^7)$



(e) MIS algorithms with a **random local graph** $(n = 10^7, m = 5 \times 10^7)$



(f) Delaunay Triangulation algorithms with a **2d in cube graph** $(n = 10^7)$

Experimental Results

- Up to 31.6x speedup on 32 processors
- Competitive with nondeterministic solutions
- On a single processor, our deterministic implementations are only 1–2.5x slower than their sequential counterparts

Conclusions

- Internal Determinism can be fast
- Some simple techniques can help
  - History independence
  - Deterministic reservations
  - Functional programming

Open questions:

- What types of problems truly benefit from a nondeterministic solution?
- Safe commutativity?

`http://www.cs.cmu.edu/~pbbs`