

Abduction

Farmer Brown is standing in the middle of his perfectly circular field feeling very content. It is midnight and there is no moon and unknown to the farmer, Martian zoologists are landing randomly at points on the circumference of his field. They land at one minute intervals, starting at midnight. As soon as there are martians at points A,B,C such that triangle ABC contains the center of the field, Farmer Brown will be teleported to the waiting space-ship and transported off to spend the rest of his life as an exhibit in a Martian zoo. What is the expected time until he is abducted?

Solution: The farmer is safe so long as there is an arc of the circle that is Martian free and spans half the circumference. Let X be the number of Martians who land altogether. Then for any integer $t \geq 1$,

$$P(X - 1 \geq t) = t(t - 1) \int_{x=0}^{1/2} x^{t-2} dx = \frac{t}{2^{t-1}}.$$

And,

$$E(X - 1) = \sum_{t=1}^{\infty} Pr(X - 1 \geq t) = \sum_{t=1}^{\infty} \frac{t}{2^{t-1}} = 4.$$

So the expected number of Martians is 5 and Farmer Brown is abducted at 12.04 AM.

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