

Democracy

The presidential elections are to be held in Anchuria. Of 20,000,000 voters only 1 percent (i.e. the regular army) support the current president Wobushko. He wants to be re-elected in a *democratic* way, which means the following. All voters are split into n_1 groups, all of equal size. Then each group can be split into n_2 smaller sub-groups of equal size, where n_2 is the same for all groups. Then each subgroup is split into n_3 equal sub-sub-groups, and so on. Each $(sub)^i$ -group chooses by majority rule one representative to represent it at level $i - 1$, and so on. (If there is a tie, the opposition wins.) Can Wobushko organize the groups and distribute his supporters so that he wins the elections?

Solution Yes, Wobushko can steal the election. Suppose in general that the number of voters is N and we write N as a product of non-trivial factors $N = N_k = n_1 \times n_2 \times \cdots \times n_k$. Then, letting $m_i = \lfloor n_i/2 \rfloor + 1$ we see that it is enough to have $M_k = m_1 \times m_2 \times \cdots \times m_k$ supporters in order to win the election. This follows by induction on k . The base case $k = 1$ is trivial. To win for general k , we have to divide the electorate into N_{k-1} groups of size n_k . Wobushko then packs M_{k-1} of these groups with m_k of his supporters. After the vote there are N_{k-1} voters altogether of which M_{k-1} are supporters of Wobushko, completing the induction.

In the numerical example we write $20,000,000 = 5^7 \times 4^4$. In which case Wobushko needs to have $3^7 \times 3^4 = 177147$ supporters which is less than 1 percent.

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