

Which door?

The rest-rooms on the 7th floor of Wean hall have just been renovated. Unfortunately, the contractors omitted to label the doors with Men/Women. A visitor to the Mathematical Sciences Department arrives outside the rest-rooms and does not wish to go in the wrong door. Standing outside the door are the famous Crane triplets: Ampule, Botule and Corpule. These guys are identical. Their own mother cannot tell them apart. It is well-known in the academic world that Ampule is a good person and always tells the truth, Botule is a mean person and always lies. Corpule is confused and half the time he tells the truth and the other half of the time, he lies. Our visitor knows that he is allowed two questions. What should they be? (Note that a question will be directed to one triplet who will answer it in his own way.)

Solution: Here are some suggested solutions to this conundrum.

(a) We use in our solution the solution to a simpler problem, in which there are only two question-answerers: a truth-teller and a liar. In this problem, we must determine which is the correct door using only one question. The solution is well known: pick one of the two answerers and ask the following question:

Q: "Which door would the OTHER answerer say leads to the men's room?"

If the response is "Door A" go through Door B and vice-versa.

We wish to reduce our situation to this situation using one question. Corpule is a confounding factor, as we never know what response he will give so it is hard to base a decision of what to do on him. However, suppose that using only one question we can ensure that our SECOND question will be asked of either Ampule or Botule. Then we can solve the problem.

So, we choose someone arbitrarily, say person P, and ask the following question:
Q0: "Of the other two, which one is most likely to give me a truthful answer when I ask them a question?"

We then ask question Q1 of whoever was NOT the answer to question Q0 and is not person P.

Q1: "One of the other two is not Corpule. What answer would that person give when asked which door leads to the men's room?"

Then you know which door leads to which room.

Let us confirm that this ensures question Q1 is asked of either Ampule or Botule. If P is Corpule, then question Q1 is not asked of the Corpule as we always switch who we are asking. If P is Botule, he will tell us that Corpule is more likely to tell the truth and we will choose the other person, namely Ampule.

If P is Ampule, he will tell us that Corpule is more likely to tell the truth and we will choose the other person, namely Botule.

Thus in all cases our second question is asked of Ampule or Botule.

(b) Here is another solution:

Definition: "Consistent" means someone who always tells the truth OR always tells a lie.

Observation: When you ask a consistent person "If I asked you binary question X, what would you answer?", their answer to this question will always be a

correct answer to question X, thus turning a liar into a truth teller.

So, here are the two questions:

Question 1, to one of the triplets (and pointing to another one): "If I asked you if this brother is Consistent, what would you answer?"

If the answer is YES, ask Question 2 of the person previously pointed to. Otherwise, ask Question 2 of the third brother.

Question 2: "If I asked you if this (pointing to one of the bathrooms) is the Men's bathroom, what would you answer?"

If the answer is YES, the bathroom pointed to is the men's bathroom, otherwise, it's otherwise.

Explanation: if the first brother being asked is consistent, then the second brother being asked is also consistent. If the first brother being asked is not consistent, then both his brothers are consistent, and therefore regardless of how he answers, question 2 is asked of a consistent person.

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