

Here comes Santa

Santa has been working hard all year and is now waiting at the bottom left corner $(0, 0)$ of Gridville, sacks loaded with presents, ready to deliver them to the children waiting at the top edge $\{(n + 1, x) : 0 \leq x \leq n\}$. Santa laid off too many elves last year and now things are running late. There are only $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ days to Xmas. The Grinch sees his chance to mess things up. Santa's reindeer love Magic Reindeer Food (made from sugar, oatmeal and glitter) and the Grinch sprinkles $F_{i,j}$ days worth of food on grid square (i, j) for $1 \leq i, j \leq n$. Magic Reindeer Food is in short supply and the Grinch can only afford to spread one days supply on each row i.e. $\sum_{j=1}^n F_{i,j} = 1$ for $1 \leq i \leq n$. If Santa's journey takes him to square (i, j) then he will have to spend $F_{i,j}$ of a day there while the reindeer's eat the food that the Grinch has left.

Santa starts at $(1, 1)$ and waits $F_{1,1}$ days and then moves to $(2, 1)$ or $(2, 2)$. In general, if Santa is at (i, j) , then after waiting $F_{i,j}$ of a day, he moves to one of $(i + 1, j - 1)$, $(i + 1, j)$, $(i + 1, j + 1)$. (Of course if $j = 1$ or n only 2 of these moves are allowed).

Santa can make all his deliveries after reaching a square (n, j) (and waiting $F_{n,j}$) because he can mount a non-deterministic delivery service from this time on. Can he make it in time, or will the Grinch ruin Xmas?