# Privacy and Network Analysis: Examples and Questions

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#### Outline

- Introduction
  - The R-U framework
  - The traditional data privacy approaches
- Networks
  - Analysis using networks
    - Knowledge management example
- Privacy in Networks
  - Why is it complicated?
  - How does privacy protection affect analysis/inference?
  - Interesting open problem

### The basic problem

- Micro data about individuals
  - Relational tuples with data about individual attributes.
     Each tuple assumed to be independent of the other.
  - Today: Network data from call data records, blogs, friendship networks etc.
- Publish micro-data
  - Maximize utility from the data
  - Subject to confidentiality constraints

# The R-U Confidentiality Map (Duncan et al, 2001)



#### Utility

Utility –example 1: Inverse of the RMSE of the estimate of a statistic such as the sample Mean

example 2: sum of tuple information loss criterion

Risk – example 1: Width of the interval at a specified confidence level of value of a Confidential variable that will lead to re-identification; example 2: value of k in K-anonymity

## The Standard Privacy Problem

#### **Variables**

#### "Solutions":

- Deleting cases
- Aggregating cases
- Deleting variables
- Adding noise
- perturbations
  - •K-anonymity
  - L-diversity

Units

## Micro-data: an example

	Non-Sensitive			Sensitive
	Zip Code	Age	Nationality	Condition
1	13053	28	Russian	Heart Disease
2	13068	29	American	Heart Disease
3	13068	21	Japanese	Viral Infection
4	13053	23	American	Viral Infection
5	14853	50	Indian	Cancer
6	14853	55	Russian	Heart Disease
7	14850	47	American	Viral Infection
8	14850	49	American	Viral Infection
9	13053	31	American	Cancer
10	13053	37	Indian	Cancer
11	13068	36	Japanese	Cancer
12	13068	35	American	Cancer

Figure 1. Inpatient Microdata	<b>Figure</b>	1. Ir	patient	Microdata
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	N	Von-Sen	Sensitive	
	Zip Code	Age	Nationality	Condition
1	130**	< 30	*	Heart Disease
2	130**	< 30	*	Heart Disease
3	130**	< 30	*	Viral Infection
4	130**	< 30	*	Viral Infection
5	1485*	$\geq 40$	*	Cancer
6	1485*	$\geq 40$	*	Heart Disease
7	1485*	$\geq 40$	*	Viral Infection
8	1485*	$\geq 40$	*	Viral Infection
9	130**	3*	*	Cancer
10	130**	3*	*	Cancer
11	130**	3*	*	Cancer
12	130**	3*	*	Cancer

Figure 2. 4-anonymous Inpatient Microdata

Source: Machanavajjhala et al., 2008

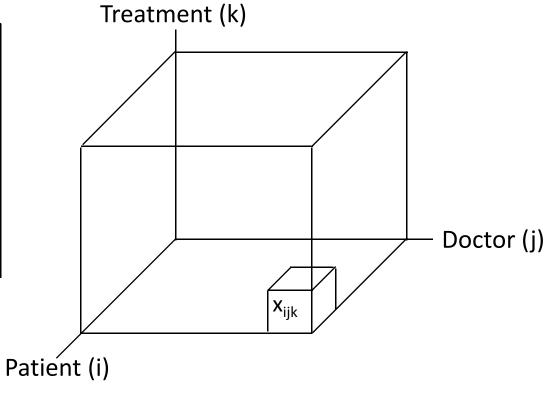
#### The Canonical 3-D Problem

Table: OfficeVisit

v#	Patient	Doctor	Treatment
122	David	Christy	Compoz
123	John	Phillips	Fungicide
124	Israel	Christy	AZT
125	John	Hill	Compoz
:	:	:	:

 $x_{ijk}$  = count of visits over

Patient i i = 1,...,IDoctor j j = 1,...,JTreatment k k = 1,...,K

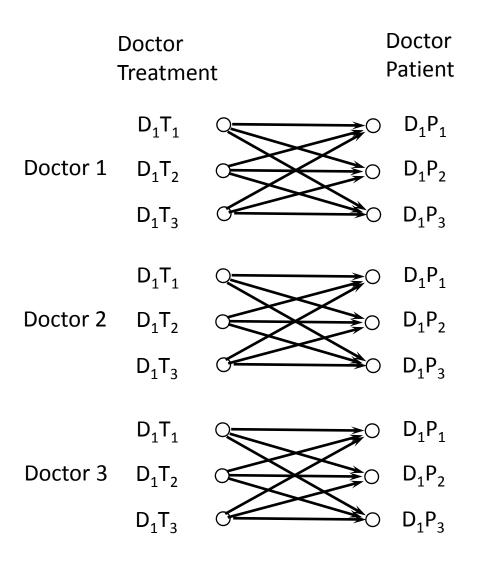


#### The "Third Projection Problem"

(Chowdhury, Duncan, Krishnan, Roehrig, Mukherjee)

- Given two 2-D projections, find bounds on cell values of the third 2-D projection
- Example: Given Patient-Doctor and Doctor-Treatment, find bounds on the sensitive table
   Patient-Treatment

#### The Decomposed Network



Arcs represent "flows" of treatments from doctor to patient.

The network splits into three smaller subgraphs.

Patient-Treatment maxima and minima are derived from flow algorithms.

Results correspond to MCA.

#### Results: Two-D Projection Bounds

Let A =  $[a_{ij}]$ , B =  $[b_{jk}]$  and C =  $[c_{ik}]$  be the two-dimensional projections of the three-dimensional table T =  $[t_{ijk}]$ .

**Proposition:** It is not possible in general to determine the entries of C given those of A and B.

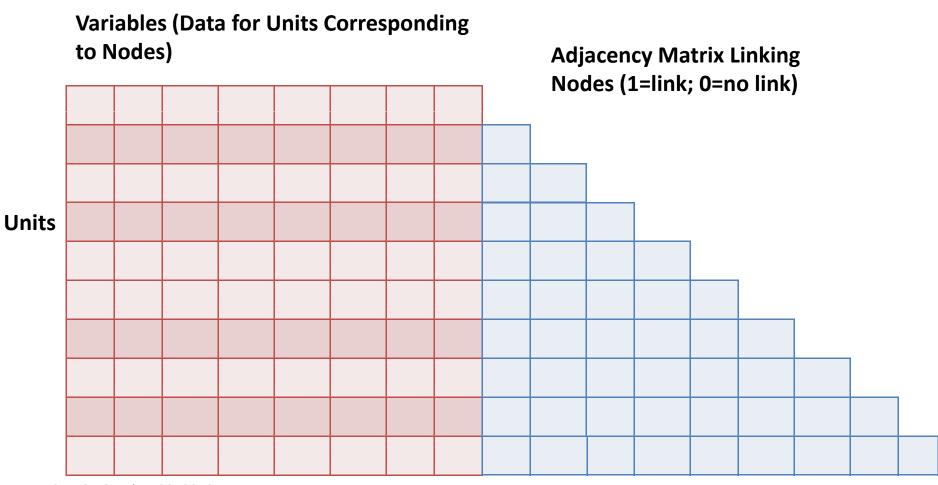
**Proposition (MCA):** Optimal upper bounds for the third projection  $C = [c_{ik}]$  are given by

$$C^{U}_{ik} = A \overline{\bigotimes} B = \sum_{j} \min(a_{ij}, b_{jk}).$$

Optimal lower bounds for C are given by

$$C_{ik}^{L} = A \otimes B = \sum_{j \text{ max}} (a_{ij} - \sum_{p \neq k} b_{jp'} 0).$$

### The Network Privacy Problem



SAMSI, October 20, 2010 11

# Society as a Graph

People are represented as nodes.

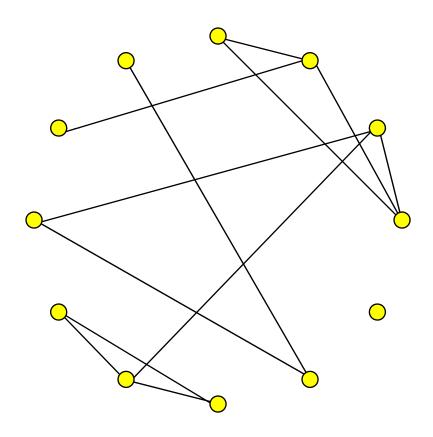
Source of next 3 slides: Rao, 2009

# Society as a Graph

People are represented as nodes.

Relationships are represented as *edges*.

(Relationships may be acquaintanceship, friendship, co-authorship, etc.)



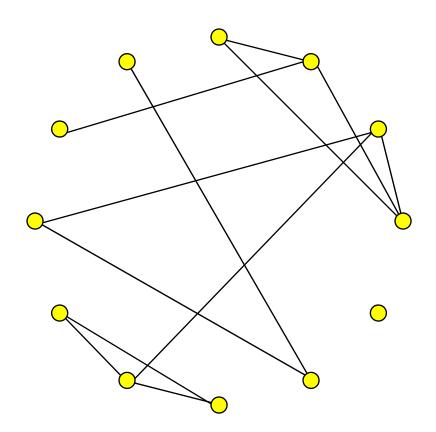
# Society as a Graph

People are represented as nodes.

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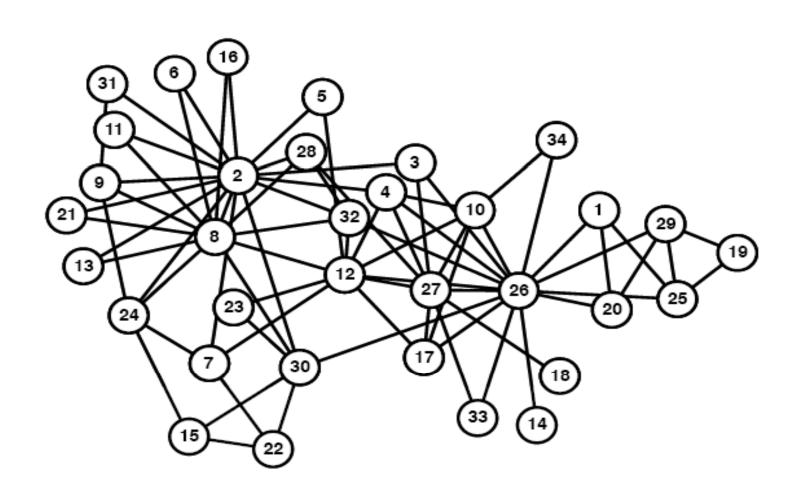
Allows analysis using tools of mathematical graph theory



#### The problem

- Publish network data
  - Maximize utility from the data
  - Subject to confidentiality constraints
- Anonymize the network
  - Naïve approach of anonymizing node labels does not work (Hay, 2010) based on assumption of some prior background knowledge
    - Degree signature attack
    - Degree signature of node and that of neighbors
    - Leading to node re-identification and edge disclosure
    - But good from the standpoint of analysis since topology is not altered

## Karate Club network- Anonymized



# Network mappings

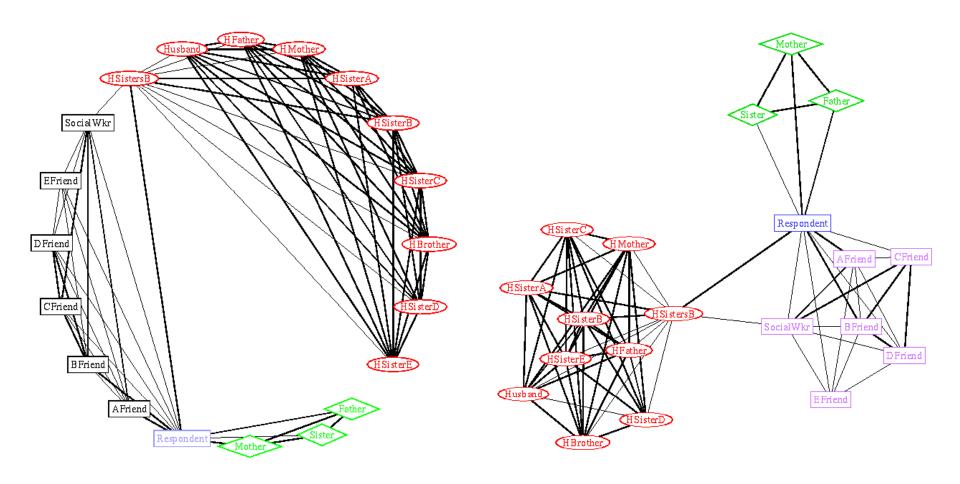
Id	Gender	Age	$\operatorname{Belt}$	Injury	
16	F	18	yellow	ankle	
6	Μ	21	white	ankle	
31	${ m F}$	42	white	elbow	
11	M	16	white	none	
9	M	19	white	clavicle	
21	M	21	white	knee	
13	F	21	$\mathbf{white}$	none	
26	M	36	black	none	
8	${ m F}$	22	brown	none	
2	M	45	black	ribs	

Edge	YearsKnown
{ 16, 8 }	2
$\{ 16, 2 \}$	3
$\{ 6, 8 \}$	10
$\{ 6, 2 \}$	11
{ 31, 2 }	20
{ 31, 9 }	1
	• •••

Alice	16
Bob	6
Carol	31
Dave	11
Ed	9
Fred	21
Gail	13
Mr. Hi	26
	•••

### But first, a network analysis discussion

## Visualization Software: Krackplot



Sources: http://www.andrew.cmu.edu/user/krack/krackplot/mitch-circle.html http://www.andrew.cmu.edu/user/krack/krackplot/mitch-anneal.html

#### Connections

- Size
  - Number of nodes
- Density
  - Number of ties that are present the amount of ties that could be present
- Out-degree
  - Sum of connections from an actor to others
- In-degree
  - Sum of connections to an actor

#### Distance

- Walk
  - A sequence of actors and relations that begins and ends with actors
- Geodesic distance
  - The number of relations in the shortest possible walk from one actor to another
- Maximum flow
  - The amount of different actors in the neighborhood of a source that lead to pathways to a target

## Some Measures of Power & Prestige

(based on Hanneman, 2001)

- Degree
  - Sum of connections from or to an actor
    - Transitive weighted degree → Authority, hub, pagerank
- Closeness centrality
  - Distance of one actor to all others in the network
- Betweenness centrality
  - Number that represents how frequently an actor is between other actors' geodesic paths

### Cliques and Social Roles

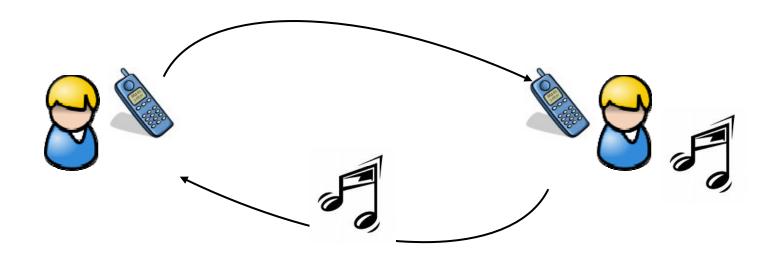
(based on Hanneman, 2001)

- Cliques
  - Sub-set of actors
    - More closely tied to each other than to actors who are not part of the sub-set
      - (A lot of work on "trawling" for communities in the web-graph)
      - Often, you first find the clique (or a densely connected subgraph)
         and then try to interpret what the clique is about
- Social roles
  - Defined by regularities in the patterns of relations among actors

# Statistical approaches to network analysis

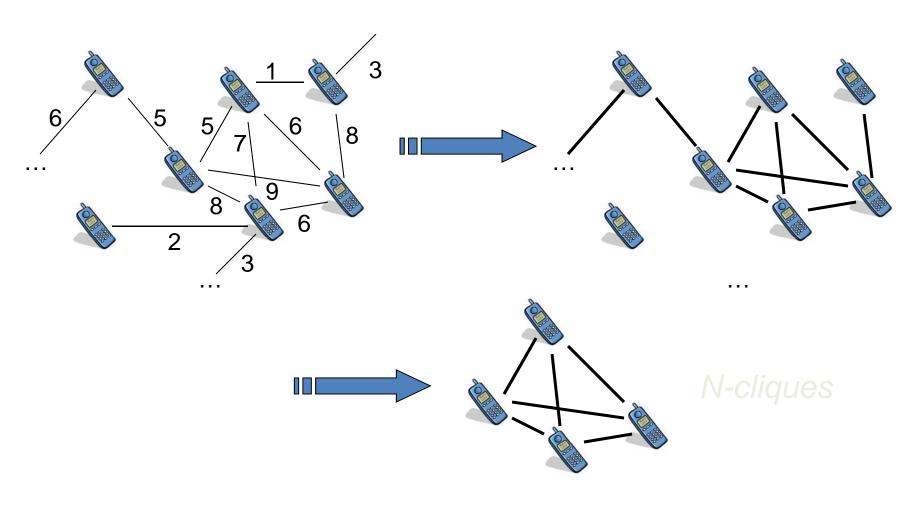
- Markov Graph-based models
  - Exponential random graph-based models
- Permutation test and regression-based approaches
  - E..g, QAP regression variants due to David
     Krackhardt at Heinz

# Example 1: Product adoption – CRBT



Caller ringback tones

# Groups



### **Exponential Random Graphs**

 Very general families for modeling a single static network observation.

$$P(N) = \exp\{\theta \cdot u(N) - \ln Z(\theta)\}\$$

- Can estimate the θ parameters by MCMC MLE
- N is a network vector, u(N) are a set of sufficient statistics to estimate the parameter theta of the model

# ERGM Example: CRBT-purchase in a cell phone network

- Classic example: (Frank & Strauss 1986)
- Once model is estimated, it can be used to predict the likelihood that a link will form between node I and node J
  - $u_1(N) = \# edges in N$
  - $u_2(N) = # 2-stars in N$
  - $u_3(N) = # triangles in N$

$$P(N) \propto \exp\{\theta_1 u_1(N) + \theta_2 u_2(N) + \theta_3 u_3(N)\}$$

# Example 2: Analyzing an Intraorganizational blogosphere

### Background

- Study conducted on an employee-only technical forum in a "top 5" Indian IT service provider
- Web-based Forum intended to serve two purposes:
  - Transfer knowledge across employees in different 'silos' by allowing anyone to post responses to queries
  - Archive posted discussions or threads for subsequent retrieval

#### Sample Query

- Query on: Singleton class and threads in Java
- Responses:
- 1. Singleton class means that any given time only one instance of the class is present, in one JVM. So, it is present at JVM level.
- 2. The thing is if two users(on two different machines which has separate JVMs) are requesting for singleton class then both can get one-one instance of that class in their JVM.

#### Sample data posting of query and responses

threadid	associateid	postedtime	messagetype	subject	message
{20070110-	138242	2007-01-10 06:41:15	Query	Panel Creation in REXX	Hi,
(20070110-	122971	2007-01-10 07:42:54	Response	Re: Panel Creation in REXX	For retaining the input panel
{20070110-	107246	2007-01-10 13:20:24	Response	Re: Panel Creation in REXX	You are not creating the
{20070110-	128623	2007-01-17 07:19:18	Response	Re: Panel Creation in REXX	No need to VPUT you can
{20070110-	129498	2007-03-01 12:31:42	Response	Re: Panel Creation in REXX	it's simple if var1 var2 are the
{20070110-	107246	2007-03-01 13:49:16	Response	Re: Panel Creation in REXX	TYPE(INPUT) is to define the
{20070110-	125034	2007-04-14 07:17:32	Response	Re: Panel Creation in REXX	You can use the command
{20070110-	107246	2007-04-14 23:43:30	Response	Re: Panel Creation in REXX	<em><strong>ADDRESS</strong></em>

### Data description

- Message level and thread-level data from forum
- Message characteristics
  - Posting time, EmployeeID, Thread, Type of message (query or response), content of message etc.
- User characteristics
  - EmployeeID, Tenure at firm, Age, Gender,
     Location, Division, Job Title

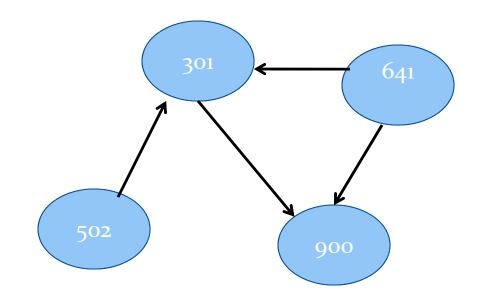
# Summary statistics of forum (8/2006-8/2007)

Statistic	Value
Total number of users participating	2974
Total number of queries	20090
Total number of responses	59038
Average responses per query	2.9
Average messages per day	162
Average time to first response	58 min
Number of users only posting queries	343
Number of users only posting responses	1377
Number of users posting queries and responses	1004

#### Network structure evolution

#### *Sequence of Actions:*

- User 301 posts a query Q1000
- Users 502, 641 post responses
- User 900 posts a query Q1001
- Users 301, 641 post responses

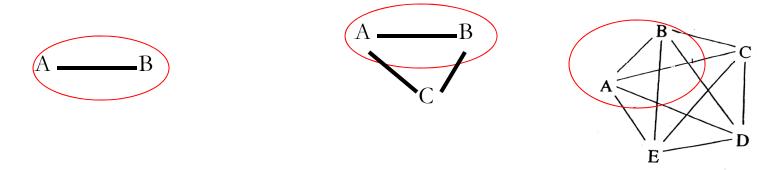


**Directed Response Graph** 

#### Simmelian Ties

Why should this matter? Theory

1908: Simmel's argument that Triads are different from Dyads, but adding more does not matter



Triads form **Groups**, with Norms, Rules, Values, Common Understandings, Pressure towards Compliance, Conformity and Cooperation

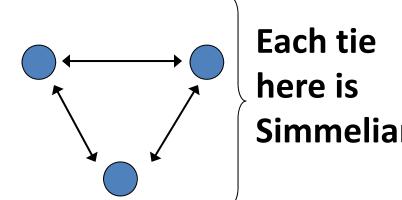
**Simmelian Decomposition:** Each network tie can be characterized as one of three mutually exclusive and exhaustive types:

Asymmetric: 
$$(a \to b)$$
  $\longrightarrow$   $\bigcirc$  Sole-Symmetric:  $(a \to b) \land (b \to a) \bigcirc$   $\longleftarrow$   $\bigcirc$  ... but not Simmelian

Simmelian: 
$$(a \rightarrow b) \land (b \rightarrow a)$$
 and

$$\exists c \ s.t. \ (c \rightarrow a) \land (a \rightarrow c)$$

$$\wedge (c \to b) \wedge (b \to c)$$



# Research Question

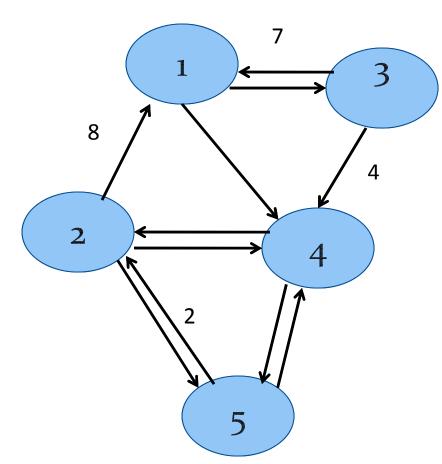
- Is the probability of response to a question posed by a node I contingent on the network structure that the node is embedded in?
  - Simmelian tie
  - Symmetric tie
  - Asymmetric tie
- Does the nature of the question (popular or not) which determines the context within which the tie was established make a difference?

## Response Matrix

#### RESPONSES

$$= \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 8 & 1 & 0 & 1 & 1 \\ 7 & 0 & 1 & 4 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 & 1 \end{pmatrix}$$

#### **Directed Response Graph**

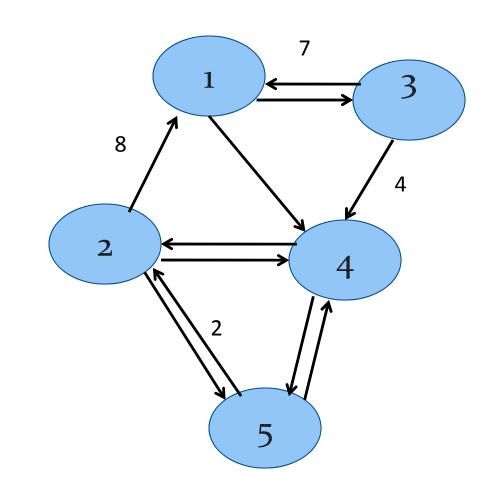


 $RESPONSES_{i,j}$ 

= number of times 'i' responds to 'j'

#### **SIMMELIAN**

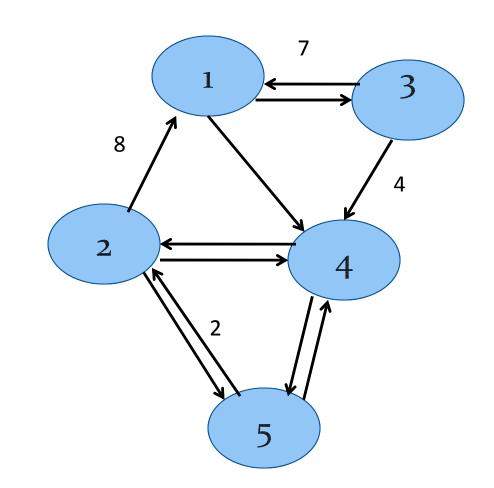
$$= \begin{pmatrix} X & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 1 & 1 \\ 0 & 0 & X & 0 & 0 \\ 0 & 1 & 0 & X & 1 \\ 0 & 1 & 0 & 1 & X \end{pmatrix}$$



 $SIMMELIAN_{i,j} = 1$  if 'i' and 'j' have a Simmelian tie

#### NON – SIMMELIAN

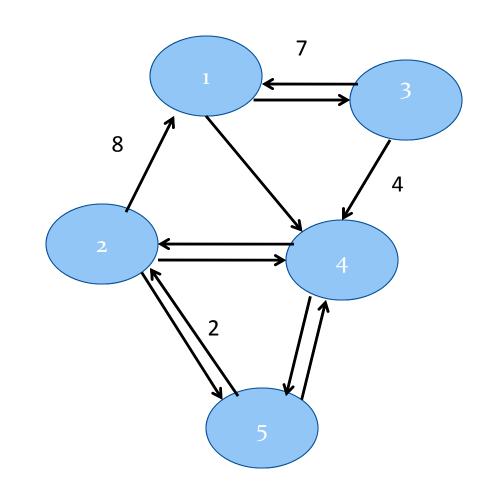
$$= \begin{pmatrix} X & 0 & 1 & 1 & 0 \\ 1 & X & 0 & 0 & 0 \\ 1 & 0 & X & 1 & 0 \\ 0 & 0 & 0 & X & 0 \\ 0 & 0 & 0 & 0 & X \end{pmatrix}$$



 $NON - SIMMELIAN_{i,j} = 1$  if 'i' and 'j' have a non-Simmelian tie

## Age difference

$$= \begin{pmatrix} 0 & 18 & 26 & 28 & 40 \\ 18 & 0 & 8 & 10 & 22 \\ 26 & 8 & 0 & 2 & 14 \\ 28 & 10 & 2 & 0 & 12 \\ 40 & 22 & 14 & 12 & 0 \end{pmatrix}$$



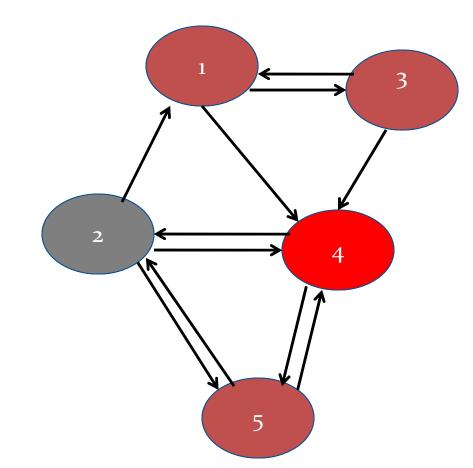
 $ABS\_AGEDIFF_{i,j}$ 

= absolute value of age difference between 'i' and 'j' (months)

#### **Locations Color Coded**

#### **SAMELOCATION**

$$= \begin{pmatrix} X & 0 & 1 & 0 & 1 \\ 0 & X & 0 & 0 & 0 \\ 1 & 0 & X & 0 & 1 \\ 0 & 0 & 0 & X & 0 \\ 1 & 0 & 1 & 0 & X \end{pmatrix},$$

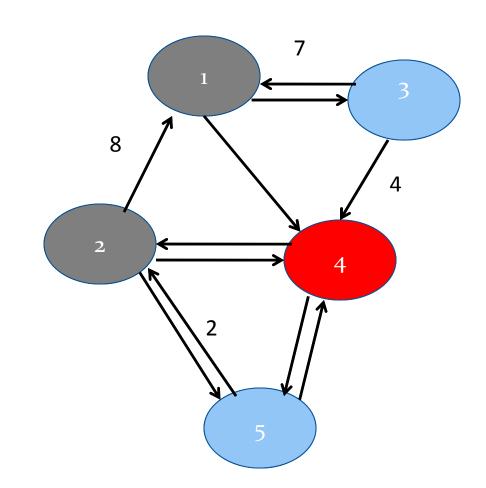


 $SAMELOCATION_{i,j} = 1$  if 'i' and 'j' are collocated

#### Verticals Color Coded

#### **SAMEVERTICAL**

$$= \begin{pmatrix} X & 1 & 0 & 0 & 0 \\ 1 & X & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 1 \\ 0 & 0 & 0 & X & 0 \\ 0 & 0 & 1 & 0 & X \end{pmatrix},$$



 $SAMEVERTICAL_{i,j} = 1$  if 'i' and 'j' are part of the same vertical

# **Empirical Methodology**

#### Want to characterize response behavior due to:

- Homophily
- Content
- Prior Network structure

## Cannot use ordinary least squares regression

- Autocorrelation induced because of structural factors
  - Some users may respond more to all others
- Unbiased, but significance tests will be incorrect
- Use QAP (Quadratic Assignment Procedure) to test for significance
  - Krackhardt (1987) reference

# QAP - Regression

- Variant of QAP (Double Semi-Partialing)
  - Dekker et al (Psychometrika, 2007)
- Divide the data into two periods
  - P1 Aug 2006 Feb 2007
  - P2 Feb 2007 Aug 2007
- Dependent variable is
  - Number of responses by A to B in period two
- Explanatory Variables:
  - Structural Properties in period one
  - Dyadic Homophily Measures

# **QAP** Regression Specification

#### **Dependent Variable**

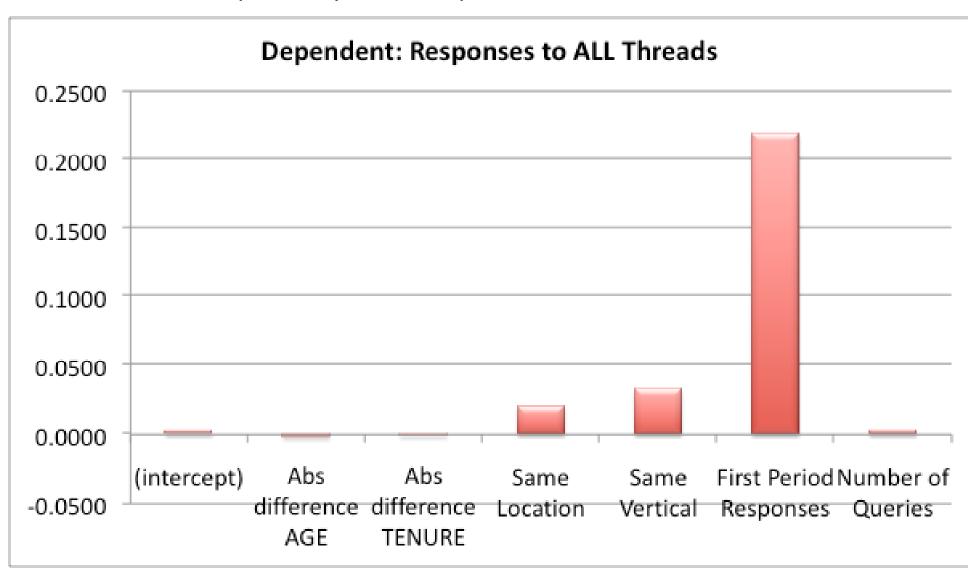
Y<sub>t</sub> = Number of responses from A to B in period 't'

#### Independent variables

- Abs(Difference between age)
- Abs(difference between tenure),
- Same location city dummy,
- Same vertical dummy,
- Number of queries posted,
- Structural Factors:
  - Simmelian and Non-simmelian of responses to:
  - (a) Low SP (Non-instrumental) threads
  - (b) High SP (Instrumental) threads

#### Dependent variable:

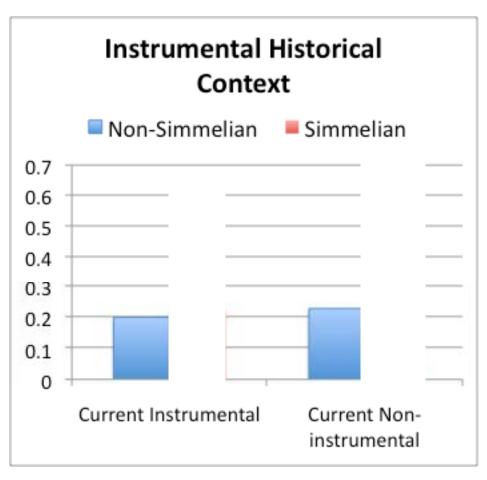
Number of response by A to B in period two

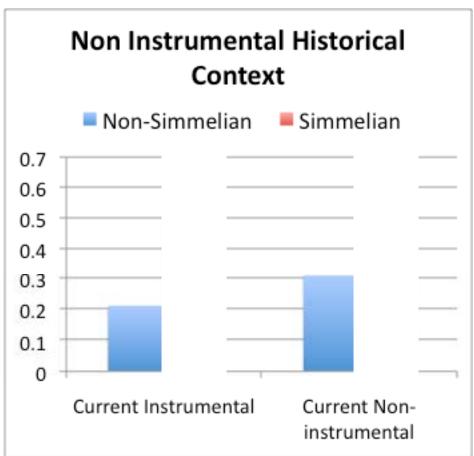


Dependent variable:

Number of response by A to B in period two

**Explanatory Variables:** 

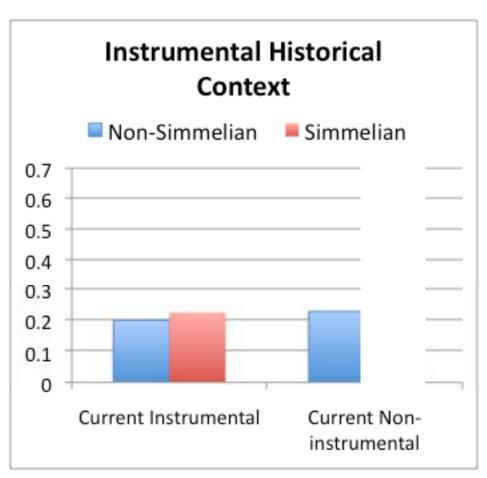


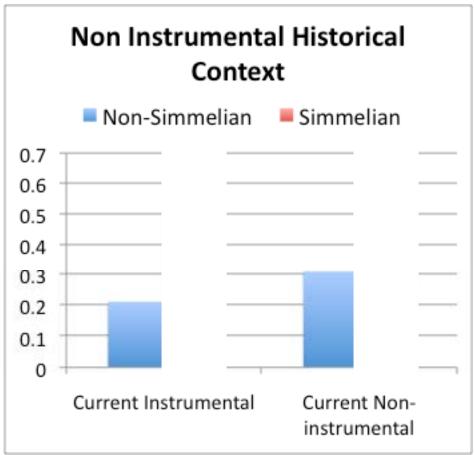


Dependent variable:

Number of response by A to B in period two

**Explanatory Variables:** 

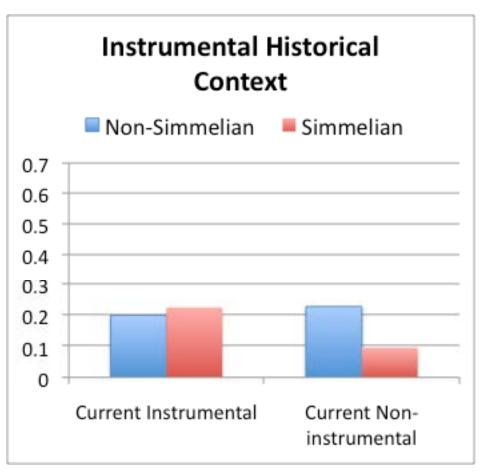


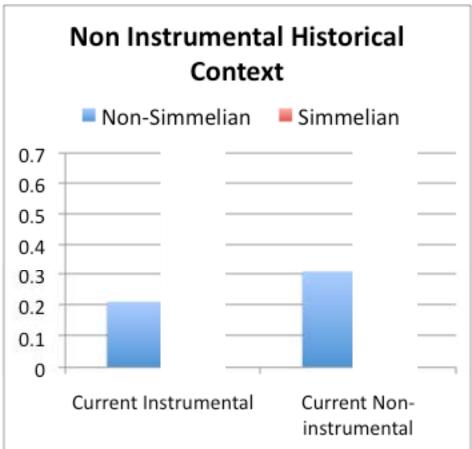


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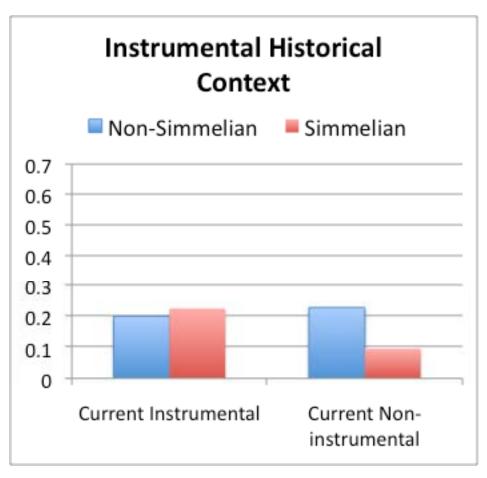


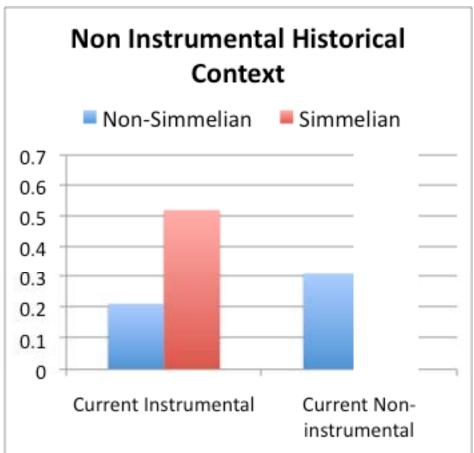


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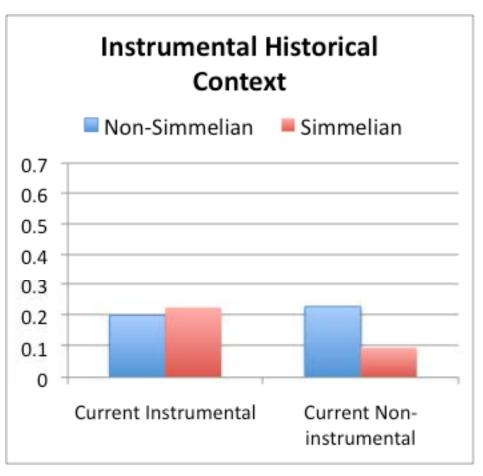


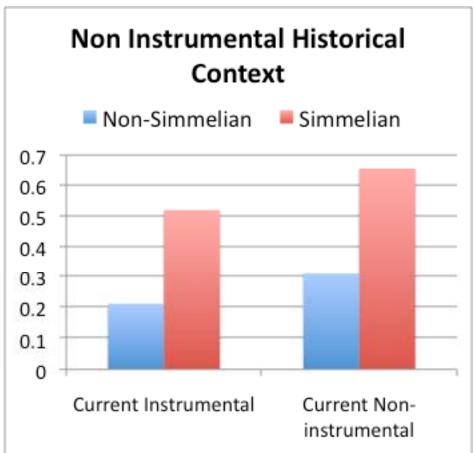


Dependent variable:

Number of response by A to B in period two

**Explanatory Variables:** 





## Other iLab network data sets

- Reliance Telecom
  - 2009; 6 months, 3 million customers, Call Data Records and Caller Ring Back Tones Purchases; Mumbai
  - 2010; 4 months, 1 million customers, call data records and caller ring back tones purchase behavior; Pune
- Vodafone Portugal
  - 1 years worth of data from Portugal
  - Call data records, churn behavior and telecom service purchase behavior
- All data have individual level attributes and data about network behaviors like with the blog data set

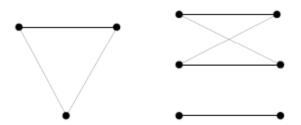
# Back to privacy and networks

- Recent interest in the literature
  - Hay et al., 2010 is a good review paper

- Focus on attacks on network data and attempts to anonymize the network through addition of "noise"
  - Directed alteration
  - Random alteration
  - generalization

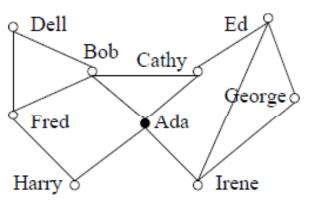
# k-degree anonymity

- The kind of attack
  - Vertex Refinement Queries

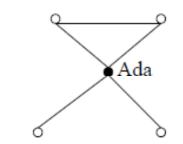


- Objective
  - The published graph
    - For every node v, there exist at least k-1 other nodes in the graph with the same degree as v
    - Choose the number of edges that are added to achieve k-degree anonymity subject to minimally affecting the graph's topology (more about this later)
- Approach
  - Add edges into the original anonymized graph to meet k-degree constraint

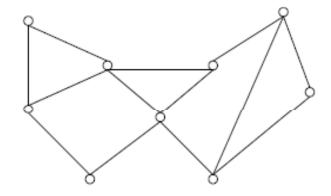
# K-neighbor anonymity to prevent sub-graph attacks



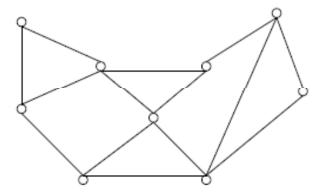
(a) the social network



(c) the 1-neighborhood graph of Ada



(b) the network with anonymous nodes



(d) privacy-preserved anonymous network

## Random alteration

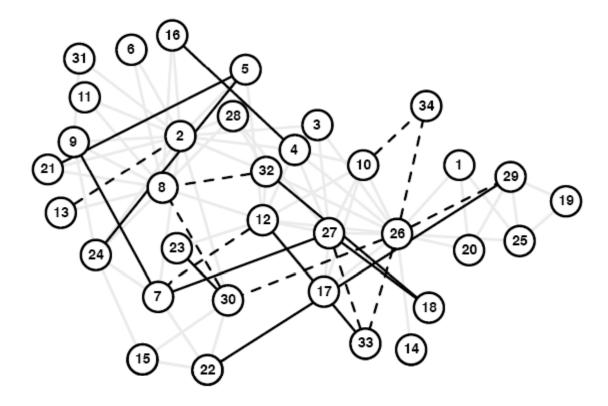
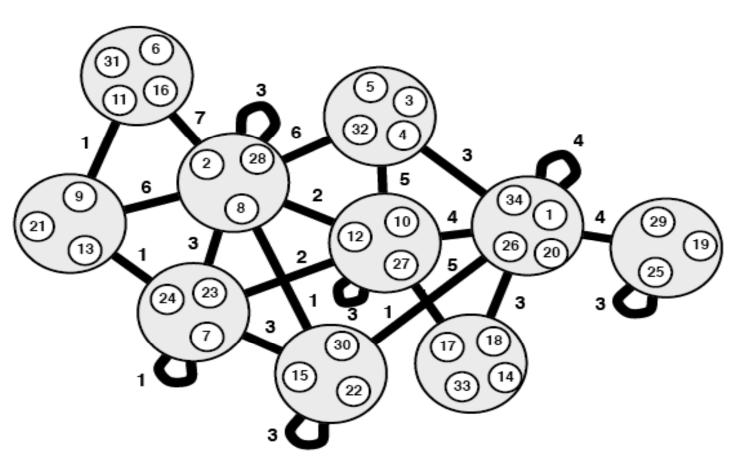


Figure 8: Examples of random alteration applied to the anonymized karate club network of Figure 2. The original edges are shown in light gray. With random alteration, edges are randomly chosen and rewired until m = 10 edges have been altered (deleted edges in dashed black; inserted edges in solid black).

# Generalization



# **Exponential Random Graphs**

 Very general families for modeling a single static network observation.

$$P(N) = \exp\{\theta \cdot u(N) - \ln Z(\theta)\}\$$

- Can estimate the θ parameters by MCMC MLE
- N is a network vector, u(N) are a set of sufficient statistics to estimate the parameter theta of the model

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- Classic example: (Frank & Strauss 1986)
- Once model is estimated, it can be used to predict the likelihood that a link will form between node I and node J
  - $u_1(N) = \# edges in N$
  - $u_2(N) = # 2-stars in N$
  - $u_3(N) = # triangles in N$

$$P(N) \propto \exp\{\theta_1 u_1(N) + \theta_2 u_2(N) + \theta_3 u_3(N)\}$$

## Addition of noise affects inference

- Note that in ERGM models sufficient statistics that are inputs to parameters estimation are number of edges, number of 2-stars and number of triangles
  - All of these would be affected when edges are added to anonymize the network
- Similar problem with QAP regression since all the dyadic variables will be affected

# Open problem

 Design a scalable anonymization technique that can be used to publish social network data such that it minimally affects the sufficient statistics used for parameter estimation of network statistics models