15-853: Algorithms in the Real World

Parallel Algorithms: Lecture 1

- Nested parallelism
- Cost model
- Parallel techniques and algorithms

Why Parallelism: Machines

Intel Xeon Eight-Core E5-2660 2.2GHz 8.0GT/s 20MB LGA2011 Processor without Fan, Retail BX80621E52660

by Intel

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Note: Not eligible for Amazon Prime. Available with free Prime shipping from other sellers on Amazon.

Only 13 left in stock.

Get it as fast as Thursday, Oct. 13.

Ships from and sold by Galactics.

- Model: Intel Xeon Processor E5-2660
- Core Count: 8
- Clock Speed: 2.2 GHz
- Cache: 20 MB
- Max Memory Bandwidth: 51.2 GB/s
- Socket: LGA2011

Used & new (29) from $47.95 + $6.44 shipping

Why Parallelism: Machines

by Rob Williams — Sunday, June 11, 2017

Intel 28-Core Xeon Platinum 8176 Dual-Socket Server Rocks Cinebench Benchmark With 112 Threads

Data Center ▸ Servers

AMD does an Italian job on Intel, unveils 32-core, 64-thread 'Naples' CPU

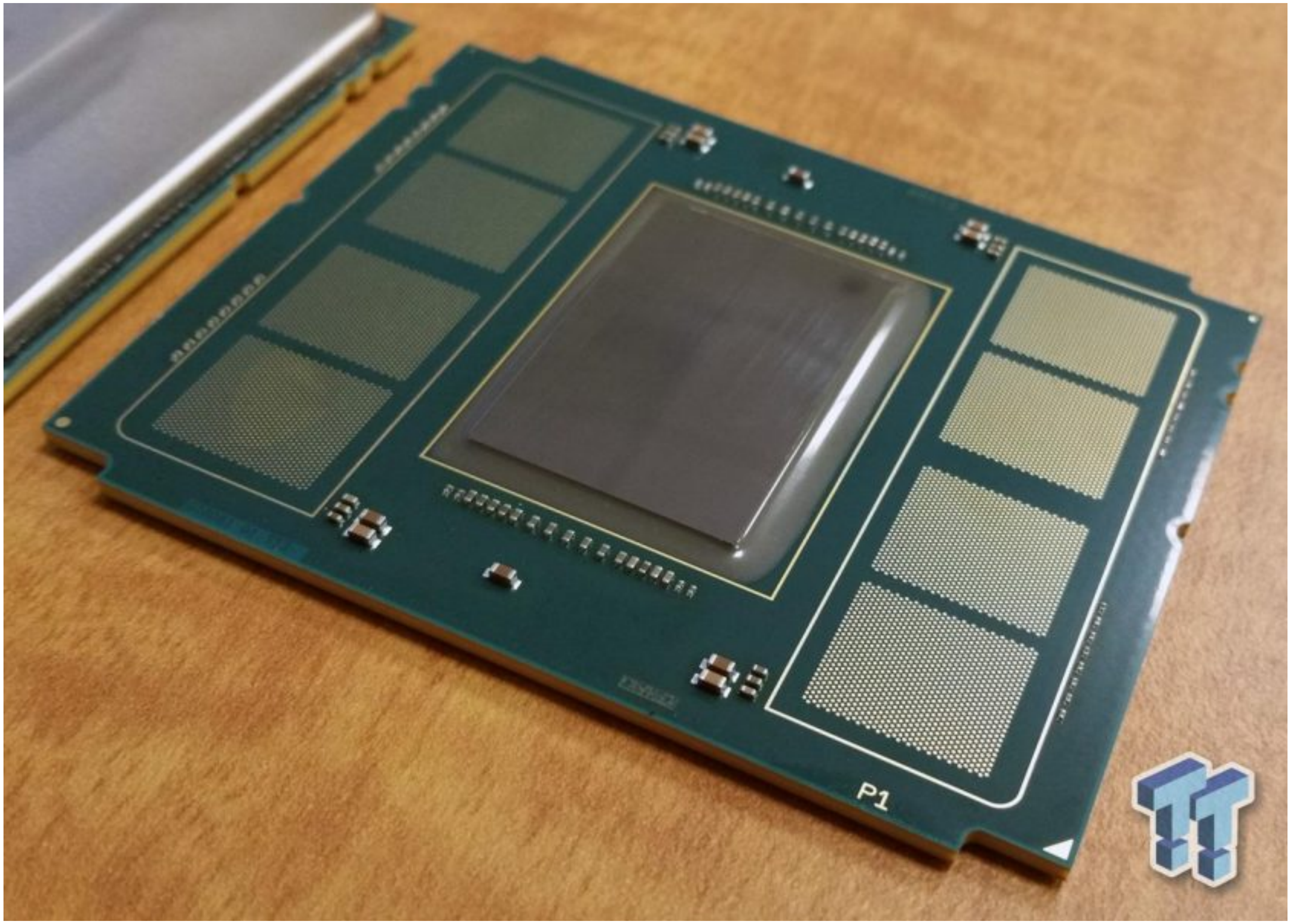
Claims to be two times faster than Chipzilla's latest data centre processor

By Chris Mellor 8 Mar 2017 at 12:35 59 SHARE ▼

112 core 4-chip server

Up to 6TByte memory

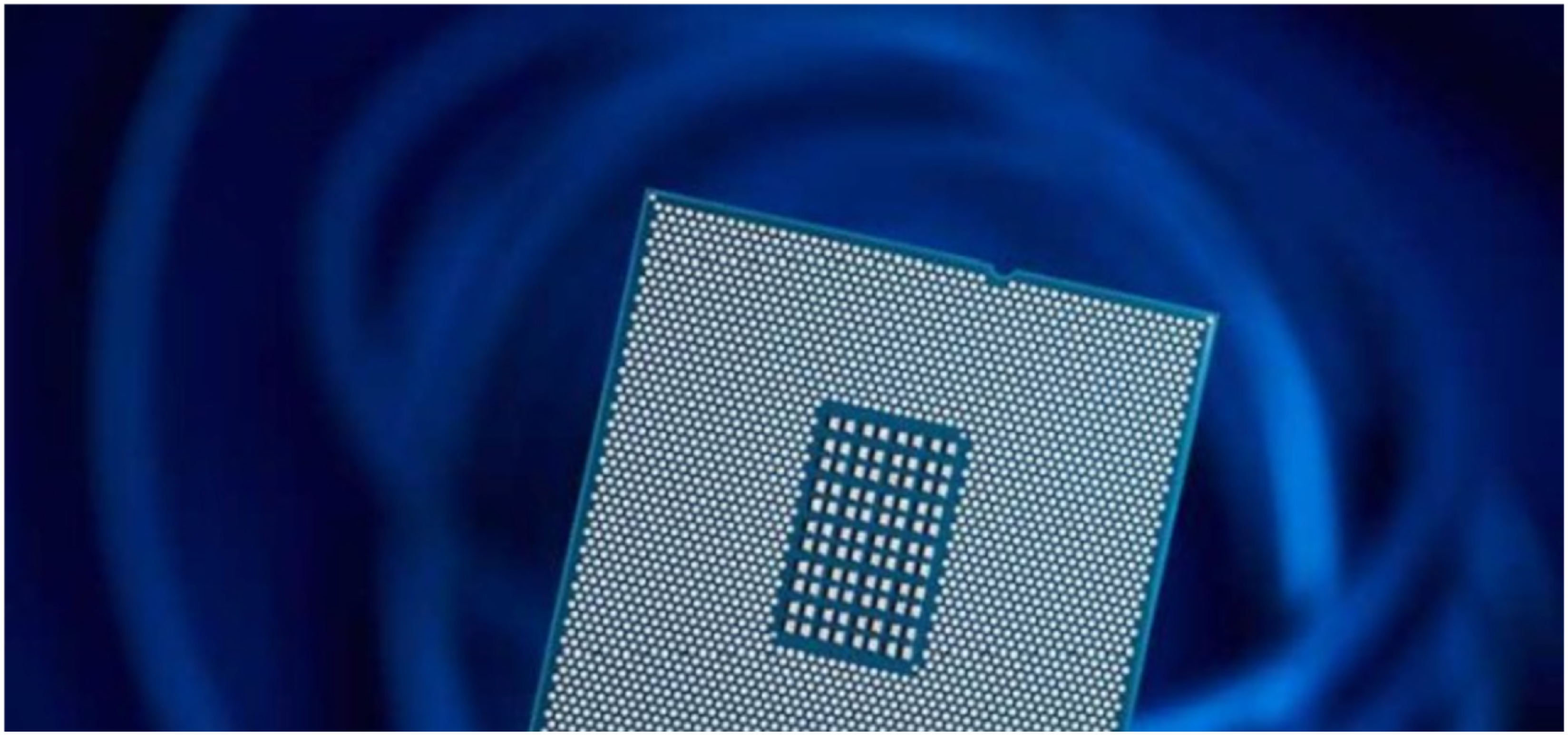
Xeon Phi: Knights Landing (64 cores)



Qualcomm readies up 48-core Centriq 2400 ARM server chip

by Zak Killian — 12:51 PM on December 9, 2016

Maybe 2017 will be the year that ARM servers finally become a thing. After demoing a 24-core server chip a little more than a year ago, Qualcomm's Datacenter Technologies subsidiary has announced the Centriq 2400 CPU. This new chip is a 48-core ARMv8 processor based on a new in-house CPU core design called Falkor, and it's compliant with ARM's Server Base System Architecture specification. Earlier in the week, Qualcomm showed off the new hardware running "a typical datacenter application" comprising Linux with Java and Apache Spark.



4992 "cuda" cores

Roll over image to zoom in

Nvidia Tesla K80 24GB GPU Accelerator passive cooling 2x Kepler GK210 900-22080-0000-000

by NVIDIA

★★★★☆ 29 customer reviews | 11 answered questions

Price: $4,295.95 + $11.55 shipping

Note: Not eligible for Amazon Prime.

In Stock.

Ships from and sold by eServer PRO.

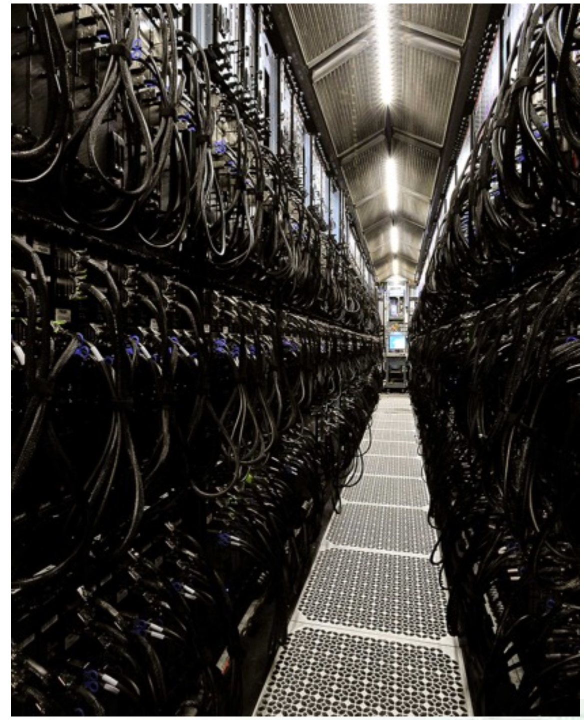
Estimated Delivery Date: Aug. 27 - Sept. 1 when you choose Expedited at checkout.

- Nvidia Tesla K80 GPU: 2x Kepler GK210
- Memory size (GDDR5) : 24GB (12GB per GPU)
- CUDA cores: 4992 (2496 per GPU)
- Memory bandwidth: 480 GB/sec (240 GB/sec per GPU)
- 2.91 Tflops double precision performance with NVIDIA GPU Boost - See more at: http://www.nvidia.com/object/tesla-servers.html#sthash.IF5LVwFq.dpuf

4 new from $4,135.00

Upgrading to a Solid-State Drive?

Learn how to install an SSD with Amazon Tech Shorts. Learn more



Up to 300K servers

LG Optimus 2X: first dual-core smartphone launches with Android, 4-inch display, 1080p video recording

Chris Ziegler
12.15.10

Dec 2010

53
Shares

Samsung Galaxy S IV to feature Exynos 28nm quad-core processor?

Written by Andre Yoskowitz @ 01 Nov 2012 18:02

Nov 2012

It has been a few weeks but there is a new rumor regarding the upcoming Samsung Galaxy S IV.

According to reports, Samsung will pack next year's flagship device with its "Adonis" Exynos processor, a quad-core ARM 15 beast that uses efficient 28nm tech.

Samsung is supposedly still testing the application processor, but mass production is scheduled for the Q1 2013 barring any delays.

SEP 4, 2014 @ 09:07 AM 11,224 VIEWS

Lenovo Announces First Octa-Core Smartphone, The Vibe X2

Jay McGregor, CONTRIBUTOR

I cover all aspects of technology and enterprise. **FULL BIO**

Opinions expressed by Forbes Contributors are their own.

Sep 2014

10-core MediaTek Helio X20 is official

NEWS

by **Robert Triggs** • May 12, 2015

May 2015

MediaTek and TSMC trialing new 7nm smartphon processor with mad CPU core count

Posted: 09 Mar 2017, 06:53, by Luis D.

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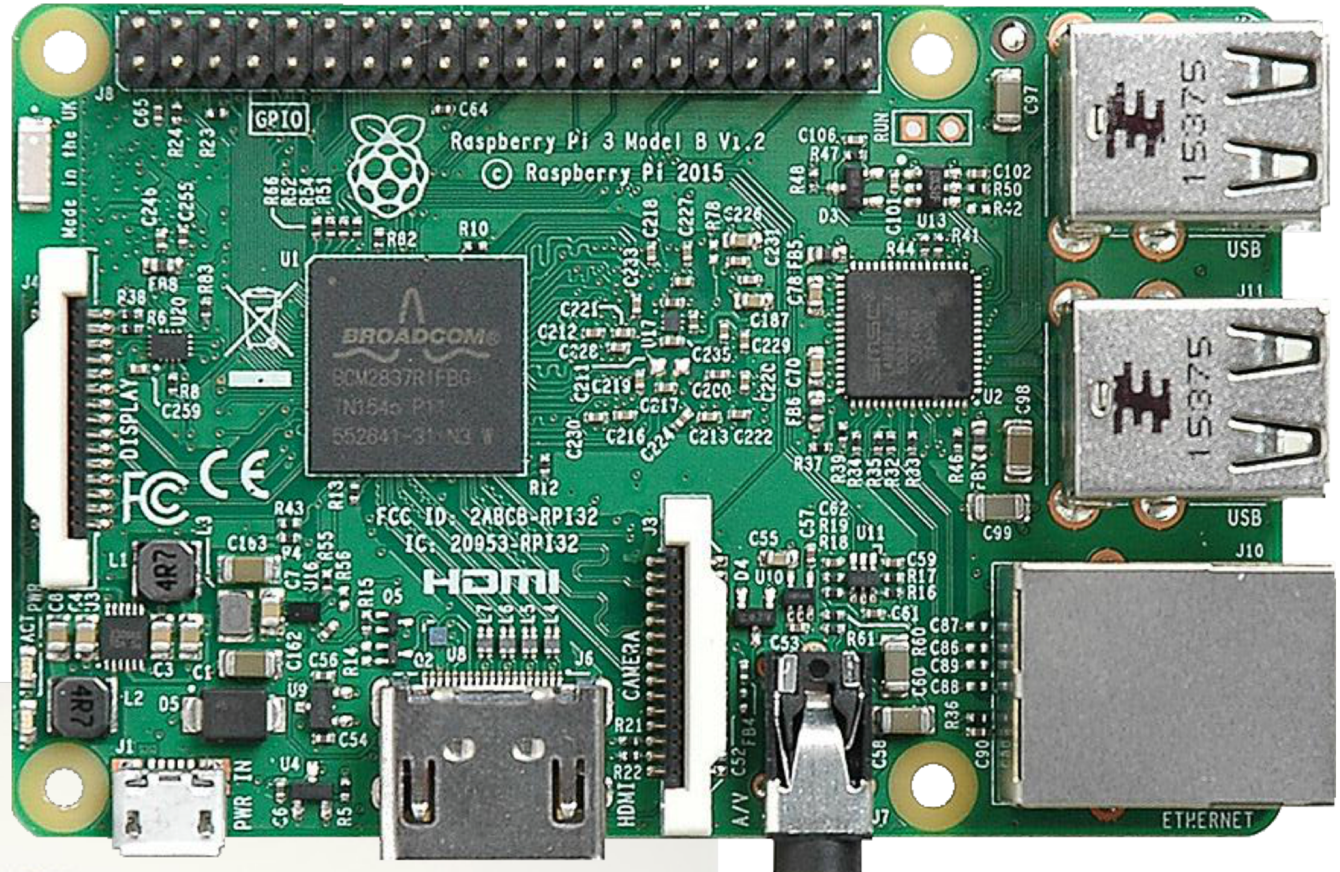
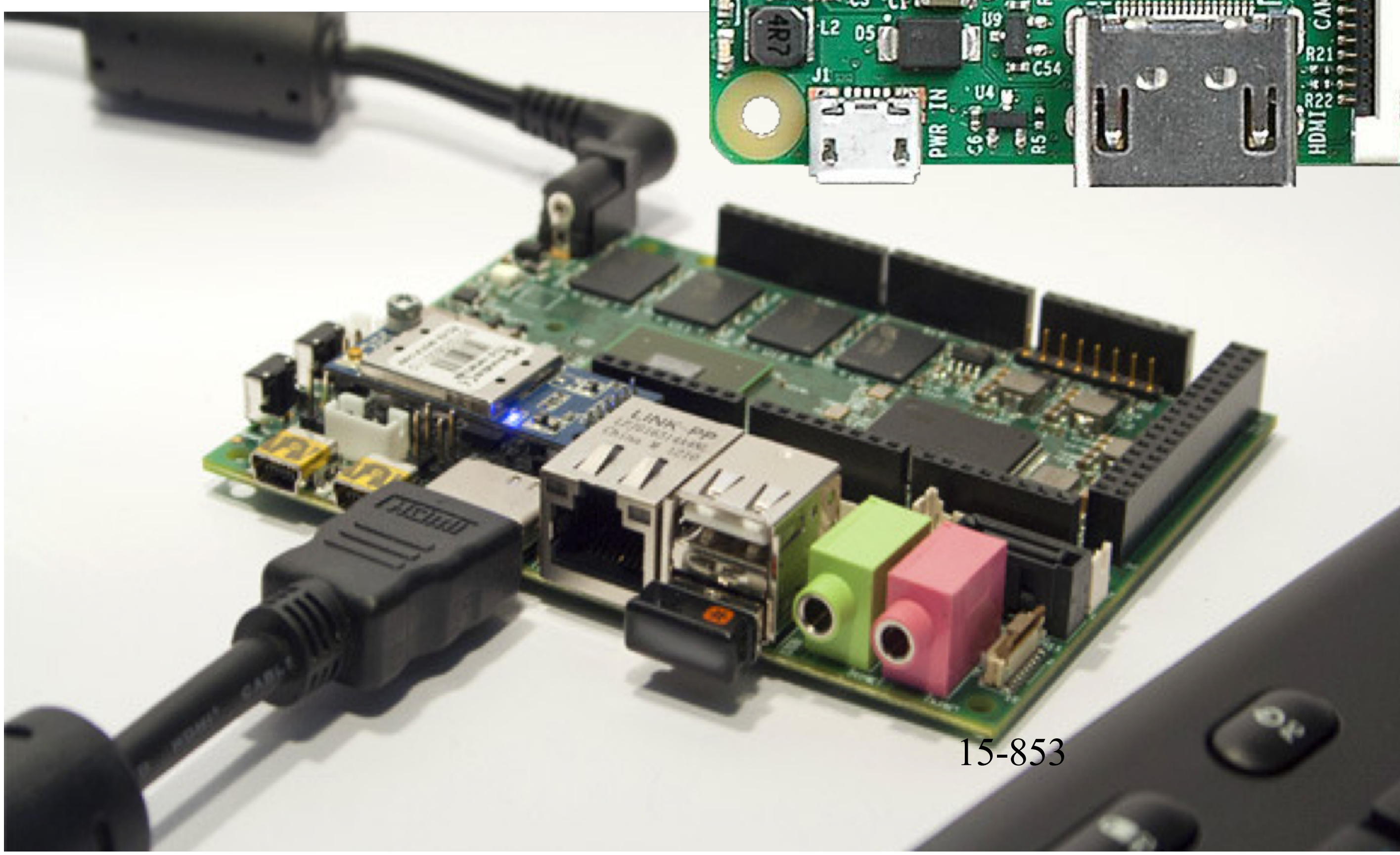
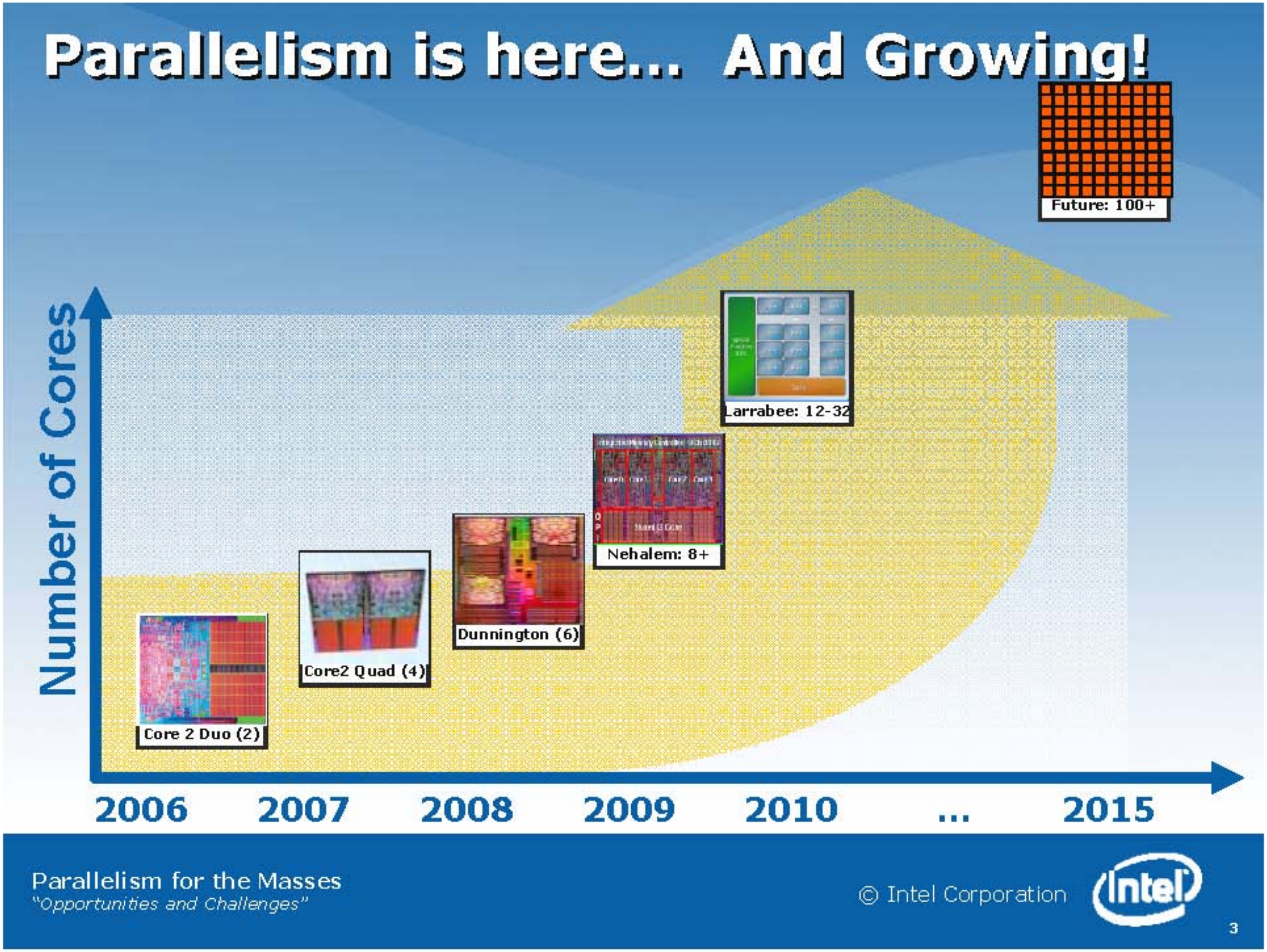
Mar 2017, 12 core



UDOO



Raspberry Pi 3



Andrew Chien, 2008

Outline (draft)

Concurrency vs. Parallelism

Concurrency example

Quicksort example

Nested Parallelism

- fork-join and parallel loops

Cost model: work and span

Techniques:

- Using collections: inverted index
- Divide-and-conquer: merging, mergesort, kd-trees, matrix multiply, matrix inversion, fft
- Contraction : quickselect, list ranking, graph connectivity, suffix arrays

Parallelism vs. Concurrency

- Parallelism: using multiple processors/cores running at the same time. Property of the machine
- Concurrency: non-determinacy due to interleaving threads. Property of the application.

		Concurrency	
		sequential	concurrent
Parallelism	serial	Traditional programming	Traditional OS
	parallel	Deterministic parallelism	General parallelism

Nested Parallelism

nested parallelism =
arbitrary nesting of parallel loops + fork-join

- Assumes no synchronization among parallel tasks except at joint points.
- Deterministic if no race conditions

Advantages:

- Good schedulers are known
- Easy to understand, debug, and analyze

Nested Parallelism: parallel loops

```
cilk_for (i=0; i < n; i++)
   B[i] = A[i]+1;
```

Cilk

```
Parallel.ForEach(A, x => x+1);
```

Microsoft TPL (C#,F#)

```
B = {x + 1 : x in A}
```

Nesl, Parallel Haskell

```
#pragma omp for
for (i=0; i < n; i++)
  B[i] = A[i] + 1;
```

OpenMP

Nested Parallelism: fork-join

```
cobegin {
  S1;
  S2;}
```

Dates back to the 60s. Used in dialects of Algol, Pascal

```
coinvoke(f1,f2)
Parallel.invoke(f1,f2)
```

Java fork-join framework

Microsoft TPL (C#,F#)

```
#pragma omp sections
{
  #pragma omp section
  S1;
  #pragma omp section
  S2;
}
```

OpenMP (C++, C, Fortran, …)

Nested Parallelism: fork-join

```
spawn S1;
S2;
sync;
```

cilk, cilk+

```
(exp1 || exp2)
```

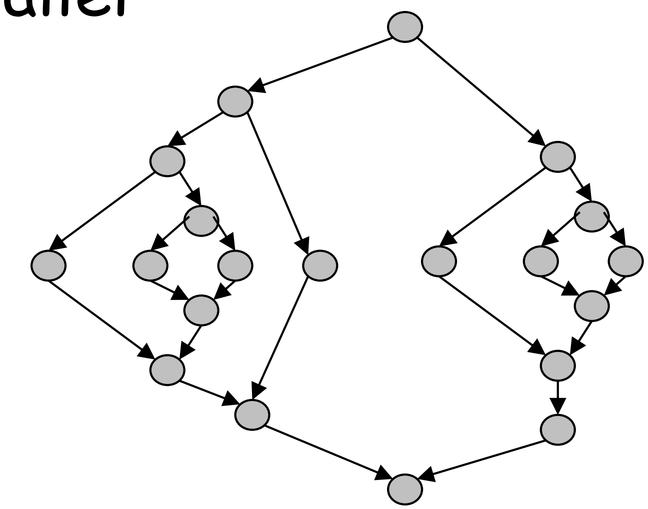
Various functional languages

```
plet
  x = exp1
  y = exp2
in
  exp3
```

Various dialects of ML and Lisp

Serial Parallel DAGs

Dependence graphs of nested parallel computations are series parallel



Two tasks are parallel if not reachable from each other.

A data race occurs if two parallel tasks are involved in a race if they access the same location and at least one is a write.

Cost Model

Compositional:

Work : total number of operations

- costs are added across parallel calls

Depth : span/critical path of the computation

- Maximum span is taken across forked calls

Parallelism = Work/Depth

- Approximately # of processors that can be effectively used.

Combining costs

Combining for parallel for:

```
parfor i in [0:n]
  f(i);
```

$$W_{\text{pexp}}(\text{pfor} \ldots) = \sum_{i=0}^{n-1} W_{\text{exp}}(\text{f(i)}) \qquad \text{work}$$

$$D_{\text{pexp}}(\text{pfor} \ldots) = \max_{i=0}^{n-1} D_{\text{exp}}(\text{f(i)}) \qquad \text{span}$$

A Formal Model: The MP-RAM

Start with the sequential Random Access Machine (RAM) model

Add a fork(n) instruction:

1. Forks n identical child copies of the process, the i-th one has i in a special **index** register
2. Suspend the parent
3. When all children finish, restart the parent with 0 in the **index** register.

Purely nested. Can be viewed as series-parallel DAG.

Work and Depth as usual

MP-PRAM: ordering

Any serialization (topological sort of DAG) is valid, and memory operations have normal semantics under this ordering.

Results can depend on ordering

Definitions (pairs of instructions):

- **Conflict**: access the same location, at least one of which is a write.
- **Concurrent:** are unordered in the DAG
- **Race**: conflict and concurrent

Race free programs always return the same result

Why Work and Span

Simple measures that give us a good sense of efficiency (work) and scalability (span).

Can schedule in $O(W/P + D)$ time on P processors.

This is within a constant factor of optimal.

Goals in designing an algorithm

1. Work should be about the same as the sequential running time. When it matches asymptotically we say it is **work efficient.**
2. Parallelism (W/D) should be polynomial $O(n^{1/2})$ is probably good enough

Example: Quicksort

```
function quicksort(S) =
if (#S <= 1) then S
else let
  a = S[rand(#S)];
  S1 = {e in S | e < a};
  S2 = {e in S | e = a};
  S3 = {e in S | e > a};
  R = {quicksort(v) : v in [S1, S3]};
in R[0] ++ S2 ++ R[1];
```

Partition

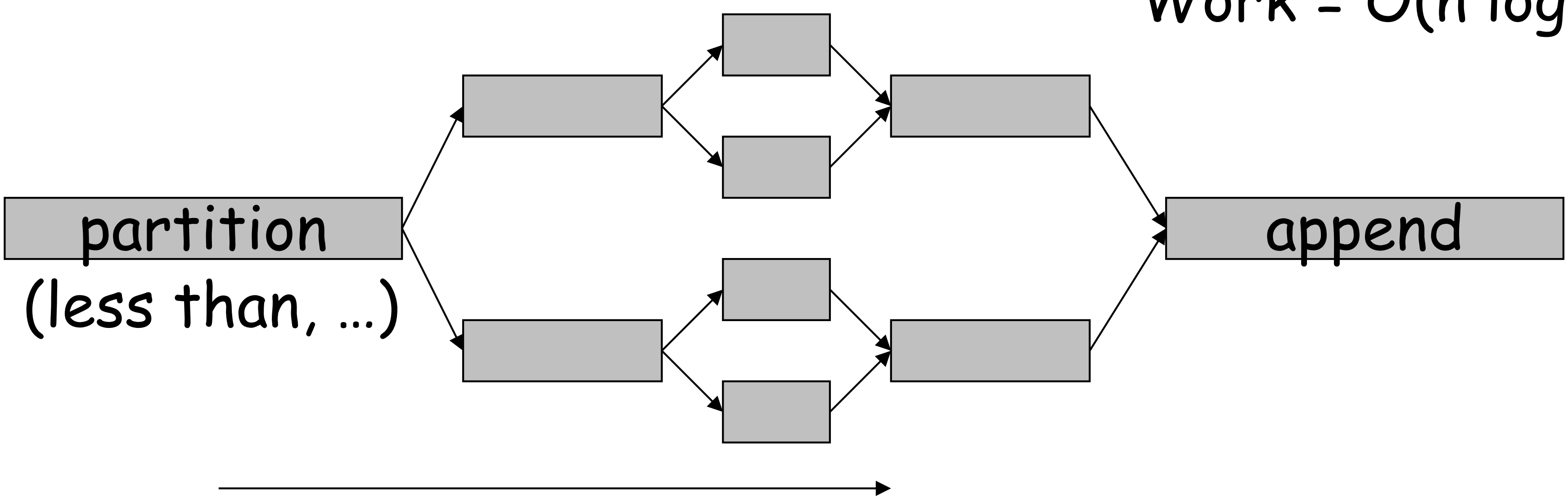
Recursive calls

How much parallelism?

Quicksort Complexity

Sequential Partition and appending
Parallel calls

Work = O(n log n)



Span = O(n)

Parallelism = O(log n)

Not a very good parallel algorithm

*All randomized
with high probability

Quicksort Complexity

Now lets assume the partitioning and appending can be done with:

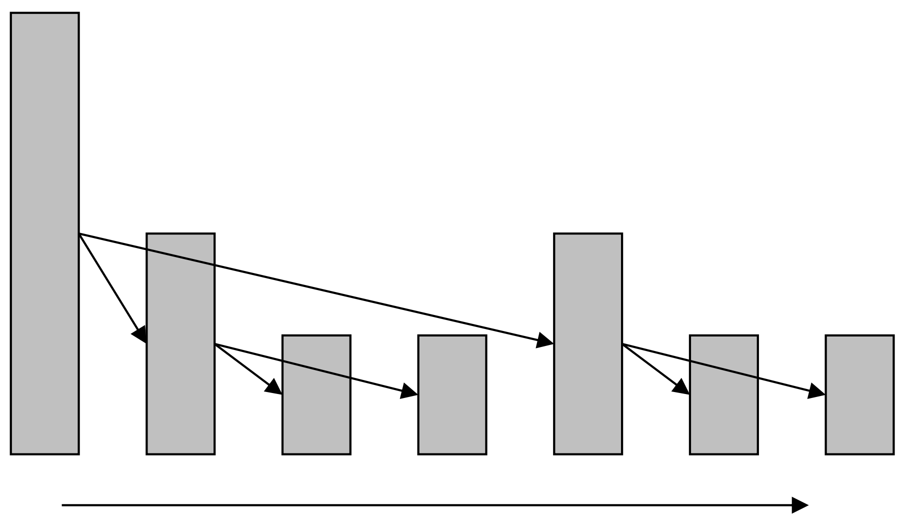
Work = $O(n)$

Span = $O(\log n)$

but recursive calls are made sequentially.

Quicksort Complexity

Parallel partition
Sequential calls



Span = O(n)

Work = O(n log n)

Parallelism = O(log n)

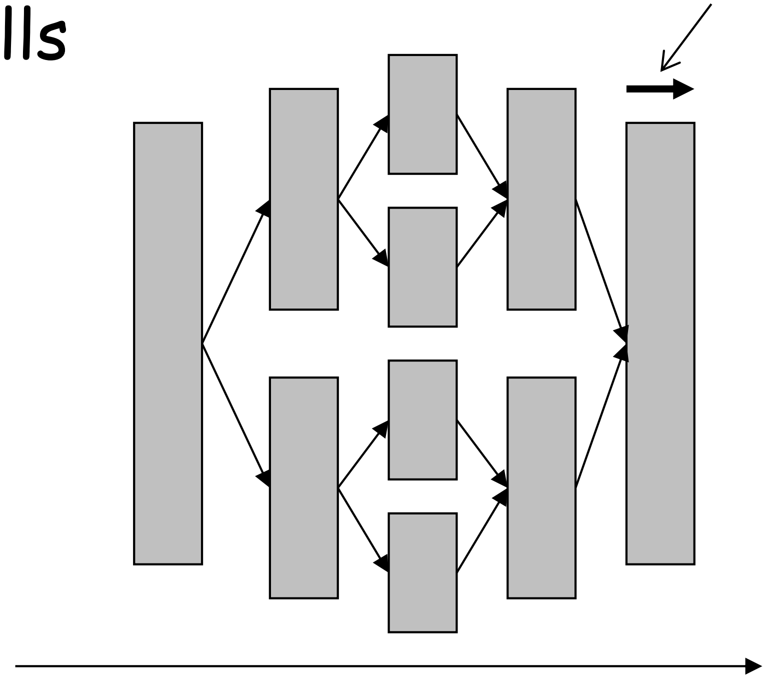
Not a very good parallel algorithm

*All randomized
with high probability

Quicksort Complexity

Parallel partition
Parallel calls

Span = $O(\lg n)$



Work = $O(n \log n)$

Span = $O(\lg^2 n)$

Parallelism = $O(n/\log n)$

A good parallel algorithm

*All randomized with high probability

Quicksort Complexity

Caveat: need to show that depth of recursion is O(log n) with high probability

Parallel selection

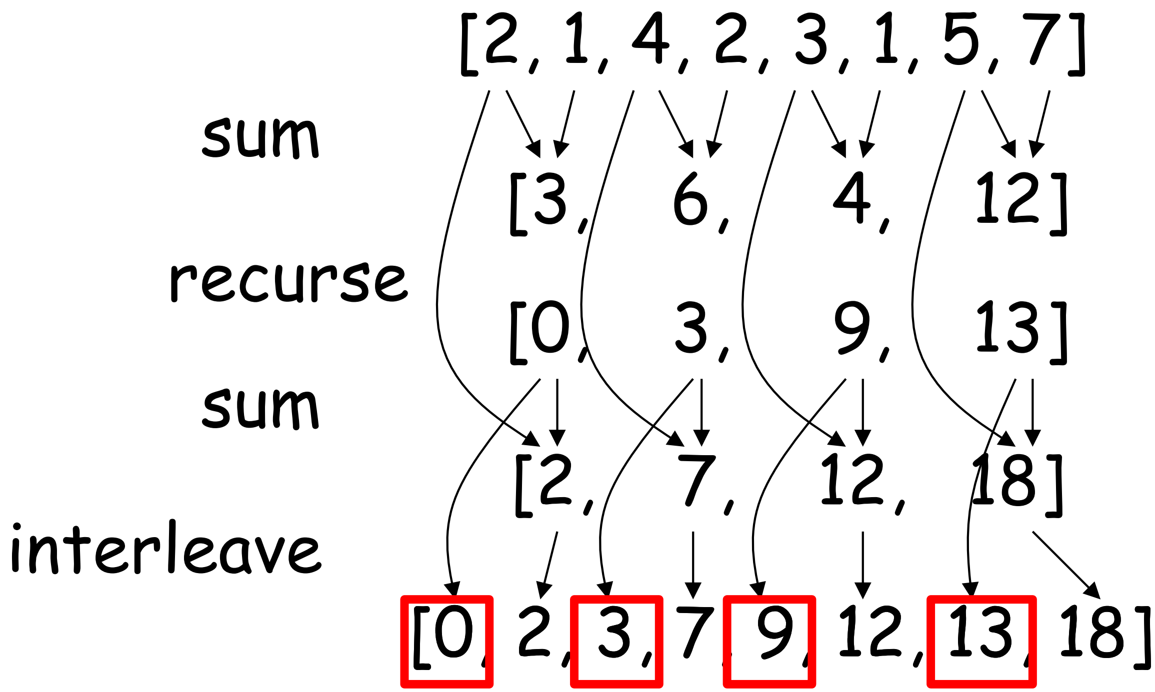
```
{e in S | e < a};
```

S = [2, 1, 4, 0, 3, 1, 5, 7]
F = S < 4 = [1, 1, 0, 1, 1, 1, 0, 0]
I = addscan(F) = [0, 1, 2, 2, 3, 4, 5, 5]

where F
R[I] = S = [2, 1, 0, 3, 1]

Each element gets sum of previous elements.
Seems sequential?

Scan


[2, 1, 4, 2, 3, 1, 5, 7]
sum
[3, 6, 4, 12]
recurse
[0, 3, 9, 13]
sum
[2, 7, 12, 18]
interleave
[0, 2, 3, 7, 9, 12, 13, 18]

Scan code

```
function addscan(A) =
if (#A <= 1) then [0]
else let
  sums = {A[2*i] + A[2*i+1] : i in [0:#a/2]};
  evens = addscan(sums);
  odds = {evens[i] + A[2*i] : i in [0:#a/2]};
in interleave(evens,odds);
```

$W(n) = W(n/2) + O(n) = O(n)$

$D(n) = D(n/2) + O(1) = O(\log n)$

Parallel Techniques

Some common themes in “Thinking Parallel”

1. Working with collections.
   - map, selection, reduce, scan, collect
2. Divide-and-conquer
   - Even more important than sequentially
   - Merging, matrix multiply, FFT, …
3. Contraction
   - Solve single smaller problem
   - List ranking, graph contraction
4. Randomization
   - Symmetry breaking and random sampling