Graph Compression

Data compression: lecture 5

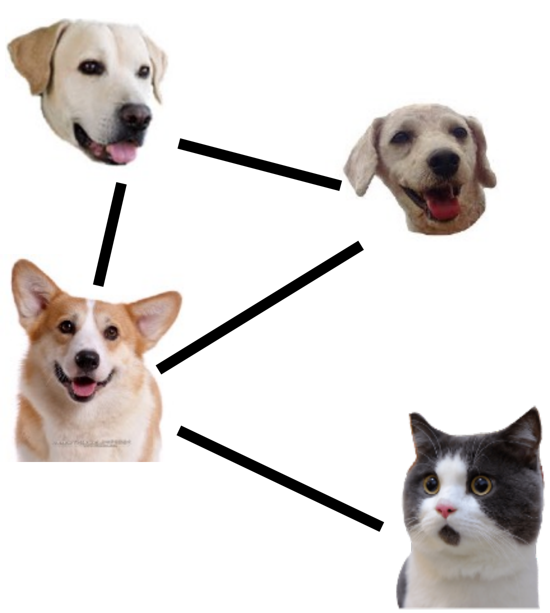
15-853, Spring 2018

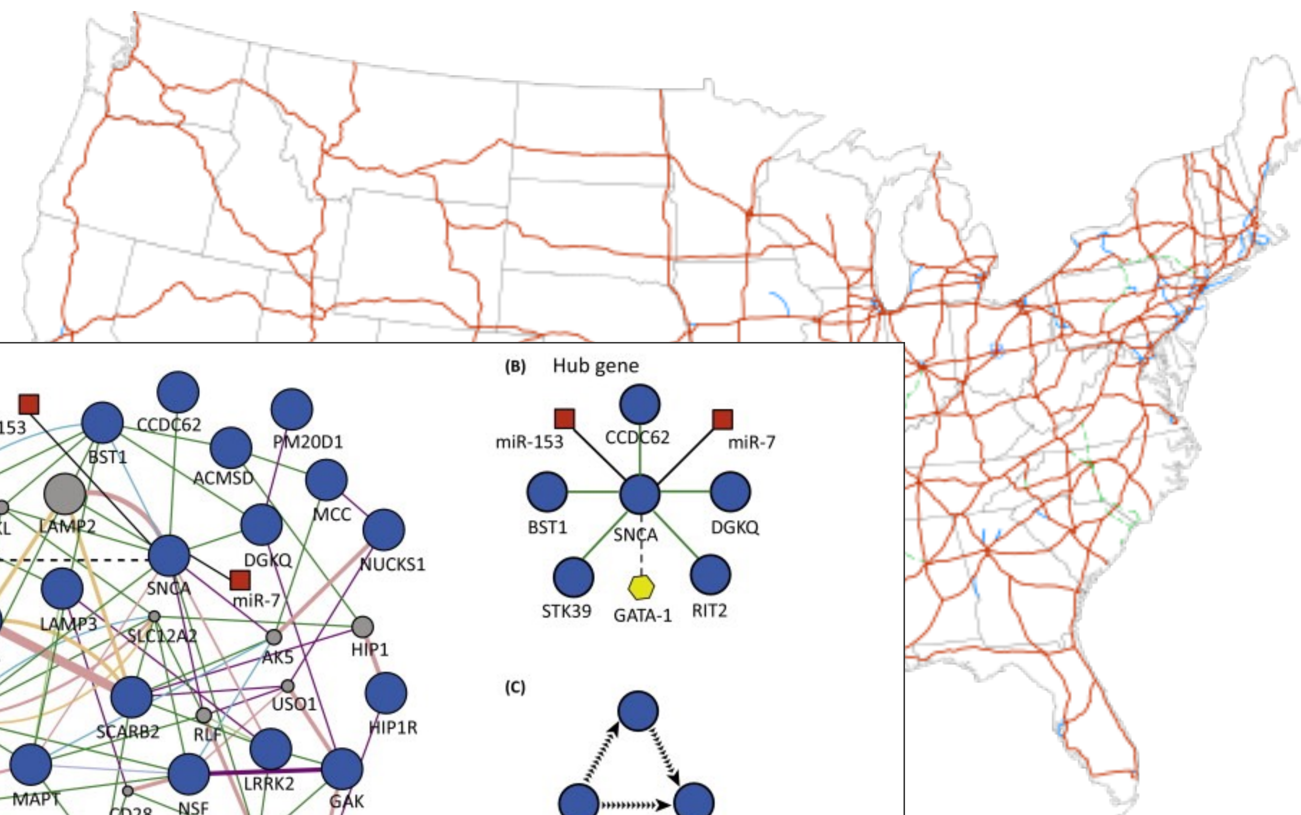
Outline

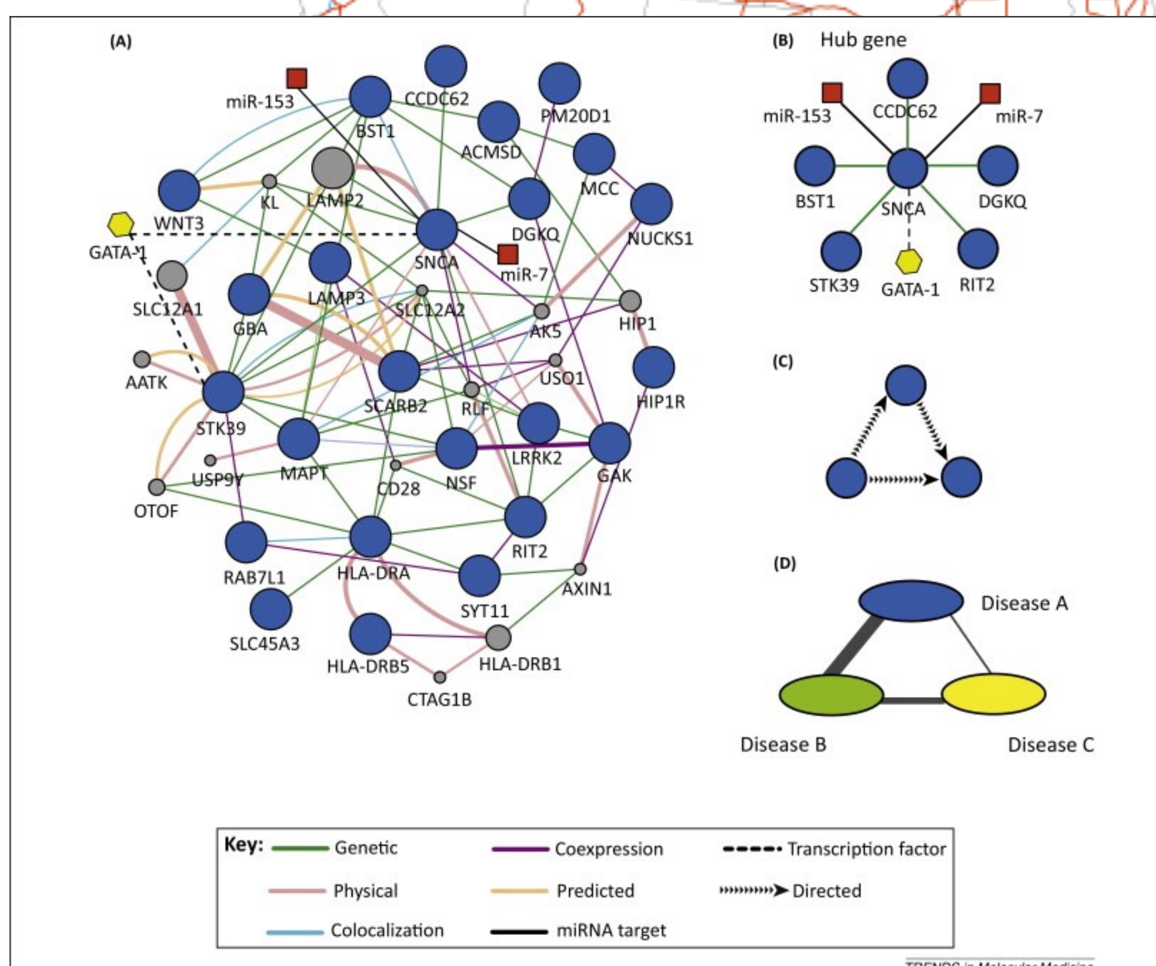
- What is a graph?
- Graph representations
- Compressing and reordering graphs
- Examples

What is a graph?

- G(V, E), usually n for #vertices, m for #edges
- Vertices model “objects”
- Edges model relationship between objects





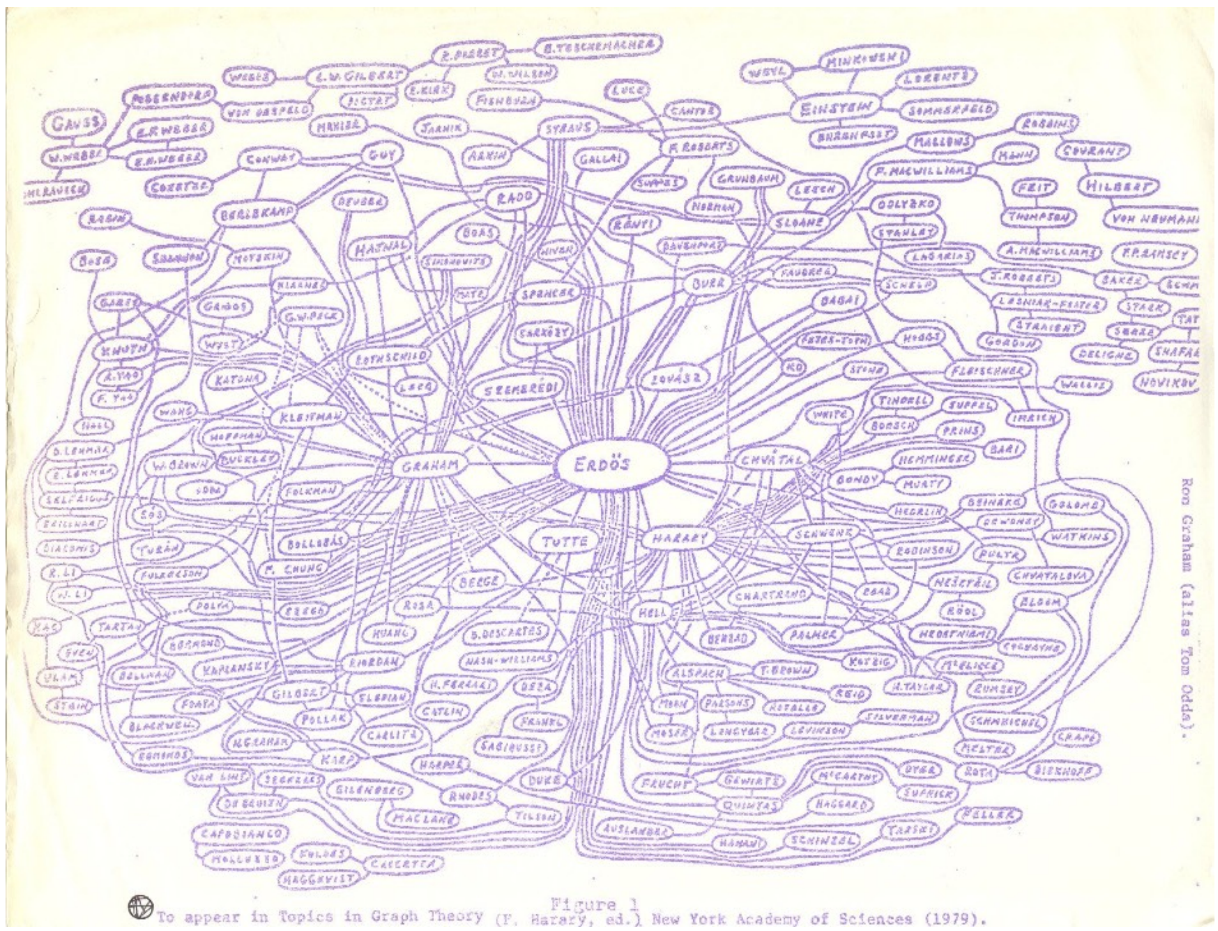


What is a graph?

- Edges can be undirected or directed

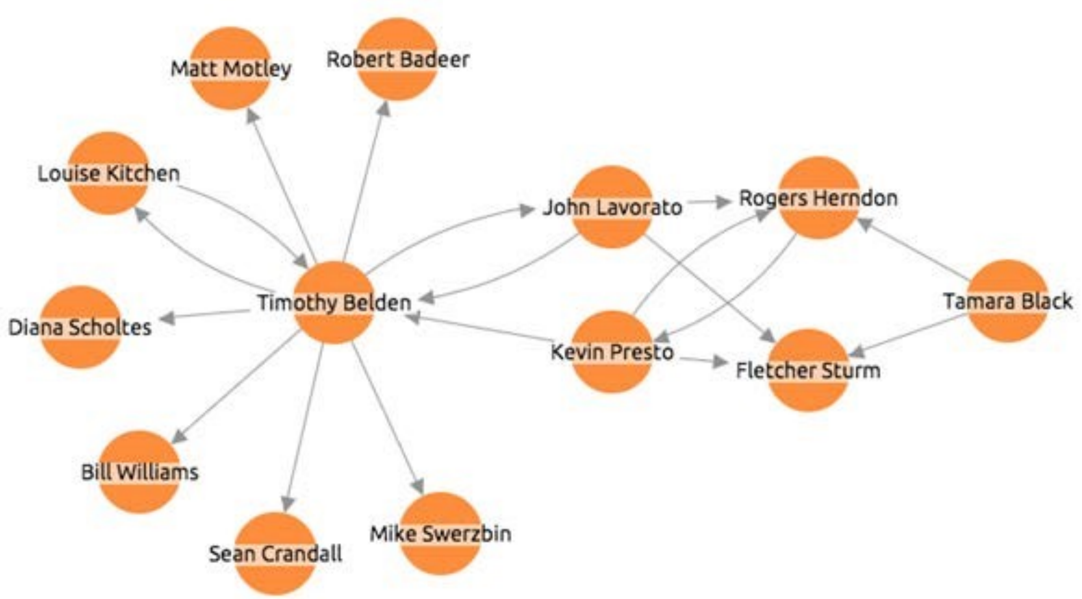
Undirected

- Coauthorship network
- Social networks (Facebook)
- Protein-protein interaction



Directed

- Hyperlink graphs
- Email graphs (enron)
- Follower graphs (twitter)



Graph sizes in 2018

Graph	|V|	|E| (symmetrized)
com-Orkut	3M	234M
Twitter	41M	1.46B
Friendster	124M	3.61B
Hyperlink2012-Host	101M	2.04B
Facebook (2011) [1]	721M	68.4B
Hyperlink2014 [2]	1.7B	124B
Hyperlink2012 [2]	3.5B	225B
Facebook (2018)	> 2B	> 300B
Google (2018)	?	?

● : Publicly available graphs

● : Private graph datasets

[1] The Anatomy of the Facebook Social Graph, Ugander et al. 2011
[2] http://webdatacommons.org/hyperlinkgraph/

Graph compression in industry

NetflixGraph Metadata Library: An Optimization Case Study

by Drew Koszewnik

Problem: running into memory issues when storing the movie property graph in memory

Solution: Compact Encoded Data Representation

We knew that we could hold the same data in a more memory-efficient way. We created a library to represent directed-graph data, which we could then overlay with the specific schema we needed.

Results

When we dropped this new data structure in the existing NetflixGraph library, our memory footprint was reduced by 90%. A histogram of our test application from above, loading the exact same set of data, now looks like the following:

Source: Netflix Tech Blog

Graph compression in industry

Compressing Graphs and Indexes with Recursive Graph Bisection

Abstract

Graph reordering is a powerful technique to increase the locality of the representations of graphs, which can be helpful in several applications. We study how the technique can be used to improve compression of graphs and inverted indexes.

Our experiments show a significant improvement of the compression rate of graph and indexes over existing heuristics. The new method is relatively simple and allows efficient parallel and distributed implementations, which is demonstrated on graphs with billions of vertices and hundreds of billions of edges.

Source: Facebook Reseach

Operations on graphs

- Static graphs:
  - scanning the whole graph (i.e. the storage cost)
  - get_neighbors(v) (in/out neighbors for digraphs)
  - is_edge(u, v) (is the (u, v) edge present in G?)
- Dynamic graphs:
  - insert/delete edges

Source: MIT-6.172 Lecture 21

Graph representations

Adjacency Matrix

- Vertices labeled from 0 to n-1
- Entry of "1" if edge exists, 0 o.w.

0	0	0	0
1	0	1	1
0	0	0	1
0	1	1	0

Edge List

(1, 0)
(1, 2)
(1, 3)
(2, 3)
(3, 1)
(3, 2)

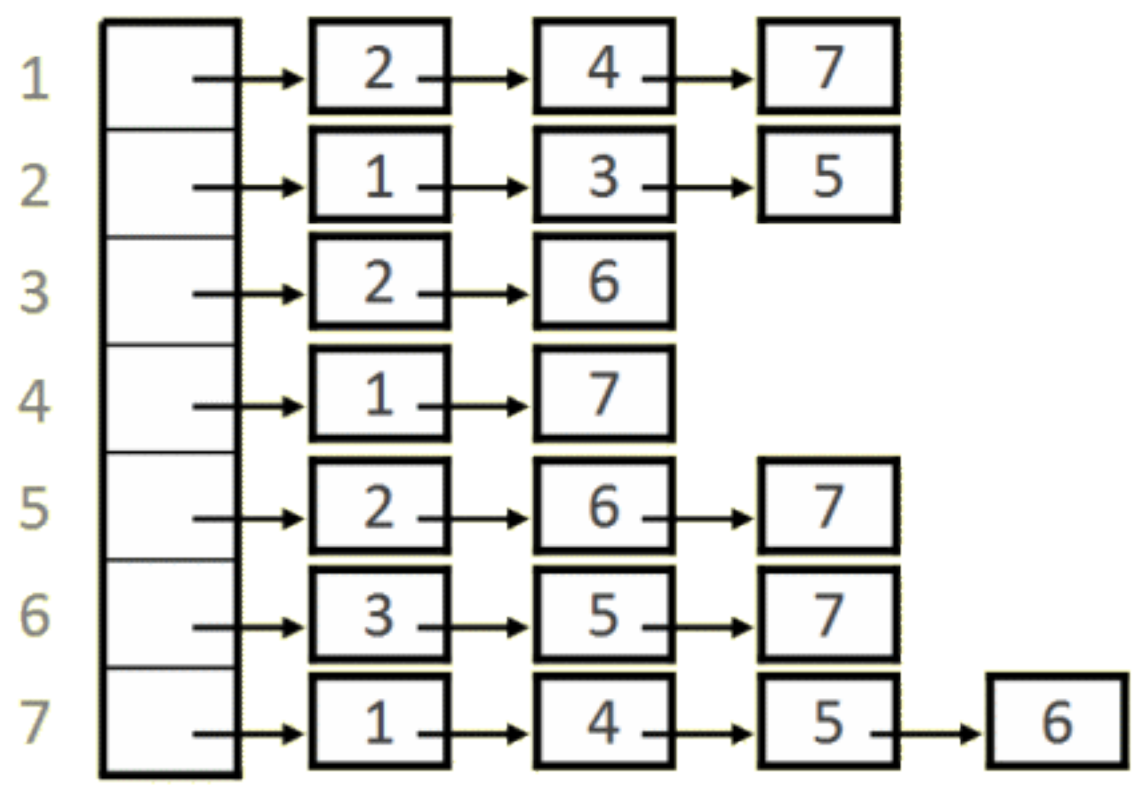
- Space requirements in terms of m and n?

Source: MIT-6.172 Lecture 21

Graph representations

Adjacency List

- Array of pointers (one per vertex)
- Each vertex points to a list of its neighbors
- Linked lists: bad cache performance, use arrays instead
  - Tradeoff: hard to insert/delete edges

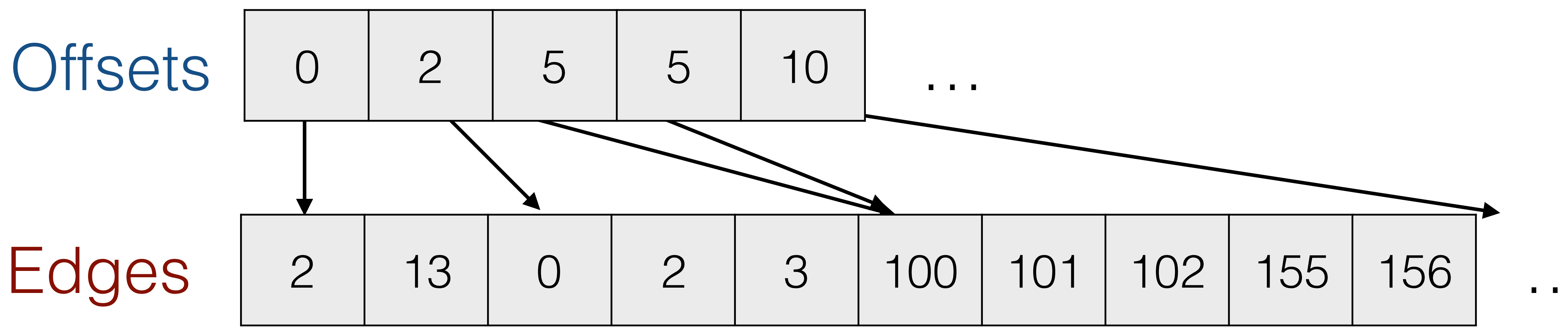


Source: MIT-6.172 Lecture 21, Image source: Stanford CS106b

Graph representations

Compressed Sparse Row (Column)

- Cache-friendly method of storing graph in memory
- Two arrays: Offsets and Edges
- Offsets[i] stores the offset where vertex i's edges start in Edges



- How do we calculate the degree of a vertex?
- Space usage?
- Jargon: CSR used for out-edges, CSC for in-edges

Source: MIT-6.172 Lecture 21

Graph representations: costs

Operation	Adjacency Matrix	Edge List	Adjacency List	CSR/CSC
scan_graph				
get_neighbors				
is_edge				
ins/del neighbor				

Source: MIT-6.172 Lecture 21

Graph representations: costs

Operation	Adjacency Matrix	Edge List	Adjacency List	CSR/CSC
scan_graph	$O(n^2)$	$O(m)$	$O(m+n)$	$O(m+n)$
get_neighbors				
is_edge				
ins/del neighbor				

Source: MIT-6.172 Lecture 21

Graph representations: costs

Operation	Adjacency Matrix	Edge List	Adjacency List	CSR/CSC
scan_graph	$O(n^2)$	$O(m)$	$O(m+n)$	$O(m+n)$
get_neighbors	$O(n)$	$O(m)$	$O(d)$	$O(d)$
is_edge				
ins/del neighbor				

Source: MIT-6.172 Lecture 21

Graph representations: costs

Operation	Adjacency Matrix	Edge List	Adjacency List	CSR/CSC
scan_graph	$O(n^2)$	$O(m)$	$O(m+n)$	$O(m+n)$
get_neighbors	$O(n)$	$O(m)$	$O(d)$	$O(d)$
is_edge	$O(1)$	$O(m)$	$O(d)$	$O(d)$
ins/del neighbor				

Source: MIT-6.172 Lecture 21

Graph representations: costs

Operation	Adjacency Matrix	Edge List	Adjacency List	CSR/CSC
scan_graph	$O(n^2)$	$O(m)$	$O(m+n)$	$O(m+n)$
get_neighbors	$O(n)$	$O(m)$	$O(d)$	$O(d)$
is_edge	$O(1)$	$O(m)$	$O(d)$	$O(d)$
insert edge				
delete edge				

Source: MIT-6.172 Lecture 21

Graph representations: costs

Operation	Adjacency Matrix	Edge List	Adjacency List	CSR/CSC
scan_graph	$O(n^2)$	$O(m)$	$O(m+n)$	$O(m+n)$
get_neighbors	$O(n)$	$O(m)$	$O(d)$	$O(d)$
is_edge	$O(1)$	$O(m)$	$O(d)$	$O(d)$
insert edge	$O(1)$	$O(1)$	$O(1)$ or $O(d)$	$O(m+n)$
delete edge				

Source: MIT-6.172 Lecture 21

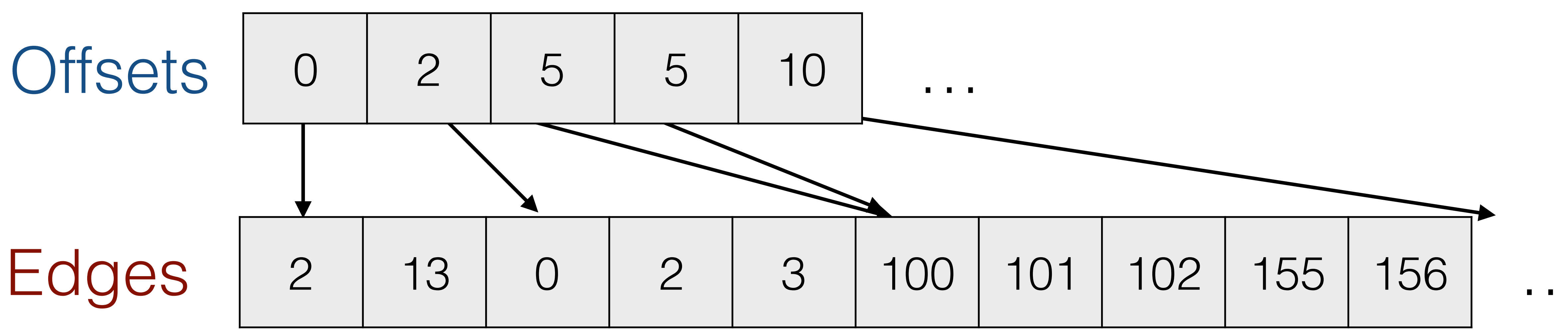
Graph representations: costs

Operation	Adjacency Matrix	Edge List	Adjacency List	CSR/CSC
scan_graph	$O(n^2)$	$O(m)$	$O(m+n)$	$O(m+n)$
get_neighbors	$O(n)$	$O(m)$	$O(d)$	$O(d)$
is_edge	$O(1)$	$O(m)$	$O(d)$	$O(d)$
insert edge	$O(1)$	$O(1)$	$O(1)$ or $O(d)$	$O(m+n)$
delete edge	$O(1)$	$O(m)$	$O(d)$	$O(m+n)$

Source: MIT-6.172 Lecture 21

Graph representations: summary

- Understand the set of operations before choosing a format
- This lecture: mostly use CSR/CSC
  - Sparse graphs (m = O(n))
  - Static algorithms
  - Need to scan over neighbors of a vertex efficiently



n + m space

Source: MIT-6.172 Lecture 21

Storing uncompressed graphs

Hyperlink2012 Graph

- n = 3.6B, m = 225B (undirected edges)
- Vertex ids fit into 4 bytes
- > 900Gb to store in CSR format

32Gb DRAM: about 300$*

So, about 9000$ of memory just to store the graph.
Doesn't include memory needed to run algorithms on it!

*Source: Hynix HMA84GR7MFR4N-UH 32GB DDR4-2400 ECC REG DIMM Server Memory

Compressing graphs

- Web graphs
- Difference encoding
- Reordering for locality

Web graphs

- Vertices are web pages
- Directed edges represent hyperlinks
- Used:
  - Understand structure of the web
  - Mine communities
  - Prioritize crawling

Entire conferences around the web and web-algorithms

Compressing web graphs

Is the web structured?

Compressing web graphs

Lots of structure!

- **Locality:** Many links stay within the same sub-domain. I.e. most links point closeby in the lexicographic ordering
- **Similarity**: Pages closeby in the lexicographic order tend to have similar sets of neighbors

- Boldi and Vigna (WWW 2004) exploit these observations about the internet in the WebGraph framework:
  - Reference coding
  - Difference coding

Source: Boldi and Vigna, "The WebGraph Framework I: Compression Techniques", WWW 2004

Compressing web graphs: techniques

Reference coding

Idea: to encode neighbors of *v*

- Find previous vertex, *ref,* which has significant overlap
- Encode edges with respect to *ref.*

Original graph:	**Reference coded:**
vertex 0: [1, 2, 4, 5, 9, 10]	vertex 0: [1, 2, 4, 5, 9, 10]
…	…
vertex 6: [1, 2, 4, 5, 9, 13]	vertex 6: ref(0), {10}, {13}

How do you find good references?

Is accessing N(v) efficient? (O(deg(v)?)

Source: Boldi and Vigna, "The WebGraph Framework I: Compression Techniques", WWW 2004

Compressing web graphs: techniques

Difference coding

Neighbor lists exhibit a high degree of *locality*

We want to store a set of integer vertex ids

N(3) = [2, 4, 1, 13, 5, 9]

Sort the elements

N(3) = [1, 2, 4, 5, 9, 13]

Store gaps instead of the actual integers

[1-3, 2-1, 4-2, 5-4, 9-5, 13-9]

= [(-)2, 1, 2, 1, 4, 4]

Compress the gaps using *integer code*s

Source: Boldi and Vigna, "The WebGraph Framework I: Compression Techniques", WWW 2004

Compressing web graphs

WebGraph Framework

Combines:

- Reference coding
- Difference encoding

The WWW paper shows that these two techniques can be used to represent a billion-edge web graph in

- out edges: 3.08 bits/edge
- in edges: 2.89 bits/edge

Why do in edges compress better?

Source: Boldi and Vigna, "The WebGraph Framework I: Compression Techniques", WWW 2004

Compressing web graphs

Many other systems and techniques:

<u>Fast and Compact Web Graph Representations</u>

- Grammar-based techniques (Re-Pair, LZ)
- Similar space as WebGraph, but faster access times

<u>Representing Web Graphs</u>

- Hierarchical representation of web graphs

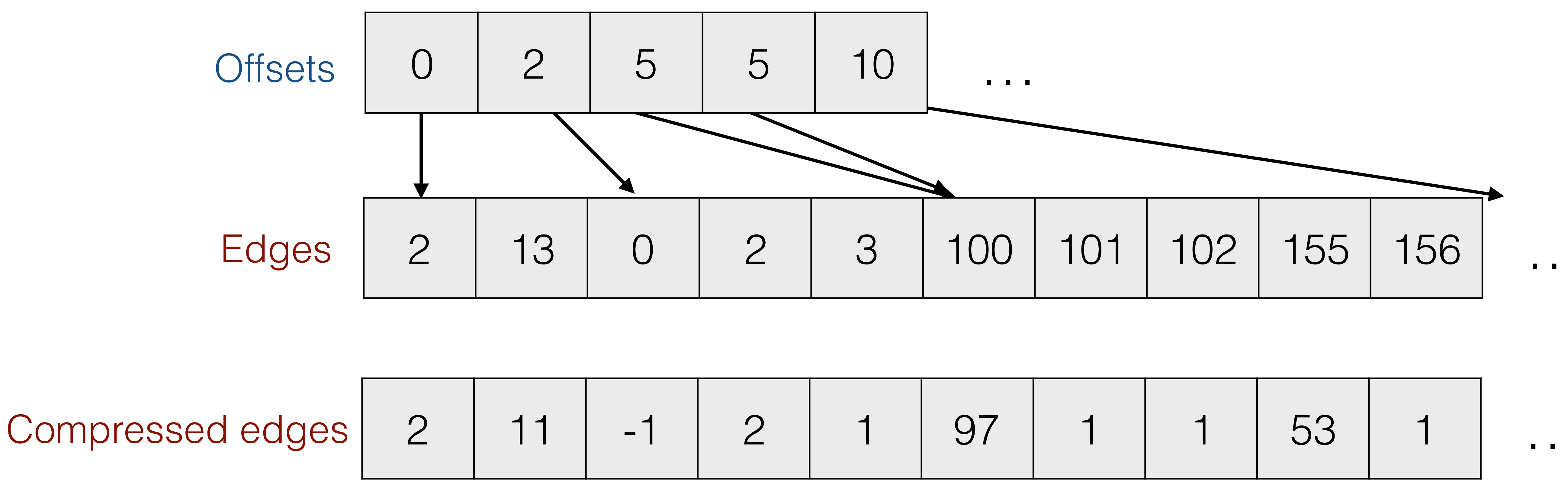
Towards Compressing Web Graphs

- Another early scheme based on copying

Compressing CSR

Use difference coding

- General purpose technique
- Compresses lists with small, regular gaps well
- Easy to see that accessing neighbors is O(deg(v))



Source: Smaller and Faster: Parallel Processing of Compressed Graphs with Ligra+

Variable length codes

k-bit codes

- Most gaps are small; want to avoid wasting 4 bytes/gap

Gaps:

8	1	1	53	1

- Ex: byte-code.

encoding(7)

0	0	0	0	0	1	1	1

continue bit

encoding(129)

1	0	0	0	0	0	0	1

0	0	0	0	0	0	0	1

= (2^0)*(block1)
+ (2^7)*(block2)

Source: Smaller and Faster: Parallel Processing of Compressed Graphs with Ligra+

Variable length codes

k-bit codes

- First gap could be negative, so first block is encoded specially (6 data bits, 1 continue bit, 1 sign-bit)
- Decoding:

```c
int read_byte_code(uint8_t* start) {
  int gap = 0;
  int shift = 0;
  while (1) {
    uint8_t b = *start++;
    gap += ((b & 0x7f) << shift);
    if (LAST_BIT_SET(b))
      shift += 7;
    else
      break;
  }
  return gap;
}
```

- Any issues with byte-codes? What if gaps are really small?

Source: Smaller and Faster: Parallel Processing of Compressed Graphs with Ligra+

Variable length codes

4-bit codes (nibbles)

- Same ideas work, encode data in blocks of k-1 bits
- Decoding cost grows in practice, more branches

What is a 1-bit code?

- Recall gamma codes: to encode a number x
  - store T, the largest power of two < x in unary
  - store a “0” (delimiter)
  - store x % T

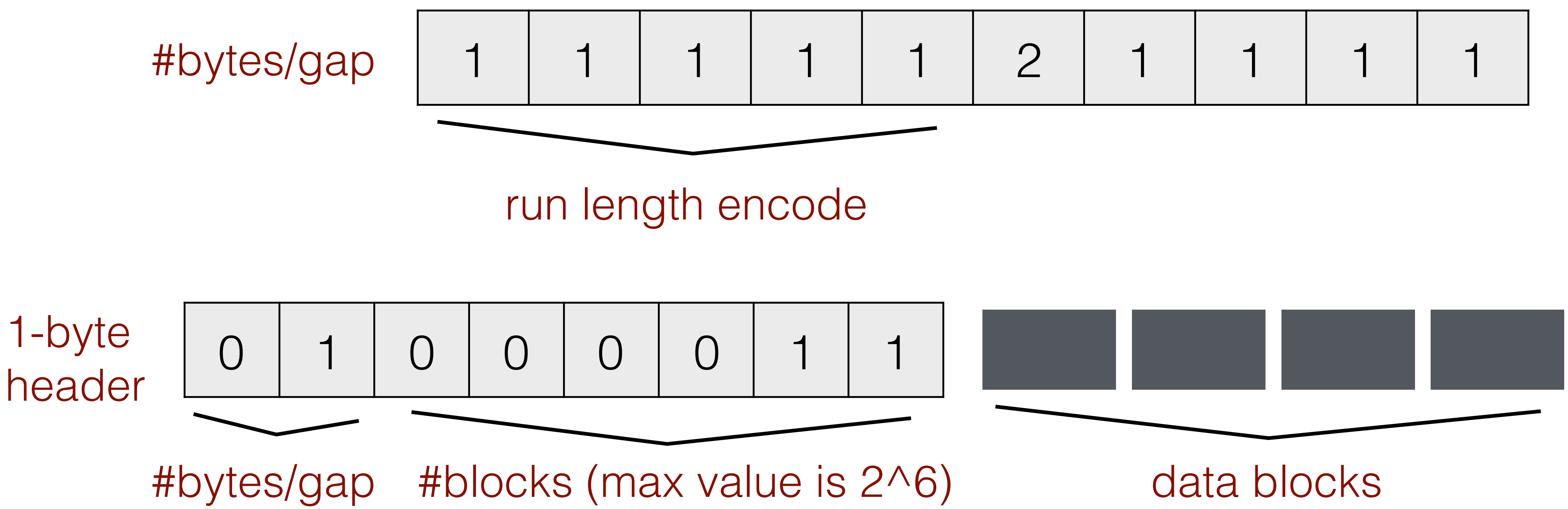
1-bit code is effectively a gamma code

Source: Smaller and Faster: Parallel Processing of Compressed Graphs with Ligra+

Variable length codes

Run-length encoded byte-codes

- Branches are costly in practice



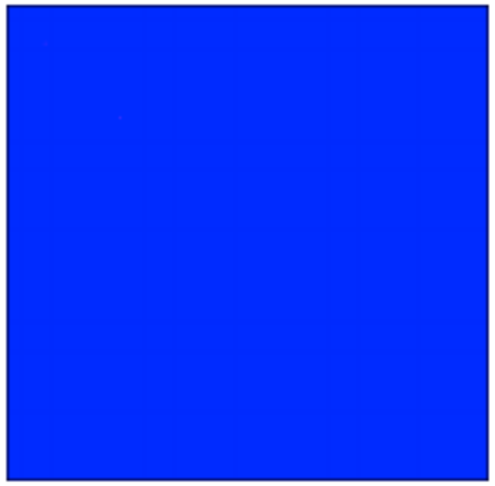
- Increases space, but decoding is cheaper (less branches)

Source: MIT-6.172 Lecture 21, Smaller and Faster: Parallel Processing of Compressed Graphs with Ligra+

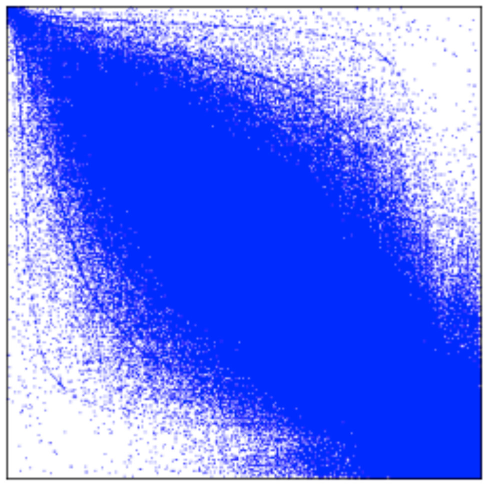
Reordering graphs for locality

What is locality?

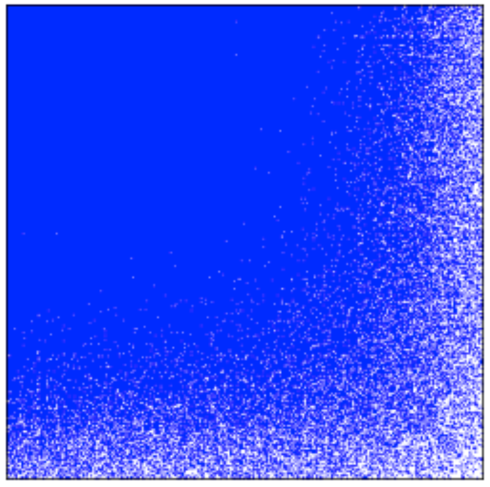
Adjacency matrix plots:



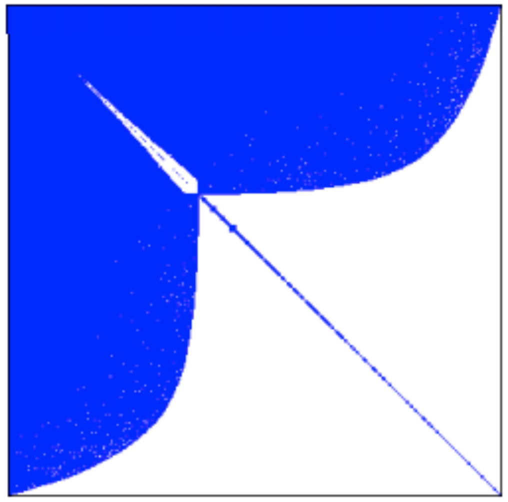
initial order: looks bad



another order; is this better?



slightly better order



lots of empty cells, is this even better?

Source: Compressing Graphs and Indexes with Recursive Graph Bisection

Reordering graphs for locality

What is locality?

I don’t really know how to define it. Maybe you know.

Here’s one idea:

- Measure the number of bits needed to difference encode all adjacency lists, and just call this locality
- Measure is known in literature as “log-gap cost“

Reordering graphs for locality

Log-gap cost

Fix some order, π

Cost of an adjlist, $f_\pi(v, out(v)) = \sum_{i=1}^{deg(v)-1} \log |\pi(v_{i+1}) - \pi(v_i)|$

Problem: find π minimizing $\sum_{v \in V} f_\pi(v, out(v))$

This problem is NP-hard

Finding the best ordering for difference-coding is hard

Related to Minimum Linear Arrangement (MLA), which comes up in VLSI design

$$\min_\pi \sum_{(u,v) \in E} |\pi(u) - \pi(v)|$$

Source: Compressing Graphs and Indexes with Recursive Graph Bisection

Reordering graphs for locality

Shingling

Originally purpose: detecting duplicate documents

- Compute a “fingerprint” of a vertex
- Order vertices that have similar fingerprints together

Jaccard coefficient: $J(A, B) = \frac{|A \cap B|}{|A \cup B|}$

Pick a hash function f (see [2] for necessary properties)

$$M_f(A) = \arg\min_{a \in A}(f(a))$$

$$P[M_f(A) = M_f(B)] = \frac{|A \cap B|}{|A \cup B|} = J(A, B)$$

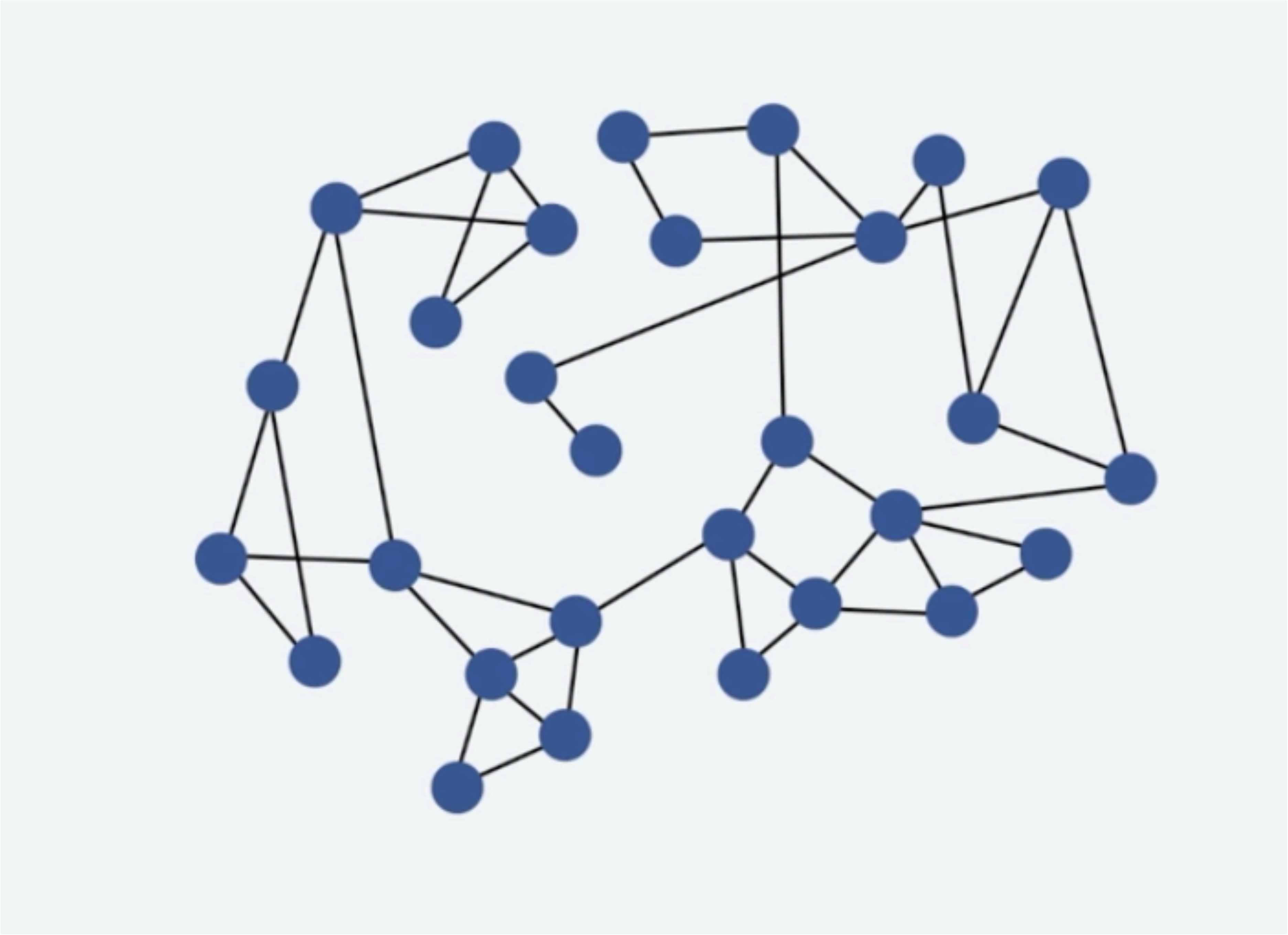
Vertices with the same shingle likely to have high Jaccard similarity

[1] On Compressing Social Networks

[2] Identifying and Filtering Near-Duplicate Documents

Reordering graphs for locality

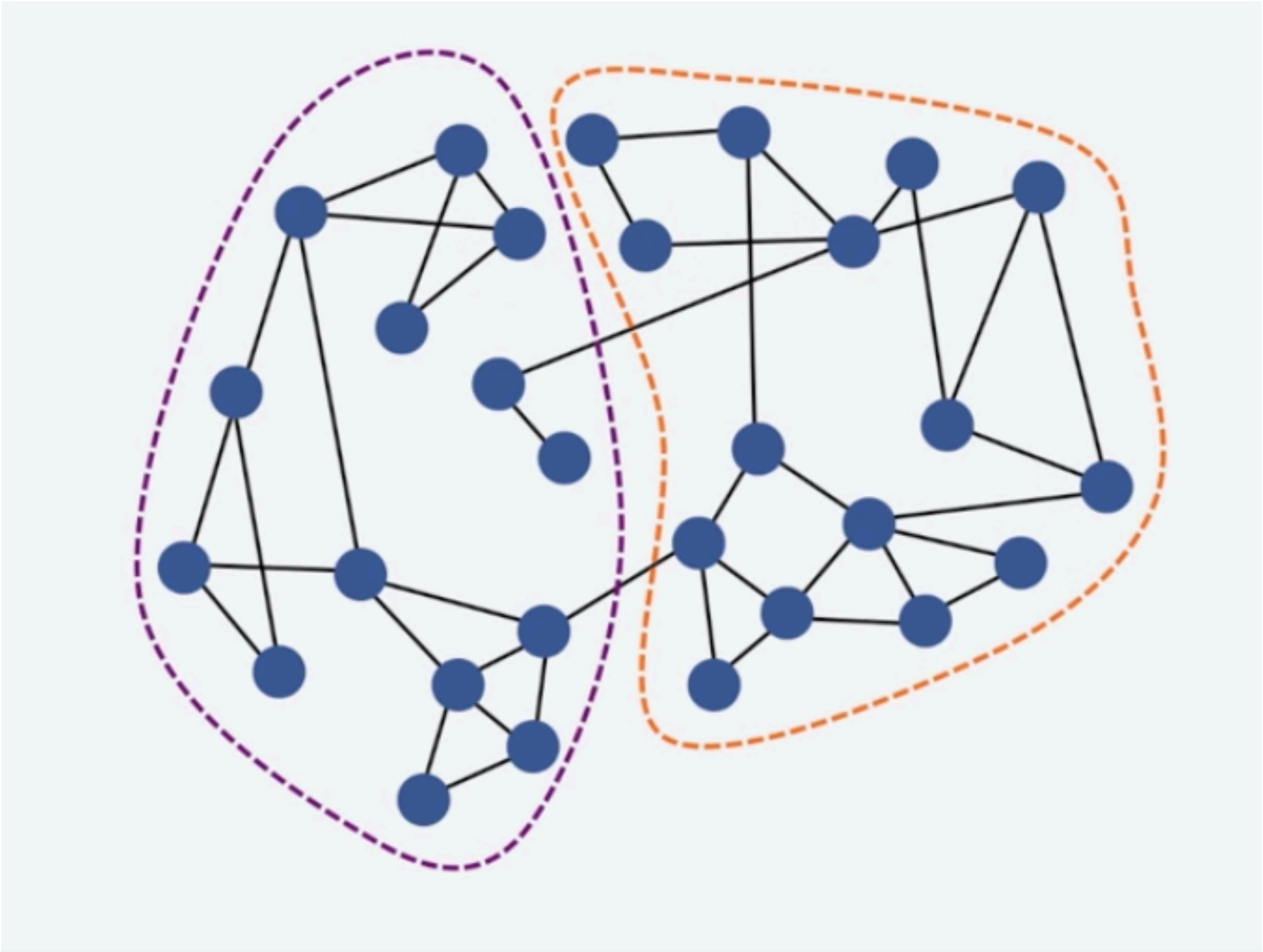
Recursive bisection



Source: Compressing Graphs and Indexes with Recursive Graph Bisection

Reordering graphs for locality

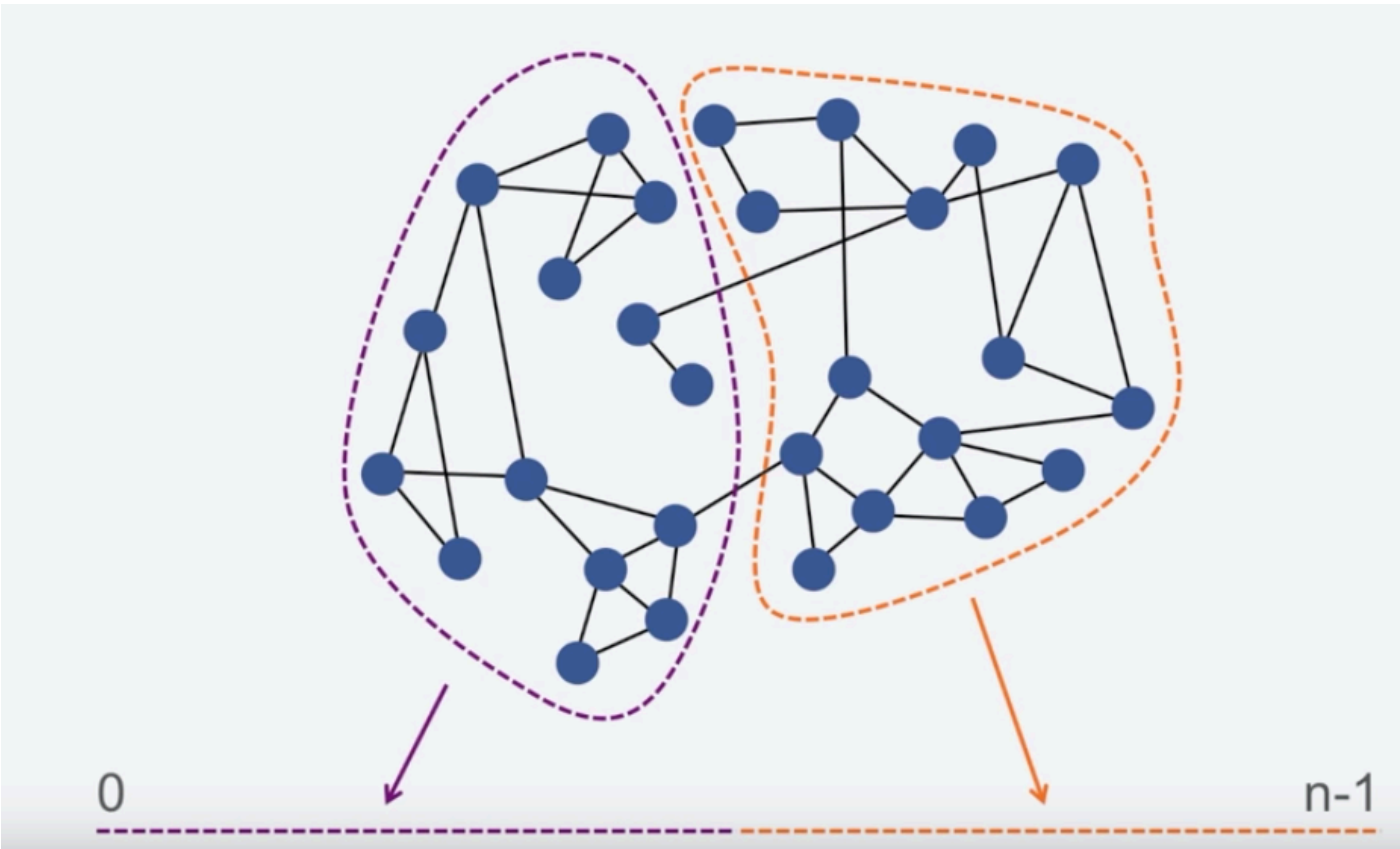
Recursive bisection



Source: Compressing Graphs and Indexes with Recursive Graph Bisection

Reordering graphs for locality

Recursive bisection

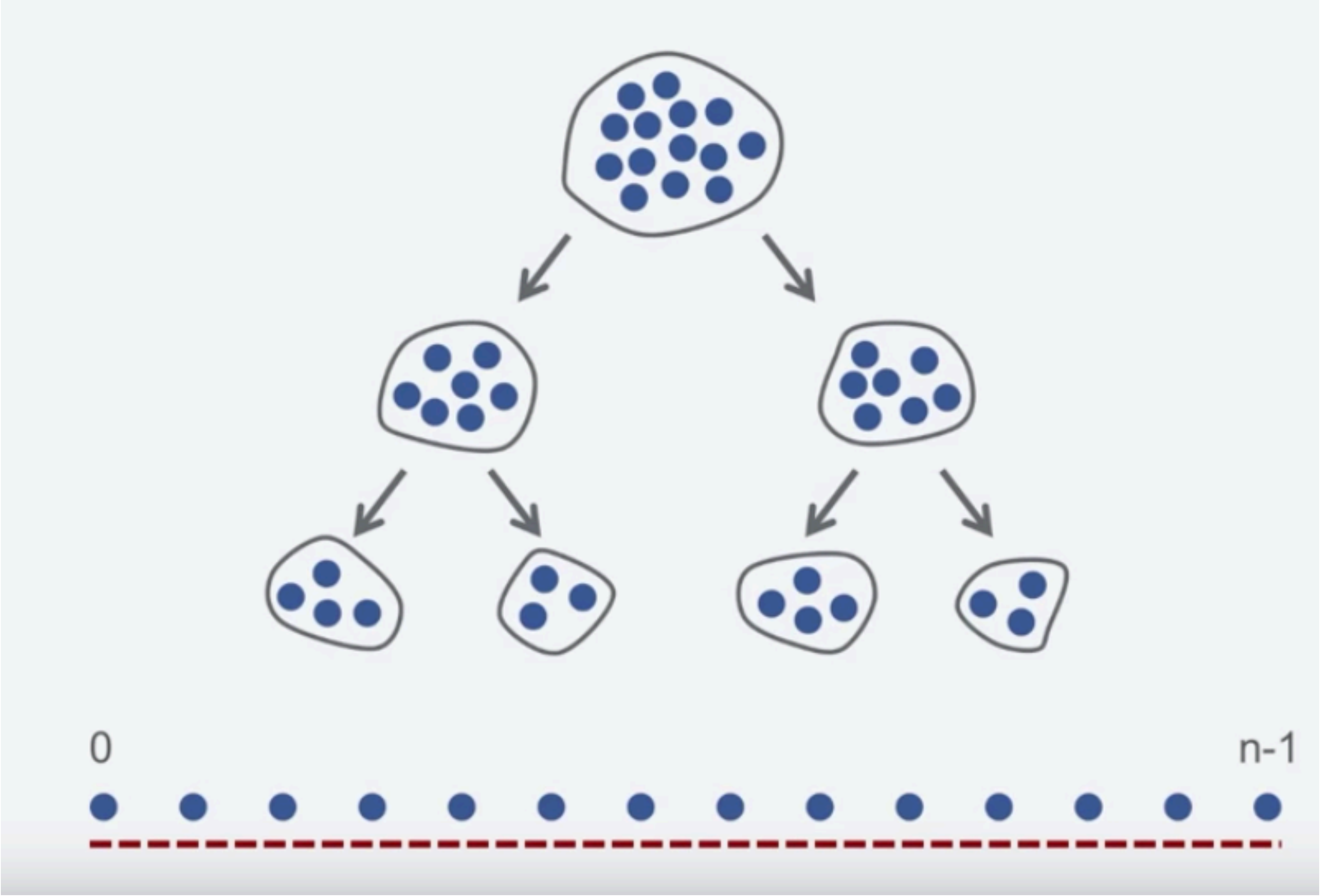


Source: Compressing Graphs and Indexes with Recursive Graph Bisection

Reordering graphs for locality

Algorithm: Bisection

- Initialize bisection randomly
- While not *converged*
  - swap two vertices that improve the optimization goal



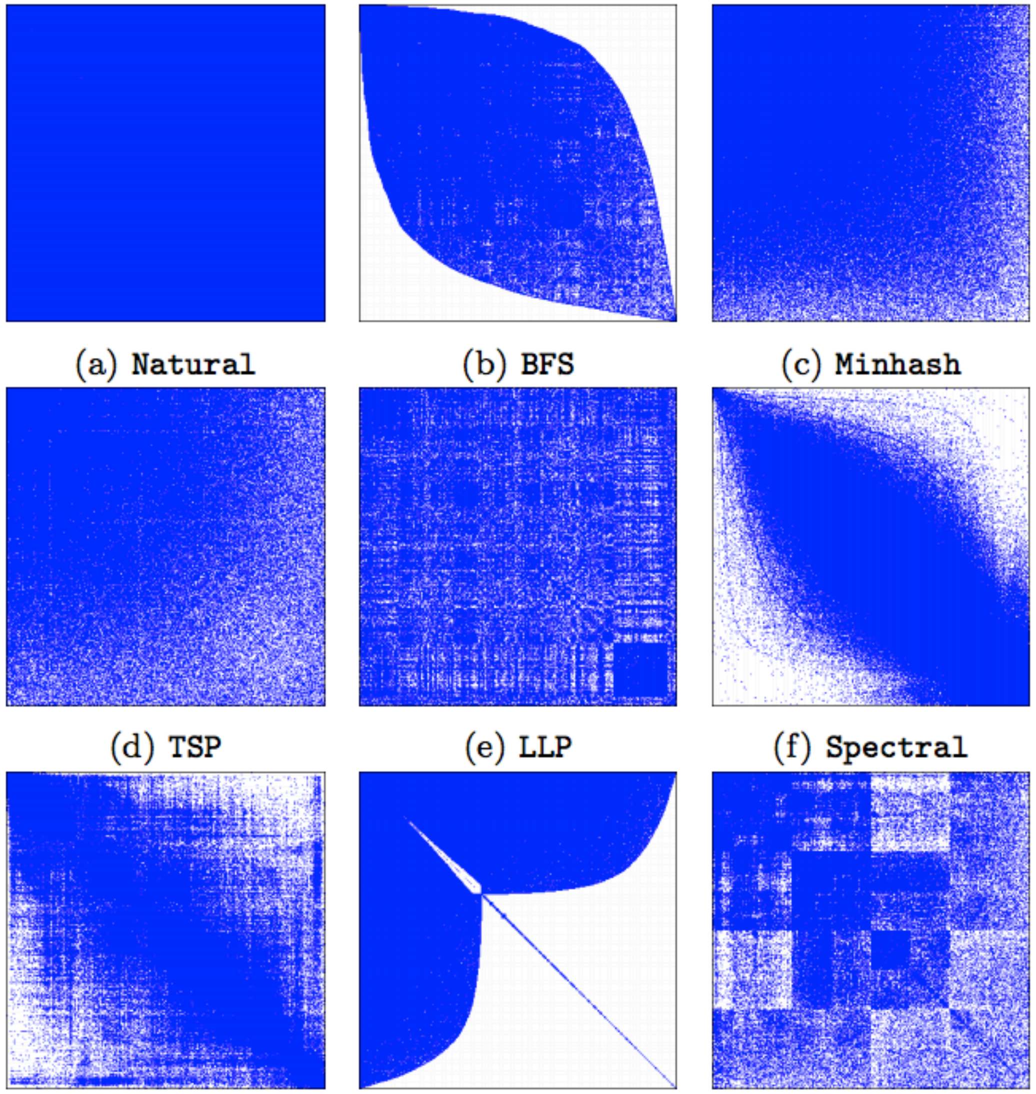
Kernighan-Lin Heuristic

Source: Compressing Graphs and Indexes with Recursive Graph Bisection

Reordering graphs for locality

Experimental results

Graph	Algorithm	LogGap	Log	BV
LiveJournal	Natural	10.43	17.44	14.61
	BFS	10.52	17.59	14.69
	Minhash	10.79	17.76	15.07
	LLP	7.46	**12.25**	11.12
	BP	**7.03**	12.79	**10.73**
Twitter	Natural	15.23	23.65	21.56
	BFS	12.87	22.69	17.99
	Minhash	10.43	21.98	14.76
	BP	**7.91**	**20.50**	**11.62**
FB-NewOrlean	Natural	9.74	14.29	14.64
	BFS	7.16	12.63	10.79
	Minhash	7.06	12.57	10.62
	TSP	5.62	11.61	8.96
	LLP	5.37	**9.41**	8.54
	Spectral	7.64	11.49	11.79
	Multiscale	5.90	9.58	9.25
	SlashBurn	8.37	13.06	12.65
	BP	**4.99**	9.45	**8.16**
FB-1B	Natural	19.63	27.22	
	Minhash	14.60	26.89	
	BP	**8.66**	**18.36**	



(a) Natural (b) BFS (c) Minhash (d) TSP (e) LLP (f) Spectral (g) Multiscale (h) SlashBurn (i) BP

spy plots for FB-NewOrlean

Edges on facebook: ~8bits/edge

Source: Compressing Graphs and Indexes with Recursive Graph Bisection

Highly compressible graph families

- Succinct data-structure: uses space that is “close” to information theoretic lower bound

lower bound: $\Omega(Z)$ succinct: $Z + o(Z)$

- Classic results: planar graphs can be represented in O(n) bits
  - Similar results for constant genus graphs

- Graphs that admit an $O(n^c)$-separator theorem in $O(n)$ bits

Source: Compact Representations of Separable Graphs

Conclusion: challenges in graph algorithms

- Compressed representations important (memory isn't free)
- Efficient representations important
  - Tradeoffs between space-efficiency and fast decoding
  - Formats should be amenable to parallelization

Real world graphs are highly compressible!

- Web graphs in a few bits/edge
- Social networks: no simple (lex) order with high locality
- Special graph families are highly compressible
- RW-graphs have much smaller separators than expected*

*Source: Compact Representations of Separable Graphs