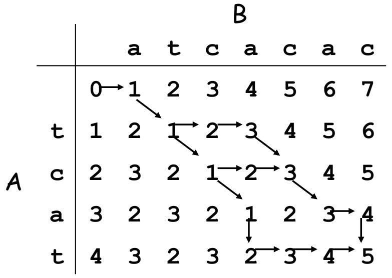
15-853:Algorithms in the Real World

Parallelism: Lecture 4
Dynamic Programming
Scheduling

Edit Distance: Dynamic Programming



		B						
		a	t	c	a	c	a	c
	0	1	2	3	4	5	6	7
A t	1	2	1	2	3	4	5	6
A c	2	3	2	1	2	3	4	5
A a	3	2	3	2	1	2	3	4
A t	4	3	2	3	2	3	4	5

→ insert
↓ delete
↘ common

Note: can be filled in any order as long as the cells to the left and above are filled.

Can follow path back through matrix to construct edits.

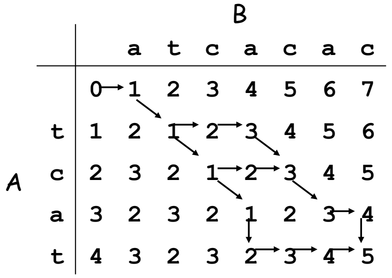
Edit Distance: Dynamic Programming

Sequential code:

```
for i = 1 to n
   M[i,1] = i;
for j = 1 to m
   M[1,j] = j;

for i = 2 to n
  for j = 2 to m
    if (A[i] == B[j])
      M[i,j] = M[i-1,j-1];
    else
      M[i,j] = 1 + min(M[i-1,j],M[i,j-1]);
```

Edit Distance: Dynamic Programming



B: a t c a c a c
0 1 2 3 4 5 6 7
t 1 2 1 2 3 4 5 6
c 2 3 2 1 2 3 4 5
a 3 2 3 2 1 2 3 4
t 4 3 2 3 2 3 4 5

→ insert
↓ delete
↘ common

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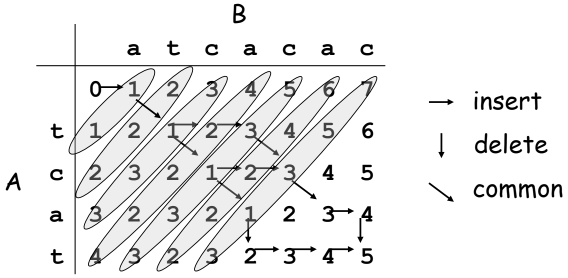
Edit Distance: Dynamic Programming



		a	t	c	a	c	a	c
	0	1	2	3	4	5	6	7
t	1	2	1	2	3	4	5	6
c	2	3	2	1	2	3	4	5
a	3	2	3	2	1	2	3	4
t	4	3	2	3	2	3	4	5

Note: can be filled in any order as long as the cells to the left and above are filled.

Can follow path back through matrix to construct edits.

Edit Distance: Dynamic Programming

Parallel code:

```
for i = 1 to n  : M[i,1] = i;
for j = 1 to m  : M[1,j] = j;

for k = 2 to n+m
  istart = max(2,k-m)
  iend = min(k,n)
  parallel for i = istart to iend
    j = k-i+2
    if (A[i] == B[j])
      M[i,j] = M[i-1,j-1];
    else
      M[i,j] = 1 + min(M[i-1,j],M[i,j-1]);
```

Scheduling

So far we have assumed "magic" schedulers that maps dynamic tasks onto processors.

Sidebar: Beyond Nested Parallelism

Assume a way to fork
  - Pairwise or multiway

What types of synchronization are allowed
  - Fork-join (nested parallelism)
  - Futures
  - Fully general

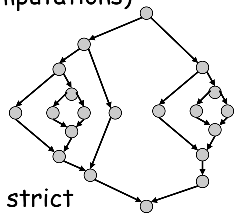
The first two can be made deterministic

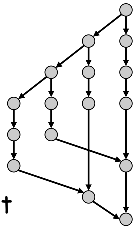
Can have a large effect on the scheduler and what can be proved about the schedules.

Strict and Fully Strict

Fully strict (fork-join, nested parallel): a task can only synchronize with its parent

Strict: a task can only synchronize with an ancestor. (X10 recently extended to support strict computations)

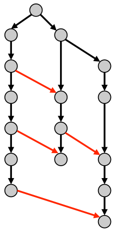

fully strict


strict

Futures

Futures or read-write synchronization variables can be used for pipelining of various forms, e.g. **producer consumer pipelines**. This cannot be supported in strict or fully strict computations.

If read always occurs “after” the write in sequential order then there is no deadlock



General

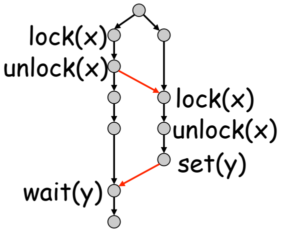
Locks

Transactions

Synch variables

Easy to create deadlock

Hard to schedule



Scheduling Outline

Theoretical results on scheduling

Graham, 1966

Greedy Schedules

Specific greedy schedules

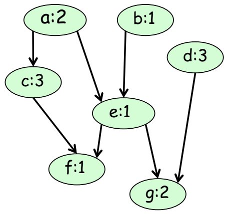
- Breadth First
- Work Stealing
- P-DFS
- Hybrid

Graham Scheduling

"Bounds on Certain Multiprocessor Anomilies", 1966
Model:
Processing Units : P_i , $1 \le i \le n$
Tasks : $T = \{T_i , \dots , T_m\}$
Partial order : $<_T$ on T
Time function : $\mu : T \rightarrow [0,\infty]$
$(T, <_T , \mu)$: define a weighted DAG

Graham Scheduling

The processors



The DAG

Graham: List Scheduling

For a task set T, a Task List L_T is some ordering $[T_{k1} , \dots , T_{km}]$ of T.

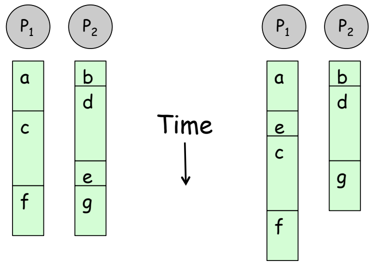
Task is **ready** when not yet started but all predecessors are finished

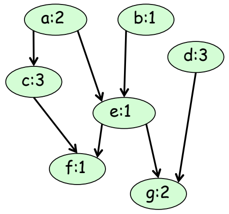
List scheduling : when a processor finishes a task it immediately takes the first ready task from L. Ties broken by processor ID.

Showed that for any L_T and L_T':

$$\frac{T(L_T)}{T(L_T')} \le 1 + \frac{n-1}{n}$$

List Scheduling





L = [a,b,c,d,e,f,g]

L' = [g,e,a,b,c,d,f]

The DAG

Some definitions

T_p : time on P processors
W : work (total weight of the DAG)
D : span (longest path in the DAG)

Lower bound on time : $T_p \geq \max(W/P, D)$

Greedy Schedules

As schedule is *greedy* if a processor cannot sit idle when a task is ready.

List schedules are greedy.

For any greedy schedule:

Efficiency = $\frac{W}{T_P} \geq \frac{PW}{W + D(P-1)}$

Parallel Time = $T_P < \frac{W}{P} + D$

Greedy Schedules

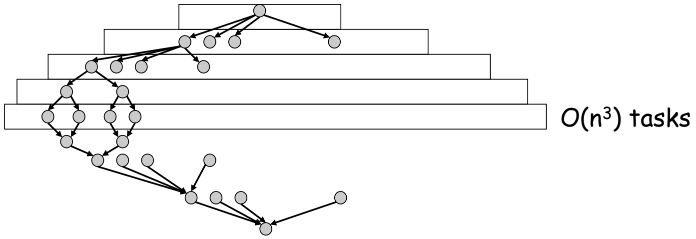
Theorem: The time taken by a greedy scheduler is

$$T_P < \frac{W}{P} + D$$

Proof: on board.

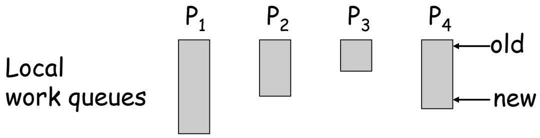
Breadth First Schedules

Most naïve schedule. Used by most implementations of P-threads.



Bad space usage, bad locality

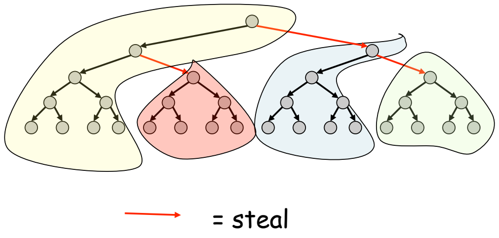
Work Stealing



- push new jobs on “new” end
- pop jobs from “new” end
- If processor runs out of work, then “**steal**” from another “old” end
- Each processor tends to execute a sequential part of the computation.

Work Stealing

Tends to schedule “sequential blocks” of tasks



Work Stealing Theory

For fork-join computations

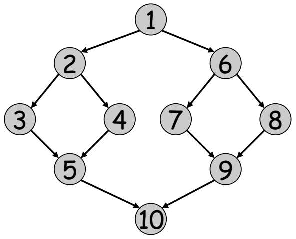
- # of steals = $O(PD)$
- Space = $O(PS_1)$
  - S_1 is the sequential space
- # cache misses on private caches = $Q_1 + O(MPD)$
  - Q_1 = sequential misses, M = cache size

Work Stealing Practice

- Used in Cilk Scheduler
  - Small overheads because common case of pushing/popping from local queue can be made fast (with good data structures and compiler help).
  - No contention on a global queue
  - Has good distributed cache behavior
  - Can indeed require $O(S_1P)$ memory
- Used in many schedulers

Parallel Depth First Schedules (P-DFS)

List scheduling based on Depth-First ordering



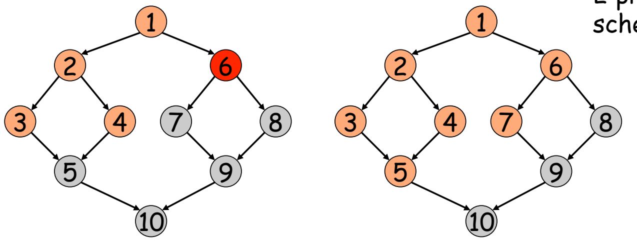
2 processor schedule

1
2, 6
3, 4
5, 7
8
9
10

For strict computations a shared stack implements a P-DFS

"Premature task" in P-DFS

A running task is premature if there is an earlier sequential task that is not complete



2 processor schedule

1
2, 6
3, 4
5, 7
8
9
10

● = premature

P-DFS Theory

For any computation:

Premature nodes at any time = $O(PD)$

Space = $S_1 + O(PD)$

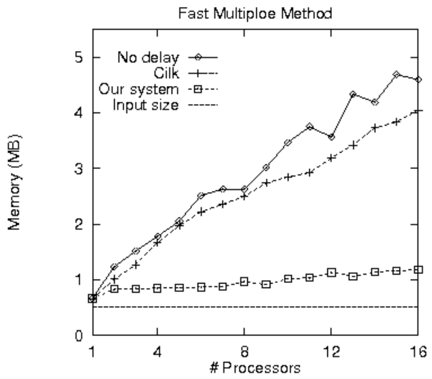
With a shared cache of size $M_1 + O(PD)$ we have $Q_p = Q_1$

P-DFS Practice

Experimentally uses less memory than work stealing and performs better on a shared cache.

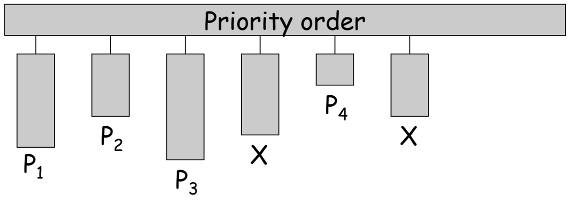
Requires some "coarsening" to reduce overheads

P-DFS Practice



Hybrid Scheduling

Can mix Work Stealing and P-DFS



Gives a way to do automatic coarsening while still getting space benefits of PDF
Also allows suspending a whole Q

Other Scheduling

Various other techniques, but not much theory

e.g.

- Locality guided work stealing
- Affinity guided self-scheduling

Many techniques are for particular form of parallelism