Algorithms and Applications (15-499)
Assignment #5 Due Date: 24 Apr 03

This assignment is only worth one half as much as assignments 1–3 since you are working on a project.
Do any three of the following four problems.

Problem 1:
Let C be a code over $\{-2 \cdots 2\}$ of blocklength 5, such that the sum of the components in any code word is 0. The distance between any two code words is the Hamming distance between them, or the number of components in which they differ. Find the rate of the code and the minimum distance between any two code words. How many errors can this code detect and how many can it correct?

Problem 2:
The ISBN is a 10-digit codeword such as 0-471-06259-6. The first digit indicates the language (0 for english), the next group specifies the publisher (471 for Wiley); the next group forms the book number assigned by the publisher. The final digit is chosen to make the entire number $x_1 \cdots x_{10}$ satisfy the single check equation: $\sum_{i=1}^{10}(ix_i) = 0 \pmod{11}$.
Note that the first 9 digits lie between 0 and 9, whereas the last digit can take any value between 0 and 10. The value 10 is represented by the letter X.

A. Calculate the check bit for the code 0-13-200809.

B. It is easy to see that the ISBN code can detect any single digit error. Show that the code can detect the transposition of any two digits (not necessarily consecutive).

C. The sixth digit in the code 0-13-28.796-X was smudged. Find the missing digit.

Problem 3:
Let C_1 and C_2 be cyclic codes of blocklength n generated by the polynomials $g_1(x)$ and $g_2(x)$ respectively. What is the generator polynomial for the smallest cyclic code containing $C_1 \cup C_2$? What is the generator polynomial for $C_1 \cap C_2$? (Note that the intersection of cyclic codes is cyclic).

Problem 4:
Suppose that there is a very inexpensive PCI board that implements an $RS(255, 223)$ Reed Solomon encoder and decoder in hardware. (This is most likely true!) The board encodes or decodes sequences of 223 bytes and can correct up to 16 errors in a sequence. You would like to use Reed-Solomon codes to protect your data against errors as it is transmitted over a wireless communication channel. Unfortunately, your radio experiments show that, at your transmission rate, bursts of errors tend to be longer than 16 bytes. Using the $RS(255, 223)$-encoder/decoder as a building block, design a system that can correct up to 64 consecutive errors in a 1020-byte transmitted message, assuming that there are no other errors in the message. You must preserve the rate of the channel.