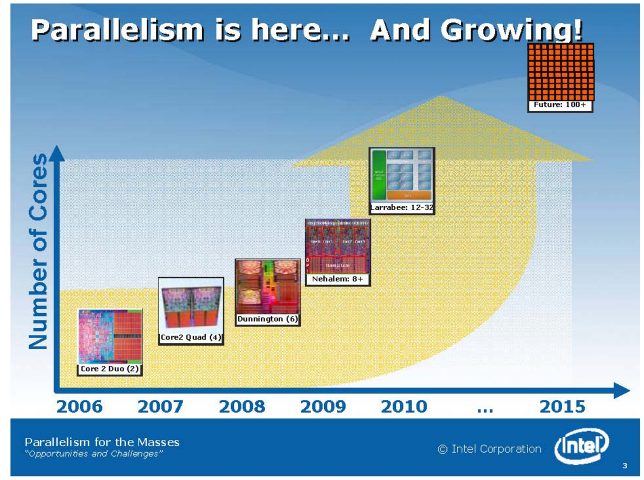
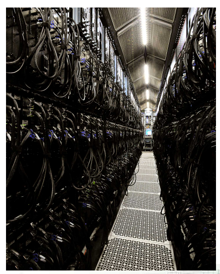
15-853:Algorithms in the Real World

Parallelism: Lecture 1
- Nested parallelism
- Cost model
- Parallel techniques and algorithms



Andrew Chien, 2008

Outline

- Concurrency vs. Parallelism
- Quicksort example
- Nested Parallelism
  - fork-join and parallel loops
- Cost model: work and span
- Techniques:
  - Using collections: inverted index
  - Divide-and-conquer: merging, mergesort, kd-trees, matrix multiply, matrix inversion, fft
  - Contraction : quickselect, list ranking, graph connectivity, suffix arrays

Parallelism in "Real world" Problems

Optimization
N-body problems
Finite element analysis
Graphics
JPEG/MPEG compression
Sequence alignment
Rijndael encryption
Signal processing
Machine learning
Data mining

Parallelism vs. Concurrency

- Parallelism: using multiple processors/cores running at the same time. Property of the machine
- Concurrency: non-determinacy due to interleaving threads. Property of the application.

		Concurrency	
		sequential	concurrent
Parallelism	serial	Traditional programming	Traditional OS
	parallel	Deterministic parallelism	General parallelism

Nested Parallelism

nested parallelism =
arbitrary nesting of parallel loops + fork-join

- Assumes no synchronization among parallel tasks except at joint points.
- Deterministic if no race conditions

Advantages:

- Good schedulers are known
- Easy to understand, debug, and analyze

Nested Parallelism: parallel loops

```
cilk_for (i=0; i < n; i++)
   B[i] = A[i]+1;
```

Cilk

```
Parallel.ForEach(A, x => x+1);
```

Microsoft TPL (C#,F#)

```
B = {x + 1 : x in A}
```

Nesl, Parallel Haskell

```
#pragma omp for
for (i=0; i < n; i++)
  B[i] = A[i] + 1;
```

OpenMP

Nested Parallelism: fork-join

```
cobegin {
  S1;
  S2;}
```

Dates back to the 70s or possibly 60s. Used in dialects of Pascal

```
coinvoke(f1,f2)
Parallel.invoke(f1,f2)
```

Java fork-join framework
Microsoft TPL (C#,F#)

```
#pragma omp sections
{
  #pragma omp section
  S1;
  #pragma omp section
  S2;
}
```

OpenMP (C++, C, Fortran, …)

Nested Parallelism: fork-join

```
spawn S1;
S2;
sync;
```

cilk, cilk+

```
(exp1 || exp2)
```

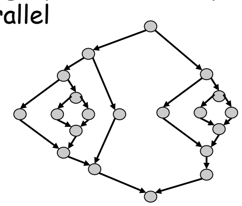
Various functional languages

```
plet
  x = exp1
  y = exp2
in
  exp3
```

Various dialects of ML and Lisp

Serial Parallel DAGs

Dependence graphs of nested parallel computations are series parallel



Two tasks are parallel if not reachable from each other.

A data race occurs if two parallel tasks are involved in a race if they access the same location and at least one is a write.

Cost Model

Compositional:

Work : total number of operations
- costs are added across parallel calls

Span : depth/critical path of the computation
- Maximum span is taken across forked calls

Parallelism = Work/Span
- Approximately # of processors that can be effectively used.

Combining costs

Combining for parallel for:

```
pfor (i=0; i<n; i++)
  f(i);
```

$$W_{\text{pexp}}(\text{pfor } ...) = \sum_{i=0}^{n-1} W_{\text{exp}}(\text{f(i)}) \qquad \text{work}$$

$$D_{\text{pexp}}(\text{pfor } ...) = \max_{i=0}^{n-1} D_{\text{exp}}(\text{f(i)}) \qquad \text{span}$$

Why Work and Span

Simple measures that give us a good sense of efficiency (work) and scalability (span).

Can schedule in O(W/P + D) time on P processors.

This is within a constant factor of optimal.

Goals in designing an algorithm

1. Work should be about the same as the sequential running time. When it matches asymptotically we say it is **work efficient.**
2. Parallelism (W/D) should be polynomial $O(n^{1/2})$ is probably good enough

Example: Quicksort

```
function quicksort(S) =
if (#S <= 1) then S
else let
  a = S[rand(#S)];
  S1 = {e in S | e < a};
  S2 = {e in S | e = a};
  S3 = {e in S | e > a};
  R = {quicksort(v) : v in [S1, S3]};
in R[0] ++ S2 ++ R[1];
```

Partition

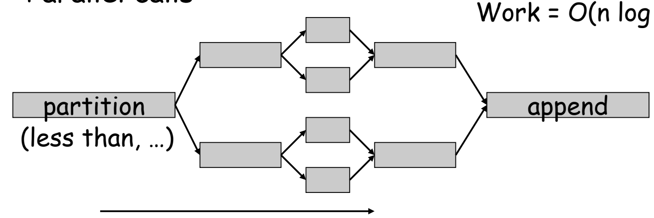
Recursive calls

How much parallelism?

Quicksort Complexity

Sequential Partition and appending
Parallel calls

Work = O(n log n)



Span = O(n)

Parallelism = O(log n)

Not a very good parallel algorithm

*All randomized with high probability

Quicksort Complexity

Now lets assume the partitioning and appending can be done with:

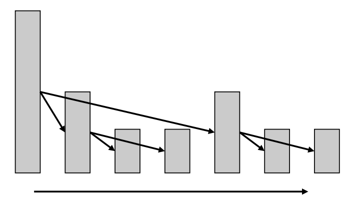
Work = O(n)

Span = O(log n)

but recursive calls are made sequentially.

Quicksort Complexity

Parallel partition
Sequential calls



Work = O(n log n)

Span = O(n)

Parallelism = O(log n)

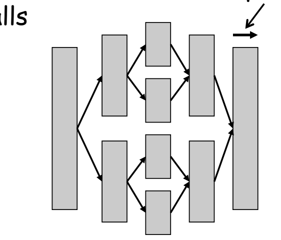
Not a very good parallel algorithm

*All randomized with high probability

Quicksort Complexity

Parallel partition
Parallel calls

Span = O(lg n)



Work = O(n log n)

Span = $O(\lg^2 n)$

Parallelism = O(n/log n)

A good parallel algorithm

*All randomized with high probability

Quicksort Complexity

Caveat: need to show that depth of recursion is O(log n) with high probability

Parallel selection

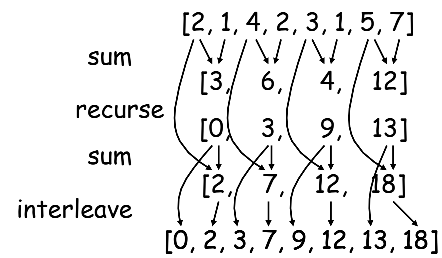
```
{e in S | e < a};
```

S = [2, 1, 4, 0, 3, 1, 5, 7]
F = S < 4 = [1, 1, 0, 1, 1, 1, 0, 0]
I = addscan(F) = [0, 1, 2, 2, 3, 4, 5, 5]

where F
R[I] = S = [2, 1, 0, 3, 1]

Each element gets sum of previous elements.
Seems sequential?

Scan



Scan code

```
function addscan(A) =
if (#A <= 1) then [0]
else let
  sums = {A[2*i] + A[2*i+1] : i in [0:#a/2]};
  evens = addscan(sums);
  odds = {evens[i] + A[2*i] : i in [0:#a/2]};
in interleave(evens,odds);
```

$W(n) = W(n/2) + O(n) = O(n)$
$D(n) = D(n/2) + O(1) = O(\log n)$

Parallel Techniques

Some common themes in "Thinking Parallel"

1. Working with collections.
   - map, selection, reduce, scan, collect
2. Divide-and-conquer
   - Even more important than sequentially
   - Merging, matrix multiply, FFT, …
3. Contraction
   - Solve single smaller problem
   - List ranking, graph contraction
4. Randomization
   - Symmetry breaking and random sampling

Working with Collections

reduce ⊙ [a, b, c, d, …
  = a ⊙ b ⊙ c ⊙ d + …

scan ⊙ ident [a, b, c, d, …
  = [ident, a, a ⊙ b, a ⊙ b ⊙ c, …

sort compF *A*

collect [(2,a), (0,b), (2,c), (3,d), (0,e), (2,f)]
  = [(0, [b,e]), (2,[a,c,f]), (3,[d])]