Algorithms in the Real World (15-853), Fall 09
Assignment #6 Due: December 3

You can look up material on the web and books, but you cannot look up solutions to the given problems. You can work in groups, but must write up the answers individually.

Problem 1: Absolute Inequalities (10pt)

Let's say we augment linear programs to allow constraints to include absolute values (*e.g.* $|x_1| + 3|x_2| \leq b$). Can we solve all such problems in polynomial time? Show why or why not. (Assume $P \neq NP$.)

Problem 2: Max Flow Dual (10 points)

The class slides described a formulation of Max-Flow using linear programming (Page 7 of the first lecture on linear programming). Assume there is no capacity inequality for the special edge x_0 (the edge from out to in can support any capacity). The same lecture also described how to convert a linear program into its dual (page 44). Please write down the dual for the max-flow problem and write down an interpretation of what the equations mean, what the variables mean, what we are optimizing, what the optimal solution will look like, and why these particular equations optimize what you claim they do. I'm not looking for an essay—short answers are best.

Problem 3: Ellipsoid Variant (10pt)

A relatively new approach to solving linear programming in (probabilistic) polynomial time is based on taking random walks to sample points in a convex polytope. Let's say you are given a black box that for a set of linear constraints returns a random point within the polytope volume (i.e., each request to the black box gives you a new and independent point selected from the distribution where all points have equal probability). Assuming this black-box runs in polynomial time, outline an algorithm that will find the optimal solution to a linear program in polynomial time. As with the ellipsoid method, it depends on finite precision—i.e. you can assume that after some number of steps there is no resolution left. You should, however, argue that something is shrinking geometrically.

Problem 4: Branch and Bound (10 points)

Solve the following problem using the implicit enumeration branch-and-bound technique (the 0-1 programming technique) describe in class. You should show the tree and mark on any leaf node why it was pruned.

$$\begin{array}{ll} \text{minimize} & 3x_1 + 2x_2 + 5x_3 + x_4 \\ \text{subject to} & -2x_1 + x_2 - x_3 - 2x_4 \leq -2 \\ & -x_1 - 5x_2 - 2x_3 + 3x_4 \leq -3 \\ & x_1, x_2, x_3, x_4 \text{ binary} \end{array}$$