Lecture 21
Grasps, fixtures, closure

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Mechanics of Manipulation

Today's outline

Review: manipulation taxonomy

- Kinematic manipulation.
  - Kinematic constraint.
  - Programmed motion; transport.
  - Kinematic models of contact.
- Static and quasistatic manipulation.
  - Pushing.
  - Grasping.
  - Fixture design.
  - Parts orienting.
- Dynamic manipulation.
- Throwing, striking, . . .

Grasping and fixturing

- Grasping and fixturing are closely related:
  - Fixture: immobilize something.
  - Grasp: immobilize something relative to the hand.

Form and force closure

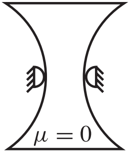
- Warning: these definitions are not used uniformly in the literature. Some papers define “form closure” to mean precisely what we mean by “force closure”!

Definition

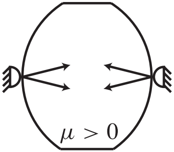
- Form closure: the object is at an isolated point in configuration space.
- First order form closure: Every nonzero velocity twist is contrary to some contact screw.
- Force closure: the contacts can apply an arbitrary wrench to the object.
- Equilibrium: the contact forces can balance the object’s weight and other external forces.

Flavors of closure

- Frictionless force closure $\equiv$ first order form closure.
- First order form closure $\longrightarrow$ form closure.
- Frictionless force closure $\longrightarrow$ force closure.
- Form closure $\not\longrightarrow$ force closure.
- Force closure $\not\longrightarrow$ form closure.



Form closure
does not imply
force closure



Force closure
does not imply
form closure

Stability

Research literature offers several aproaches:

- Define *stability* to be first order form closure.
  - Conservative—numerous contacts required.
- Define *stability* to be force closure.
  - Sometimes too optimistic. Remember the piranha.
  - Nguyen's result offers rationale for using either first-order form closure or force closure:

Theorem (Nguyen 1989)

Every force closure grasp with hard point contacts can be stabilized by modeling each fingertip contact point as a three-axis spring.

- But, Nguyen's notion of stability isn't always the right one. There is much subsequent work to consult.
- Also see closely related work on *grasp metrics*.

Issues in fixture and grasp design

- Mishra, Schwartz and Sharir (1987) laid out three problems:
  - *Analysis:*
    Given an object, a set of contacts, and possibly other information, determine whether closure applies.
  - *Existence:*
    Given an object, and possibly some constraints on the allowable contacts, does a set of contacts exist to produce closure?
  - *Synthesis:*
    Given an object, and possibly some constraints on the allowable contacts, find a suitable set of contacts.

Analysis

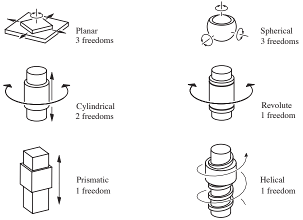
- Force closure: check positive linear span of friction cones.
- Frictionless force closure or first order form closure: check positive linear span of contact normals.
- Higher order form closure: beyond the scope of the course. See, e.g., Rimon and Burdick 1995.

Existence

- Given an object, does a force closure grasp exist?
  - Put fingers everywhere. Check whether positive linear span is all of wrench space.
- Are there are any shapes that do *not* have force closure grasps?

Theorem (Mishra, Schwartz and Sharir)

For any bounded shape that is not a surface of revolution, a force closure (or first order form closure) grasp exists.



Synthesis

- Consider a finger to be *redundant* if it can be deleted without reducing the positive linear span of all the fingers

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procedure GRASP
    put fingers "everywhere"
    while redundant finger exists
        delete any redundant finger
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- *Everywhere* means a dense sampling of the object boundary.
- Clearly the algorithm generates a grasp for any object not a surface of revolution, if the sampling is dense enough. But how many fingers does it take?

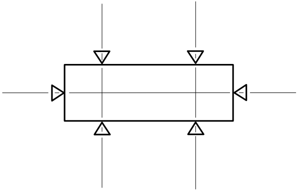
How many fingers?

Theorem (Steinitz)

Let X be a set of points in $\mathbf{R}^d$, with some point p in the interior of the convex hull of X. Then there is some subset Y of X, with $2d$ points or less, such that p is in the interior of the convex hull of Y.

Theorem (Mishra, Schwartz, and Sharir)

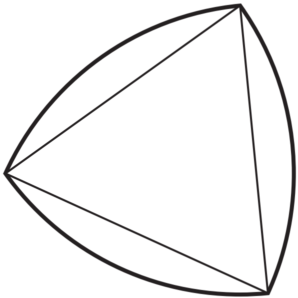
For any surface not a surface of revolution, GRASP *yields a grasp with at most 6 fingers in the plane, at most 12 fingers in three space.*



- In the absence of coincidences among the initial sampling of contact normals, how many fingers will GRASP terminate with?

Problem: grasping Reuleaux's triangle

- *Reuleaux's triangle* is a figure of constant diameter. Each edge is a circular arc centered on the opposite vertex.
- Consider frictionless force closure.
- If only parallel jaw grippers are used, show that six fingers are required.
- Construct a four-finger grasp. (Hint: don't use parallel jaw grippers!)



Implementation

How do we implement, e.g., force closure?

- Moment labeling? Force dual? NO! Those are graphical methods. We want analytical methods. So we adapt the standard numerical tools to polyhedral cones in twist and wrench space. Some handwritten notes follow.