15-750: Graduate Algorithms April 7, 2017

Lecture 30: Parallel Algorithms I: Prefix-Sum, List-Ranking

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1 Motivation

Let $\oplus$ be an associative binary operation:
Defination: All prefix sums
input : $[a_0, ..., a_{n-1}]$
output: $[a_0, a_0 \oplus a_1 \oplus a_3, ..., a_0 \oplus ... \oplus a_{n-1}]$
Prescan output: $[I, a_0, a_0 \oplus a_1, ..., a_0 \oplus ... \oplus a_{n-2}]$
Application: Packing Memory

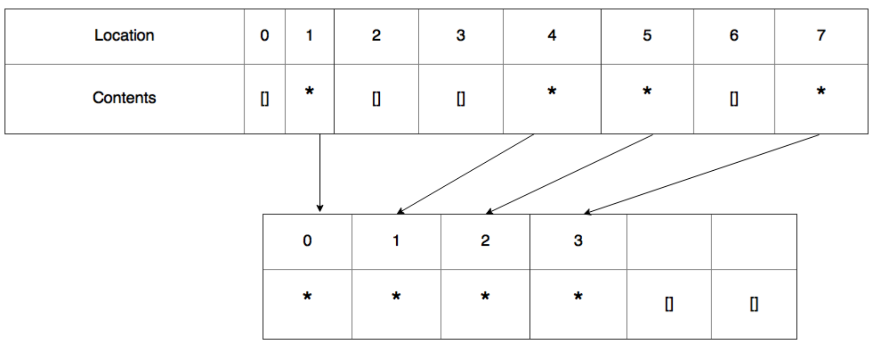

Figure 1: Packing memory

2 Prescan

input : (3,1,7,0,4,1,6,3) $\oplus$ addition
Here is the algorithm:
Algorithm:

1. Compute tree of partial sums
2. Set root to zero
3. DOWN!
   (a) Right child $\Leftarrow$ Parent $\oplus$ Left child
   (b) Left-child $\Leftarrow$ Parent

Here is an example:
For this algorithm
$T(n) = \omega(\log n), W(n) = O(n)$

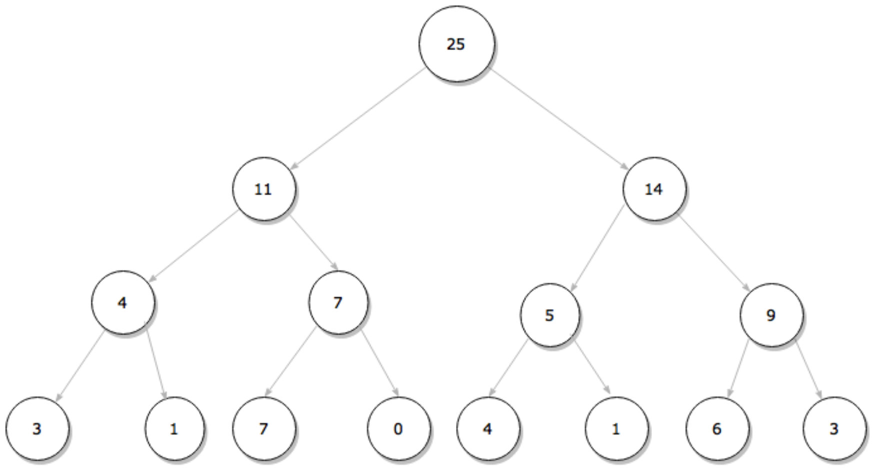

Figure 2: Original tree

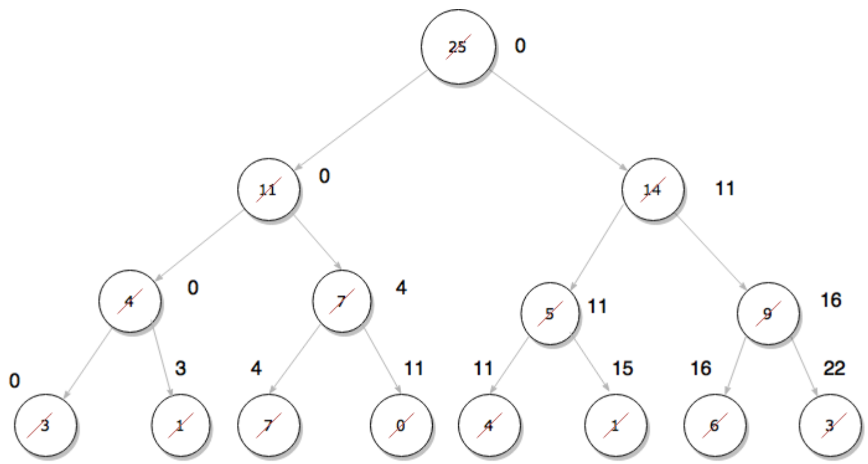

Figure 3: After prescan

3 List Ranking

Input: linked list
Output: a mark on each node such that mark = distance from head or mark = distance to tail
Assume:

1. Pointers are in consecutive memory
2. We know location of head and tail
3. Pointers are in arbitrary order

4 Wyllie's Algorithm

Algorithm 1 Wyllie's

In parallel rank(!) $= 1$; rank (tail) $=0$
In parallel while succ(head)$\neq$ nil do
if succ(!) $\neq$ nil do **then**
    rank (!) $=$ rank (!) $\oplus$ rank (succ(!))
    succ (!) $=$ succ (succ(!))
end if

Using n processors and a CREW model for memory, this algorithm does $\omega(n \log n)$ work in $\omega(\log n)$ time. Our goal is to reduce it to $\omega(n)$ work in $\omega(\log n)$ time.

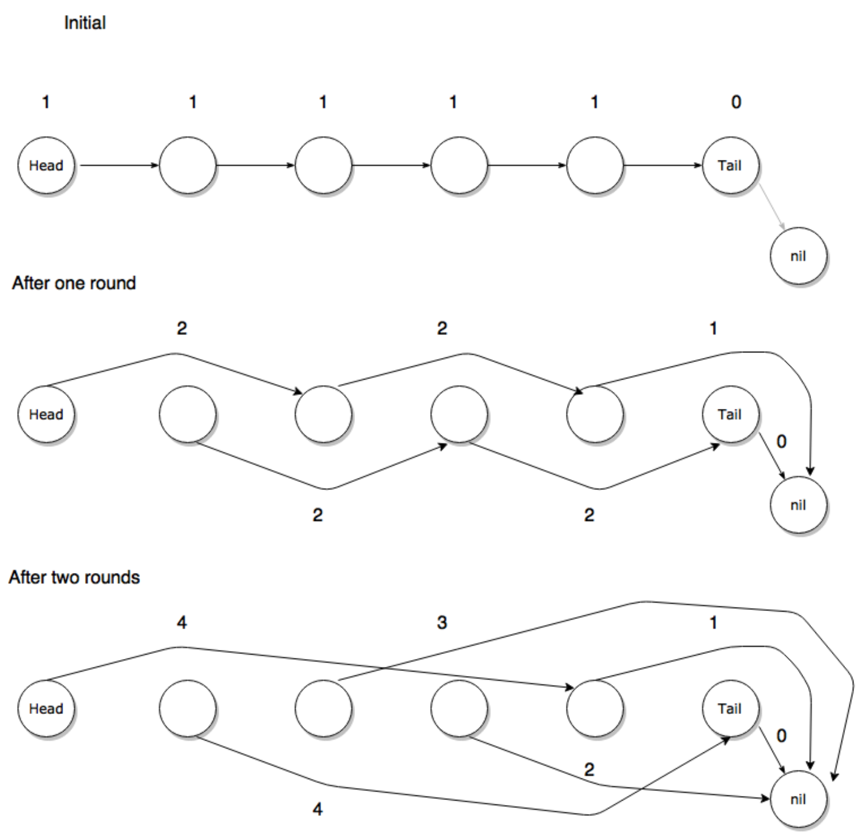

Figure 4: Wyllie's Algorithm

5 Random-Mate

Contraction Phase

1. Each live node randomly picks a sex
2. If $F \to M \to X$ then $F \to X$, M dies

3. Stop when head points to NILL (Only head is alive)

5.0.1 How many rounds needed?

Theorem 5.1. *The contraction phase stops in $c \log n$ rounds with high probability.*

Proof. Let P_i = Event that node i is still alive after one round
Note: If node i is some other node besides head, then $Prob(P_i) = \frac{3}{4}$
Let P_i^k = Event that node i is still alive after k rounds.
Note: $Prob(P_i^k) = (\frac{3}{4})^k i$ not head
Set $k = c \log_(\frac{4}{3})n$
$Prob(P_i^k) = \frac{1}{(\frac{4}{3})^k} \leq \frac{1}{(\frac{4}{3})^{(c \log_{\frac{4}{3}} n)}} = \frac{1}{n^c}$
Let P^k = Event that some non-head node is still alive. Assume that $node_0$ is the head.
$P^k = P_1^k \cup P_2^k \cup ... \cup P_n^k$
$Prob[P^k] = Prob[P_1^k \cup ... \cup P_n^k]$
$\leq Prob[P_1^k] + ... + Prob[P_n^k]$
$\leq n.\frac{1}{n^c} = \frac{1}{n^{c-1}}$
If we set $c = 2$ then the contraction phase stops with probability
$\leq \frac{1}{n}$ □

In the expansion phase we run contraction phase "backwards".

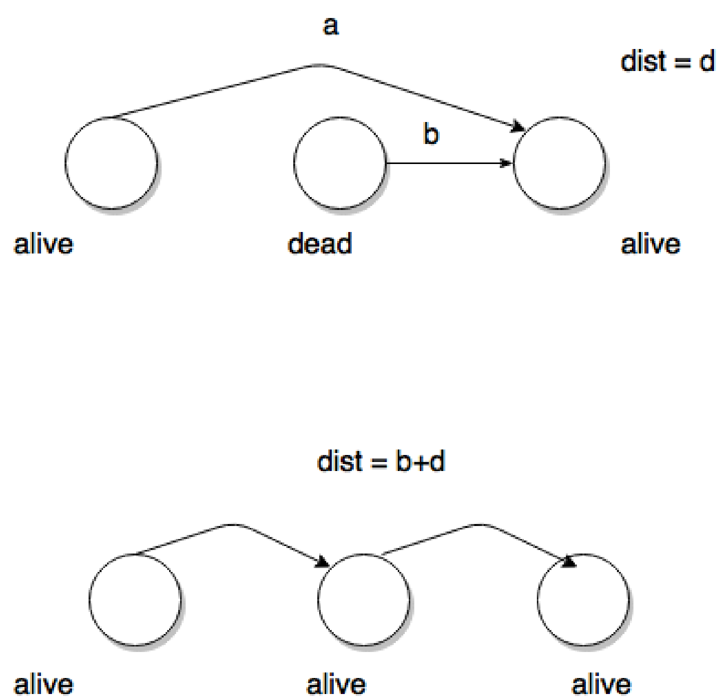

Figure 5: