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Exploring a Graph using BFS

Methods

1) DFS
2) BFS
3) Random walks

Today

Consider BFS

Low Diameter Decomp

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$G=(V,E)$ $d(v) \equiv \text{degree}(v)$

$W \subseteq V$ $\quad Vol(W) = \sum_{v \in W} d(v)$

note $Vol(V) = 2m$

Def $\partial W = \{(x,y) \mid x \in W \ \& \ y \notin W\}$

Isoperimetric number $(W) = \dfrac{|\partial W|}{Vol(W)}$

Prob Input: $G=(V,E)$, $x \in V$, $\beta < 1$

Find small $x \in W \subseteq V$ s.t. $\dfrac{|\partial W|}{Vol(W)} \leq \beta$

We will consider several algorithms

Simplest Idea BFS

Grow a ball centered at x until IsoNum $\leq \beta$.

Def $B(x,r) = \{y \in V \mid d(x,y) \le r\}$

GrowBall(G, β, x)

Set $r=0$

While $\frac{|\partial B_r|}{Vol(B_r)} > \beta$ set $r = r+1$

Return B_r, $R = r$

Claim $R \le O\left(\frac{\log n}{\beta}\right)$

If $r < R$ then $|\partial B_r| > \beta Vol(B_r)$

Note $\overline{\partial B_r} = \{y \mid (x,y) \in E, x \in B_r \ \& \ y \notin B_r\}$

$Vol(\overline{\partial B_r}) \ge |\partial B_r|$

thus $Vol(B_{r+1}) \ge (1+\beta) Vol(B_r)$

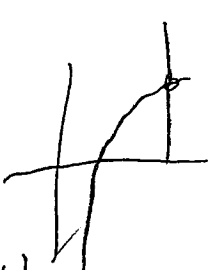
$2m \ge Vol(B_r) \ge (1+\beta)^r$

$r \log(1+\beta) \le \log 2m$

$r \le \frac{\log m + 1}{\beta}$

$\log 2 = 1$

$\log(1+\beta) \ge 1$ if $0 < \beta < 1$



Ball Decomp(G, β)

While $V \neq \emptyset$ do

1) Pick $x \in V$

2) Remove B_r = Grow Ball(G, β, x) & ∂B_r

Return Balls.

Thm Ball Decomp returns a partition of V

1) Dia of each cluster $O(\frac{\log n}{\beta})$

2) # of inter cluster edges $\leq \beta m$

Application of Low Dia Decompo

1) Low Stretch Spanning Trees, LSST.

2) Spanners

3) Hop. sets

4) LSST used in fast linear system solvers

5) LSST $\Rightarrow$

1) Maximum Flow

2) Image Processing

3) Graph Cuts