2/1/16
15-750

Coloring a Fence

Dynamic Programming

Picket Fence

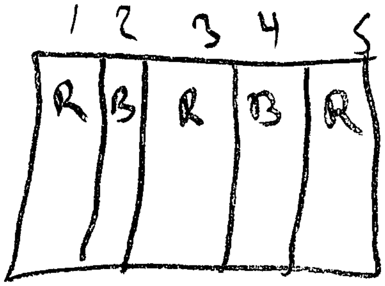
<u>Input:</u> Fence ≡ Pickets $P_1 \cdots P_n$

Requested colors $C_1 \cdots C_n$

<u>Operations:</u> Stroke(i, j, c) ≡ colors pickets $P_i \cdots P_j$ color c

<u>Output:</u> min stroke seq coloring the fence.

<u>eg</u>



$S(1,5,R), S(2,2,B), S(4,4,B)$

Subproblems

Naive: $\#(i,j) = \overset{\text{min}}{\#}$ strokes i to j

$\#(i,j,c)$ = min # strokes given free background color c.

Recurrence:

$$\#(i,j,c) = \begin{cases} 0 \text{ if } j<i \\ 0 \text{ if } i=j \ \& \ c_i=c \\ \min \begin{cases} \text{a) } \min\limits_{\substack{i \le k \le j \\ c_k = c}} C(i,k-1,c) + C(k+1,j,c) \\ \text{b) } C(i,j-1,c_j)+1 \quad c_j \neq c \end{cases} \end{cases}$$

Claim $\#(i,j,c) \geq \text{Opt}$

Recurrence generates a coloring of $\#(i,j,c)$ strokes

Claim $\#(i,j,c) \leq \text{Opt}$

pf induction on $t = j-i$

$t \leq 1$ done

assume true $\leq t$ suppose $j-i = t+1$

Let $S = \{S_0, \ldots, S_k\}$ be opt seq of strokes.

Case 1 S uses background color c. Say P_k

$\exists$ partition of S into S' & S''

st S' coloring of $P_i \cdots P_{k-1}$ using bg c.

S'' " $P_{k+1} \cdots P_j$ " "

by induct $\#(i, k-1, c) \leq |S'|$

$\#(k+1, j, c) \leq |S''|$

thus $\#(i,j,c) \leq |S|$

Case 2   BG color not used

WLOG 1) P_j only painted once
2) first stroke paints P_j

$S_1 = \text{stroke}(i, j, C_j)$

$S_2 \cdots S_k$ paints $P_1 \cdots P_{k-1}$ using bg color C_j

$\#(i,j,C) \le \#(i, j-1, C_j) + 1 \le (k-1)+1 \le |S|$

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timing   subprobs
(i,j,c) $O(n^2 \cdot c)$ $c = \#$ of colors.

cost per problem $O(n)$

$O(n^3 \cdot c)$