

# Ball Growing Using Exponential Delay Alg

15-750  
2/10/16

ExpDelay( $G, \beta$ )

- 1) Each  $v \in V$  draws  $X_v \sim \text{Exp}(\beta)$
  - 2) Each  $v \in V$  computes  $S_v = X_{\max} - X_v$
  - 3) Each  $v \in V$  starts a BFS at time  $S_v$ 
    - a) If at time  $S_v$   $v$  is not owned then  
"v owns v."
    - b) Each vertex owned by first arrival
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$u \in \text{Cluster}(v)$  if  $v = \text{argmin}_y (\text{dist}(u, y) + X_y)$

Note: At time  $X_{\max}$  all vertices owned.

Thus radius all clusters  $\leq X_{\max} \leq \frac{c \log n}{\beta}$  hp

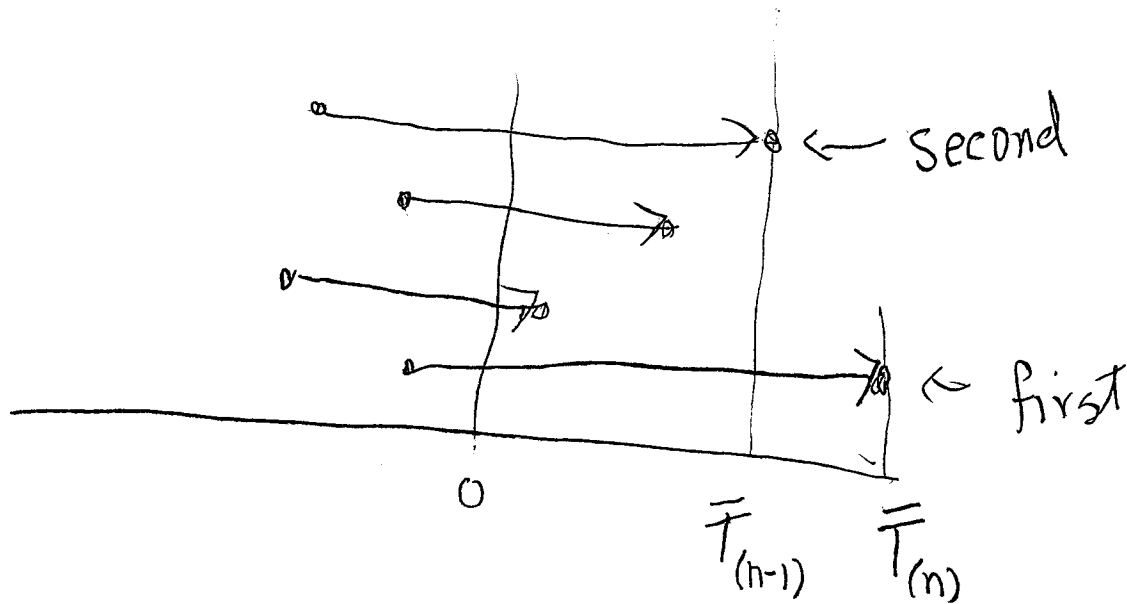
Question what is prob an edge is intercluster?  
ie edge is cut.

Let  $e \in E$  &  $c \in e$  its center

Let  $T_1, \dots, T_n$  be random variable of time  $V_i$   
would arrive at  $c$ .

$$T_i = X_{\max} - X_i + d(V_i, c)$$

Consider  $\bar{T}_i = X_{\max} - T_i = -d(V_i, c) + X_i$   
(early arrivals)



$$S_{n-1} = T_{(n)} - \bar{T}_{(n-1)}$$

by memoryless prop  $S_{n-1} \approx \text{Exp}(\beta)$

$$\text{Prob}[S_{n-1} < 1] = 1 - e^{-\beta} < \beta$$

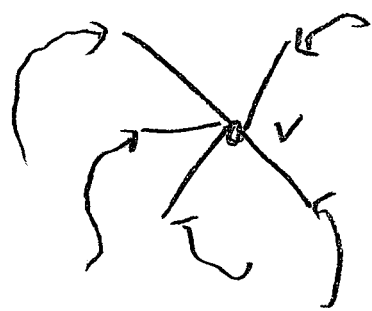
Claim  $e^{-\beta} > \beta$

$$e^{-\beta} = 1 - \beta + \frac{\beta^2}{2} - \frac{\beta^3}{3!} + \dots > 1 - \beta$$

$$\cancel{\text{iff}} 1 - e^{-\beta} < \beta$$

Question How many clusters will a vertex see?

- 1) It will belong to one.
- 2) How many edges will it have with distinct clusters?



Consider early arrivals to  $v$ !

