## 9.1 Median and selection

Median is a special case of the selection algorithm. The sequential algorithm $select(S, k)$ to find the $k$-th smallest element from the set $S$ of $n$ elements is as follows:

1. Group elements of $S$ in to groups of $5$ elements.
2. Find the median of each of the $n/5$ groups with direct comparisons; call this the set $M$.
3. Recursively find the median of the set $M$. Call this element $c$.
4. Determine the number of elements ($r$) that $c$ is greater than in $S$.
5. If $r > k$, then from each of the 5-groups whose median is greater than $c$, we can throw away the larger 3 elements of the group.
6. If $r < k$, then from each of the 5-groups whose median is lesser than $c$, we can throw away the smaller 3 elements of the group.
7. If $r = k$, return $c$, if not use a recurisve call on the reduced set of elements.

Note that if $r \neq k$ in the above function, $3/10$-th of the elements are thrown away and a recursive call is made on $7/10$-th of the elements. Work: $W(n) = cn + W(n/5) + W(7n/10) \implies W(n) = O(n)$. However, the depth $D(n) = D(n/5) + D(7n/10) + O(\log n) = O(n^\alpha)$, $\alpha \approxeq 0.82$. We want to bring the depth down to a polylogarithmic function.

### 9.1.1 A variant of the select algorithm

Instead of dividing the elements in to groups of $5$, divide them in to groups of $\log n$ size each. The median of these $\log n$ elements can be computed using the earlier version of selection/median algorithm in depth less than $c_0 \log n$ using $c_1 \log n$ work. While $3/10$ fraction of elements were thrown away in each round in the earlier algorithm, now about a quarter of the elements are thrown away in each round.

Work: $W(n) = W(n/\log n) + W(3n/4) + c_1 n = O(n)$.
Depth: $D(n) = D(n/\log n) + D((3n/4) + c_0 n$.

The depth is still not a polylogarithmic function. To make it so, we can take advantage of the fact the number of groups is only $n/\log n$ and use a sorting algorithm to sort up the medians of these groups to find the median of medians. This would involve only $O((n/\log n)\log(n/\log n)) = O(n)$ work and $O(\log^2 n)$ depth.

Work: $W(n) = O(n) + W(3n/4) + c_1 n = O(n)$.
Depth: $D(n) = O(\log^2 n) + D((3n/4) + c_0 n = O(\log^3 n)$.