Kinematics

15-494 Cognitive Robotics
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Outline

Kinematics is the study of how things move.

- Homogeneous coordinates
- Kinematic chains
  - Description of the AIBO as a collection of chains
- Reference frames
- Kinematics and PostureEngine classes
- Forward kinematics: calculating limb positions from joint angles. (Straightforward math.)
- Inverse kinematics: calculating joint angles to achieve desired limb positions. (Hard.)

Homogeneous Coordinates

- Represent a point in N-space by an (N+1)-dimensional vector. Extra component is an inverse scale factor.
  - In “normal” form, last component is 1.

$$\vec{v} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - Points at infinite distance: last component is 0.
- Allows us to perform a variety of transformations using matrix multiplication:

  Rotation, Translation, Scaling

- Tekkotsu uses 3D coordinates (so 4-dimensional vectors) for everything.

NEWMAT

- Tekkotsu uses the NEWMAT package for vector and matrix arithmetic.

```
NEWMAT::ColumnVector v(4), w(4);
v = Kinematics::pack(5.75, 30.0, 115);
w << 17 << -4.2f << 100 << 1;

NEWMAT::Matrix T(4,4);
T = v * w.t();
```

Transformation Matrices

- Let θ be rotation angle in the x-y plane.
  Let dx, dy, dz be translation amounts.
  Let 1/s be a scale factor.

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & dx \\ -\sin\theta & \cos\theta & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & s \end{bmatrix}$$

$$T\ \vec{\mathrm{v}} = \begin{bmatrix} x\cos\theta + y\sin\theta + dx \\ -x\sin\theta + y\cos\theta + dy \\ z + dz \\ s \end{bmatrix} = \begin{bmatrix} (x\cos\theta + y\sin\theta + dx)/s \\ (-x\sin\theta + y\cos\theta + dy)/s \\ (z + dz)/s \\ 1 \end{bmatrix}$$

Transformations Are Composable

- To rotate about point p, translate p to the origin, rotate, then translate back.

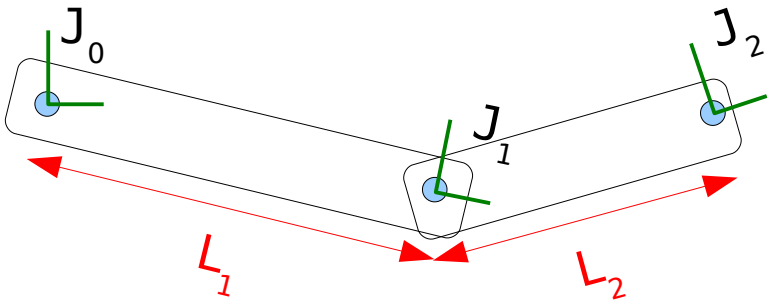
$$Translate(p) = \begin{bmatrix} 1 & 0 & 0 & p.x \\ 0 & 1 & 0 & p.y \\ 0 & 0 & 1 & p.z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rotate(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RotateAbout(p,\theta) = Translate(p) \cdot Rotate(\theta) \cdot Translate(-p)$$

Kinematic Chains

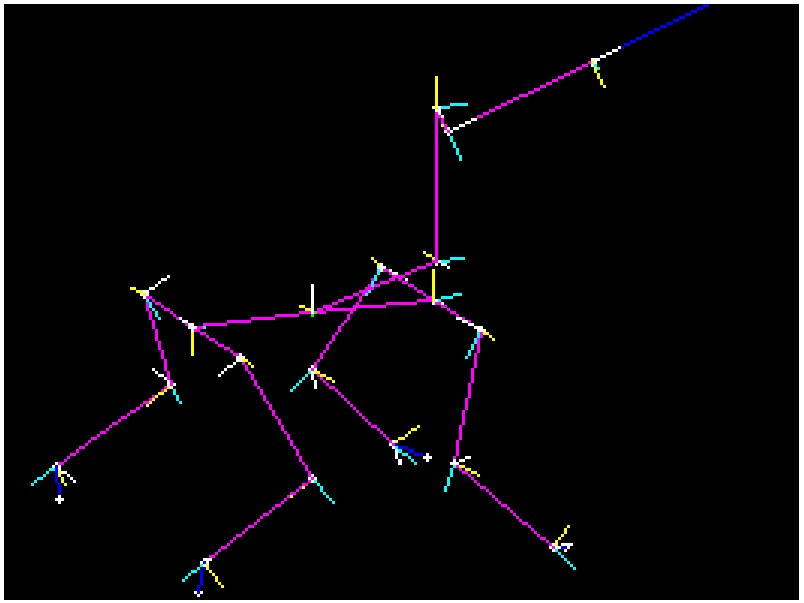
- Sequence of joints separated by links.

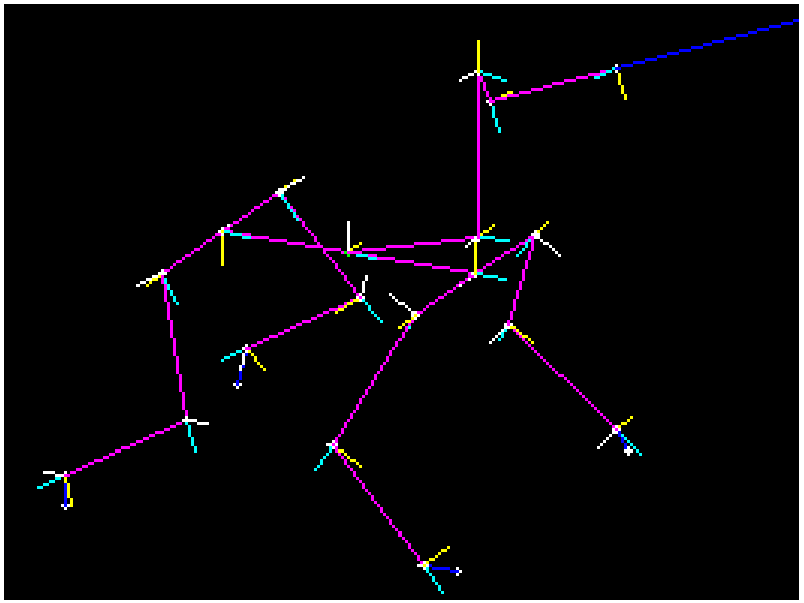


- We can use transformation matrices to calculate the position of the tip of the chain (joint J_2) from the joint angles θ_0, θ_1 and the link lengths L_1, L_2.
- Each joint has a rotation transform; each link has a translation transform.

AIBO Kinematic Chains

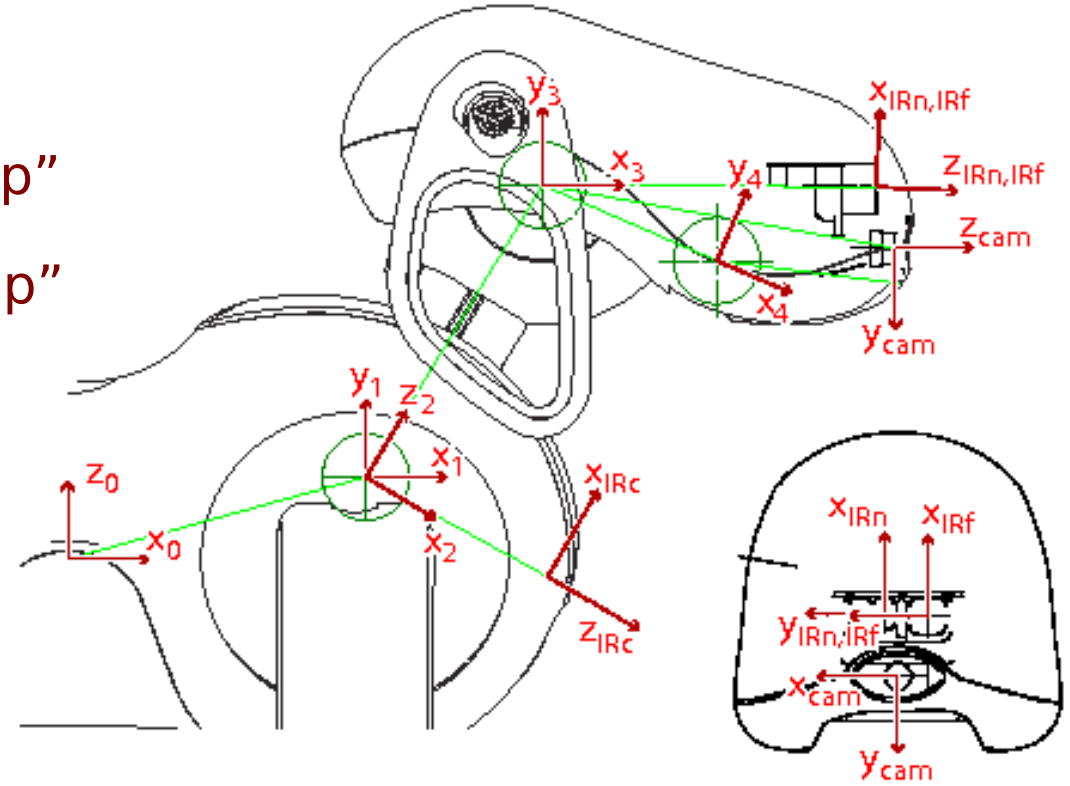
- The AIBO has 9 kinematic chains:
  - 4 for the legs
  - 1 for the head (ending in the camera), 1 for the mouth
  - 3 for the IR range sensors
- All chains begin at the center of the body (base frame).





Reference Frames

- Every joint and every link has an associated reference frame.
- Denavit-Hartenberg conventions: all joints move about their reference frame's z-axis.
- The head chain:
  - Base frame 0 z_0 = “up”
  - Tilt joint 1 y_1 = “up”
  - Pan joint 2
  - Nod joint 3
  - Camera 4

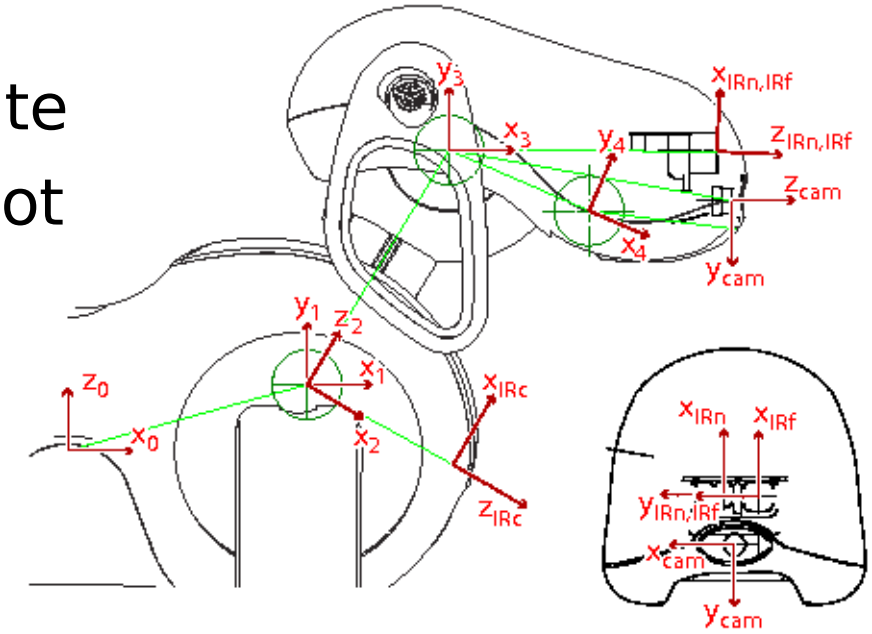


Joint vs. Link Reference Frames

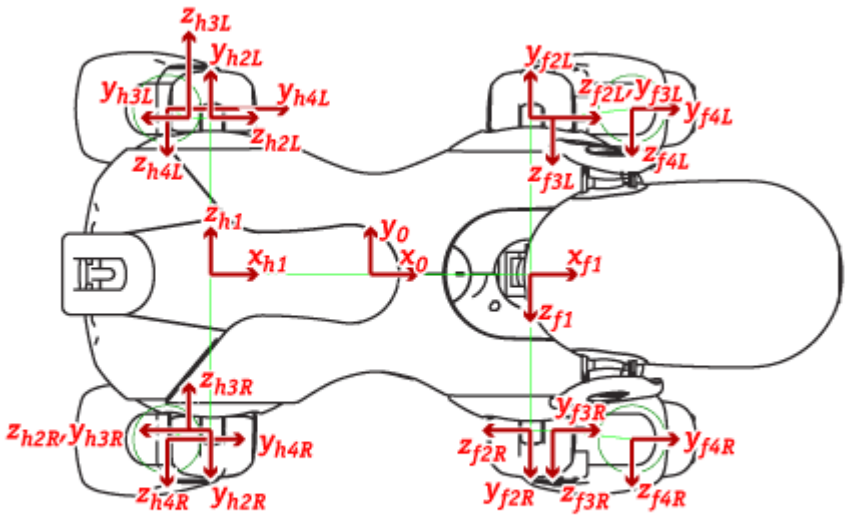
- The joint reference frame does not rotate with the joint. The link reference frame does.

- The x_1, y_1, z_1 joint axes remain fixed with respect to the base frame when the head tilts up or down.

- The x_2, z_2 joint axes rotate with the tilt angle (but not the pan angle.)



Leg Reference Frames



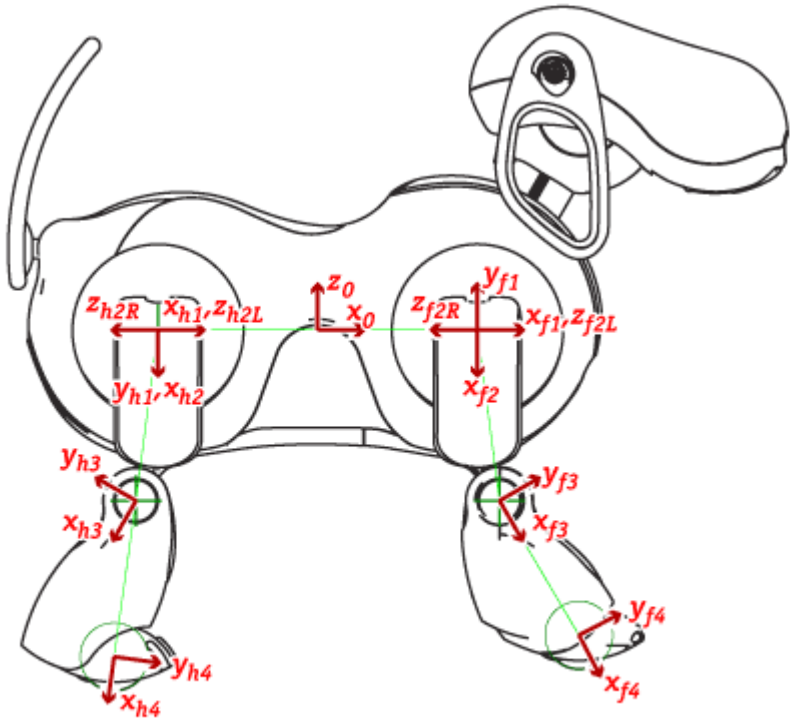
ERS-7 Legs

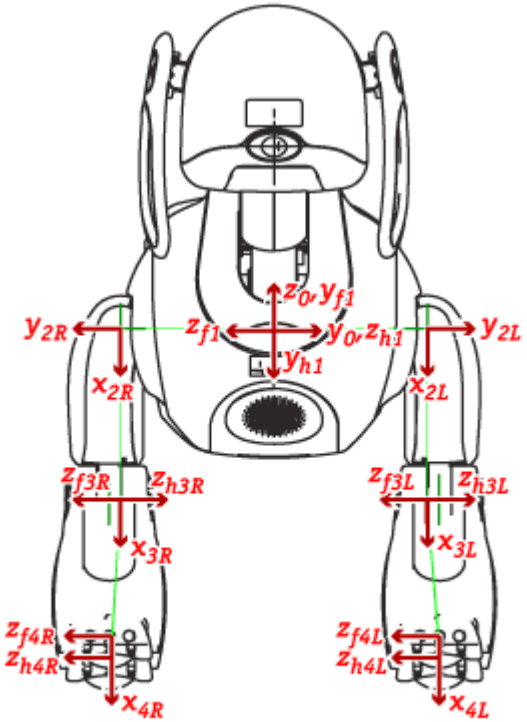
	Δx	Δy	Δz
1. - shoulder	65	0	0
2. - elevator	0	0	62.5
3. - knee	69.5	0	9
f4. - ball	*69.987*	*-4.993*	4.7
h4. - ball	*67.681*	*-18.503*	4.7

Diameter of ball of foot is *23.433*mm

Each link offset is relative to previous link

The shins shown in this diagram appear to be slightly distorted compared to a real robot. Corresponding measurements have been taken from actual models.





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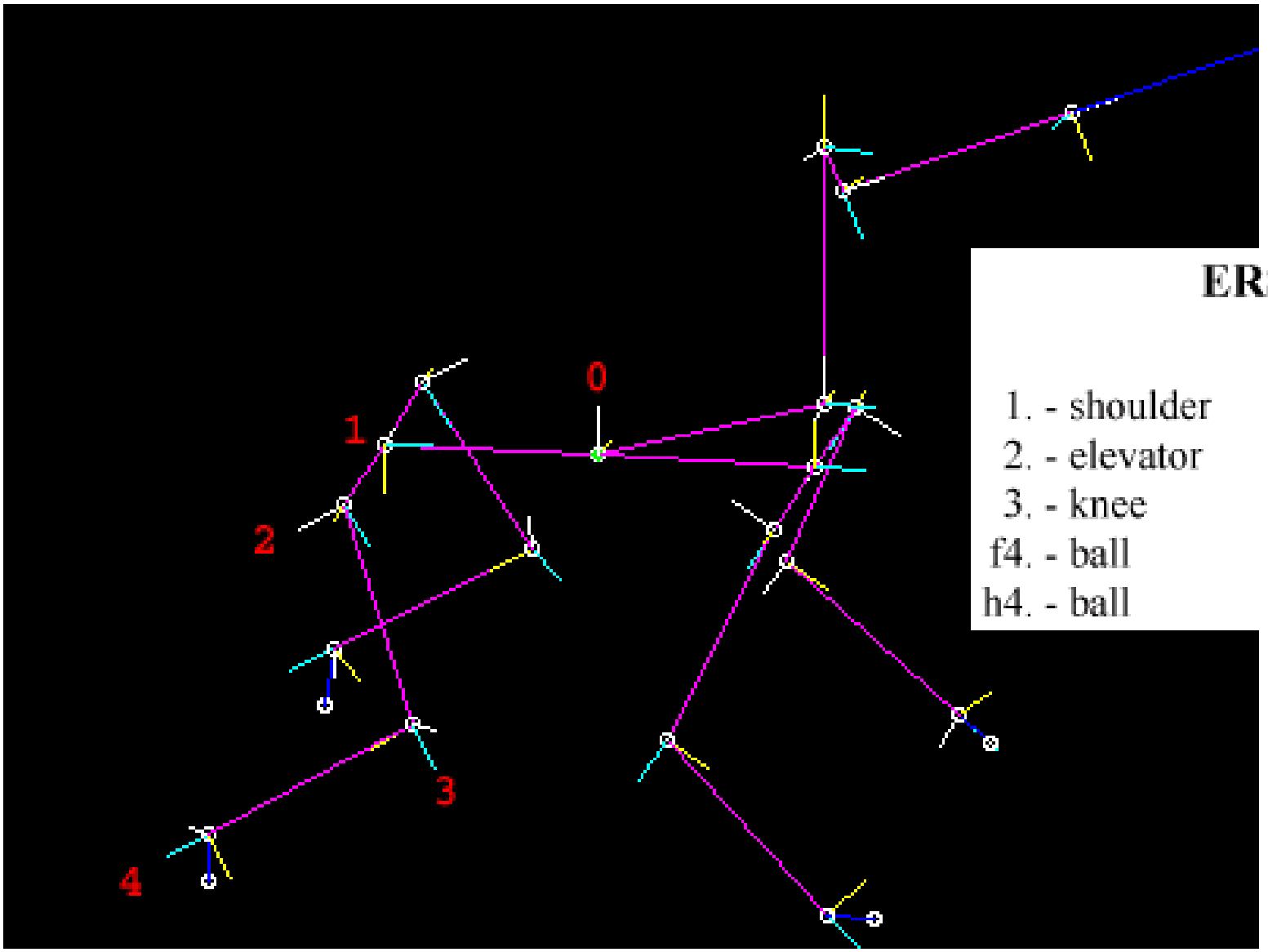

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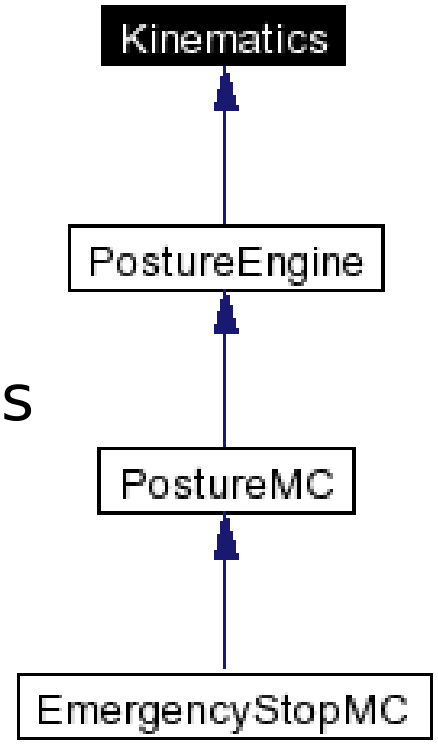
Reference Frame Naming Conventions

- Use a similar offset-based indexing scheme as for joint names in motion commands and world state vectors:
  - BaseFrameOffset
  - HeadOffset + TiltOffset
  - CameraFrameOffset
  - LFrLegOffset + ElevatorOffset
- Note: the distinction between joint and link reference frames is made in the function name, not the reference frame name:
  - jointToBase(HeadOffset+TiltOffset)
  - linkToBase(HeadOffset+TiltOffset)

Kinematics Class

- Tekkotsu uses the ROBOOP package for kinematics calculations.
- The Kinematics class provides access to ROBOOP functionality for forward kinematics.
- Global variable **kine** holds a special Kinematics instance:
  - Joint values reference WorldState
- PostureEngine is a child of Kinematics so it can do kinematics calculations too. It adds inverse kinematics.
  - Joint angle results are stored in the PostureEngine instance.



Converting Between Reference Frames

- Most common conversion is between the base frame (body coordinates) and a limb frame.

  baseToJoint(), baseToLink(), jointToBase(), linkToBase()

  NEWMAT::ReturnMatrix jointToBase(unsigned int joint)

- General conversion functions:

  jointToJoint(), jointToLink(), linkToJoint(), linkToLink()

  NEWMAT::ReturnMatrix jointToJoint(unsigned int ij,
  unsigned int oj)

Reference Frame Conversion 1

- Transform Base to Base:

```
cout.precision(3);
cout << setw(7) << kine->jointToBase(BaseFrameOffset);
cout  << endl;
```

- Result:

1.000	0.000	0.000	0.000
0.000	1.000	0.000	0.000
0.000	0.000	1.000	0.000
0.000	0.000	0.000	1.000

Reference Frame Conversion 2

- Translate base frame to head tilt frame:

```
const float headtilt = state->outputs[HeadOffset+TiltOffset];
cout << "Head tilt is " << headtilt * 360/(2*M_PI)
     << " degrees." << endl;

NEWMAT::Matrix TtiltJ(kine->jointToBase(HeadOffset+TiltOffset));
NEWMAT::Matrix TtiltL(kine->linkToBase (HeadOffset+TiltOffset));

cout << "tilt jointToBase= " << endl << setw(7) << TtiltJ << endl;
cout << "tilt linkToBase= "  << endl << setw(7) << TtiltL << endl;
```

At ~Zero Degree Tilt Angle

```
Head tilt is 1.25 degrees.

tilt jointToBase=
  1.000   0.000   0.000  67.500
  0.000   0.000  -1.000   0.000
  0.000   1.000   0.000  19.500
  0.000   0.000   0.000   1.000

tilt linkToBase=
  1.000  -0.022   0.000  67.500
  0.000   0.000  -1.000   0.000
  0.022   1.000   0.000  19.500
  0.000   0.000   0.000   1.000
```

ERS-7 Head

	Δx	Δy	Δz
1. - $tilt_0$	67.5	0	19.5
2. - pan_1	0	0	0
3. - nod_2	0	0	80
4. - jaw_3	40	-17.5	0
cam. - $camera_3$	81.06	-14.6	0
IRn. - $NearIR_3$	76.9	*1.917*	*2.795*
IRf. - $FarIR_3$	76.9	*1.052*	*-8.047*
IRc. - $ChestIR_0$	109.136	-3.384	0

$x_3 \angle x_4 = -23.6294°$

At ~ -30 Degree Tilt Angle

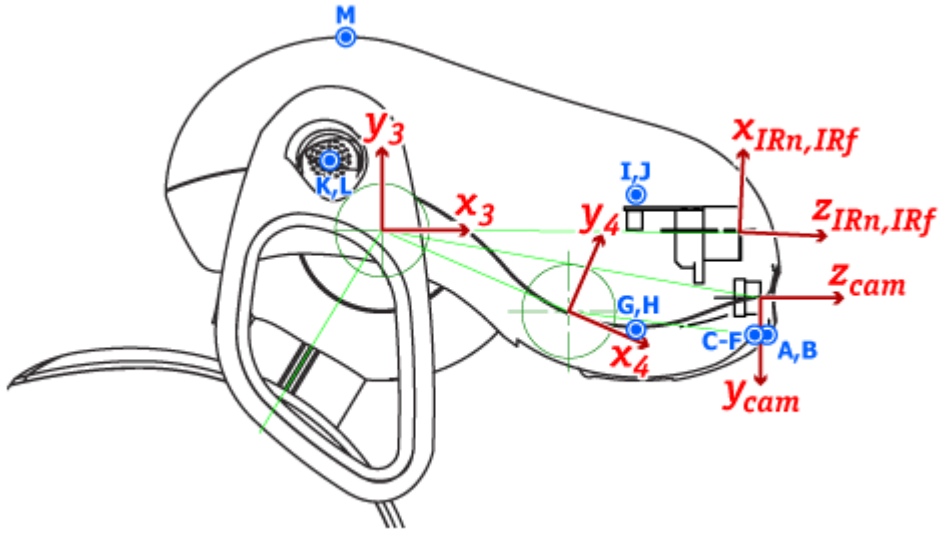
```
Head tilt is -29.5 degrees.

tilt jointToBase=
  1.000   0.000   0.000  67.500
  0.000   0.000  -1.000   0.000
  0.000   1.000   0.000  19.500
  0.000   0.000   0.000   1.000

tilt linkToBase=
  0.871   0.492   0.000  67.500
  0.000   0.000  -1.000   0.000
 -0.492   0.871   0.000  19.500
  0.000   0.000   0.000   1.000
```

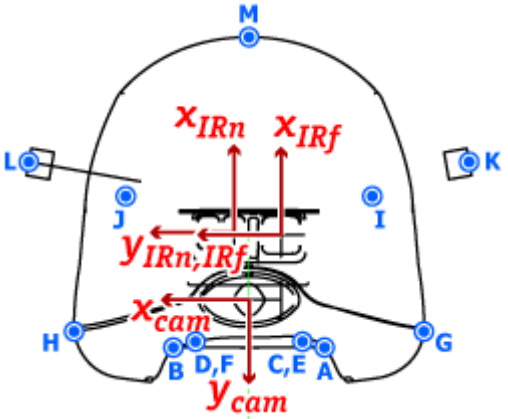
cos(-30°) = 0.866
sin(-30°) = 0.500

- The tilt joint reference frame doesn't rotate with tilt.
- The tilt link reference frame does rotate.

Interest Points

- Interest points on the head, legs, and body are predefined for use in kinematics calculations.





Interest Points:
A - LowerLeftLowerLip$_4$
B - LowerRightLowerLip$_4$
C - UpperLeftLowerLip$_4$
D - UpperRightLowerLip$_4$
E - LowerLeftUpperLip$_3$
F - LowerRightUpperLip$_3$
G - LowerLeftSnout$_3$
H - LowerRightSnout$_3$
I - UpperLeftSnout$_3$
J - UpperRightSnout$_3$
K - LeftMicrophone$_3$
L - RightMicrophone$_3$
M - HeadButton$_3$

Leg Interest Points

Interest Points:

- A - Toe{L,R}{Fr,Bk}Paw_4
- B - Lower{Inner,Outer}Front{L,R}{Fr,Bk}$Shin_3$
- C - Lower{Inner,Outer}Middle{L,R}{Fr,Bk}$Shin_3$
- D - Lower{Inner,Outer}Back{L,R}{Fr,Bk}$Shin_3$
- E - Middle{Inner,Outer}Middle{L,R}{Fr,Bk}$Shin_3$
- F - Upper{Inner,Outer}Front{L,R}{Fr,Bk}$Shin_3$
- G - Upper{Inner,Outer}Back{L,R}{Fr,Bk}$Shin_3$
- H - Lower{Inner,Outer}Front{L,R}{Fr,Bk}$Thigh_2$
- I - Lower{Inner,Outer}Back{L,R}{Fr,Bk}$Thigh_2$
- J - Upper{Inner,Outer}Front{L,R}{Fr,Bk}$Thigh_2$
- K - Upper{Inner,Outer}Back{L,R}{Fr,Bk}$Thigh_2$
- L - Upper{L,R}$Chest_0$
- M - Lower{L,R}$Chest_0$
- N - {L,R}{Fr,Bk}$Belly_0$
- O - Lower{L,R}$Rump_0$
- P - Upper{L,R}$Rump_0$

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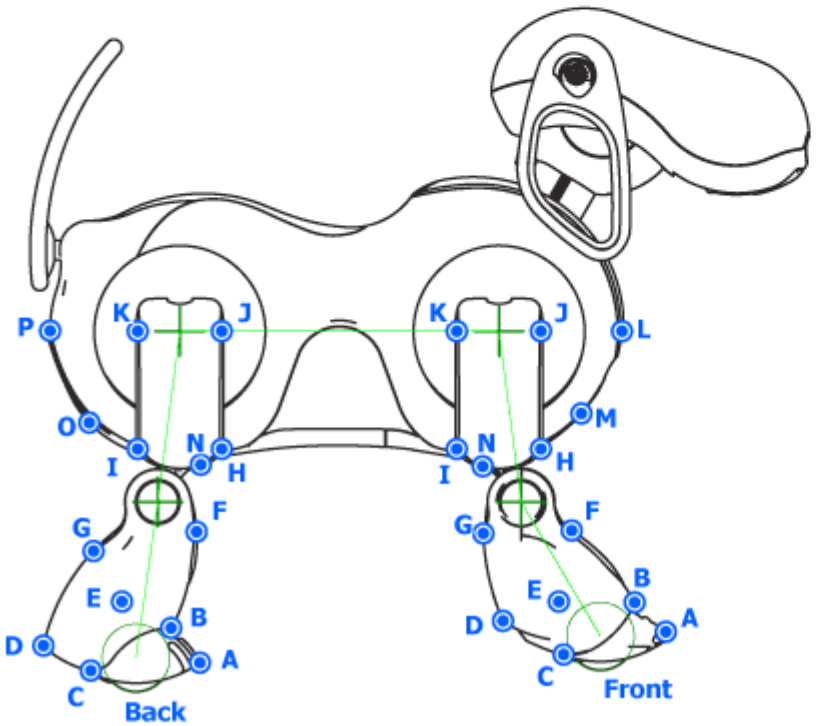
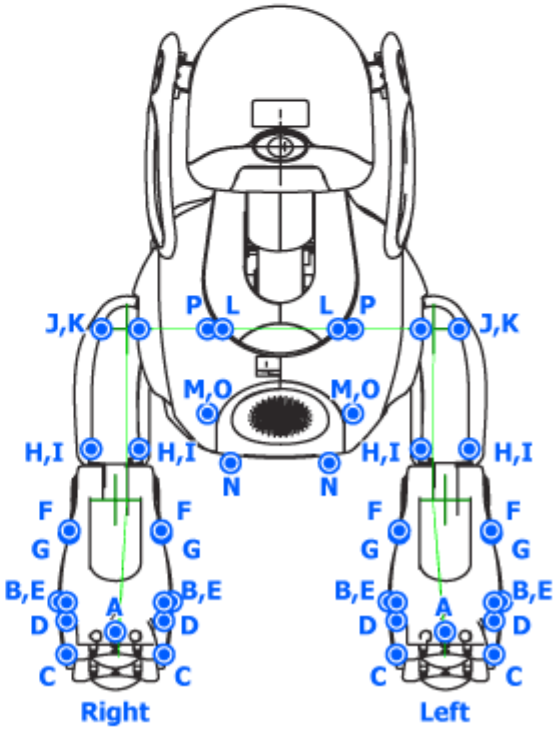




Retrieving Interest Points

- Each interest point is attached to a link:

```
void getInterestPoint(const std::string &name,
                      unsigned int &link,
                      NEWMAT::Matrix &coords)
```

  - Returns the link associated with the named interest point, and its coordinates in the link's reference frame.

- Interest points can be expressed in any reference frame:

```
NEWMAT::ReturnMatrix
    getJointInterestPoint(unsigned int joint,
                          const std::string &name)
```

Forward Kinematics: Measure Distance Between Front Paws

```
virtual void processEvent(const EventBase&) {

  NEWMAT::ColumnVector Pleftpaw =
    kine->getJointInterestPoint(BaseFrameOffset,
                                "LowerInnerMiddleLFrShin");

  NEWMAT::ColumnVector Prightpaw =
    kine->getJointInterestPoint(BaseFrameOffset,
                                "LowerInnerMiddleRFrShin");

  const float distance = (Pleftpaw-Prightpaw).NormFrobenius();

  std::cout.setf(ios::fixed);
  std::cout.precision(1);
  std::cout << "Inter-paw distance is "
            << setw(5) << distance << " mm." << std::endl;

}
```

Inverse Kinematics: lookAtPoint

- Inverse kinematics finds the joint angles to put an effector at a particular point in space.
- Hard problem:
  - solution space can be discontinuous
  - can be highly nonlinear
  - multiple solutions may be possible
  - maybe no solution (so find closest approximation)
- Example: lookAtPoint(x,y,z,distance)
  - point described in base frame coordinates
  - calculates head joint angles

Stare At Paw Behavior 1

```
class DstBehavior : public BehaviorBase {
private:
  MotionManager::MC_ID relax_id, headpointer_id;
  static const float head_to_paw_distance = 30.0;  // millimeters

public:
  DstBehavior() : BehaviorBase("DstBehavior"),
                  relax_id(MotionManager::invalid_MC_ID),
                  headpointer_id(MotionManager::invalid_MC_ID){}

  virtual void DoStart() {
    BehaviorBase::DoStart();

    relax_id =
      motman->addPersistentMotion(SharedObject<PIDMC>
                (LFrLegOffset,LFrLegOffset+JointsPerLeg,0.0));

    headpointer_id =
      motman->addPersistentMotion(SharedObject<HeadPointerMC>());

    erouter->addListener(this,EventBase::sensorEGID);
  }
```

Stare At Paw Behavior 2

```
virtual void processEvent(const EventBase&) {

  NEWMAT::ColumnVector Ptarget =
   kine->getJointInterestPoint(BaseFrameOffset,"ToeLFrPaw");

  MMAccessor<HeadPointerMC>(headpointer_id)->
   lookAtPoint(Ptarget(1),Ptarget(2),Ptarget(3),
               head_to_paw_distance);

}
```

General Inverse Kinematics

- Inverse kinematics solver included in PostureEngine:

```
solveLinkPosition(const NEWMAT::ColumnVector &Ptgt,
                  unsigned int link,
                  const NEWMAT::ColumnVector &Peff)
```

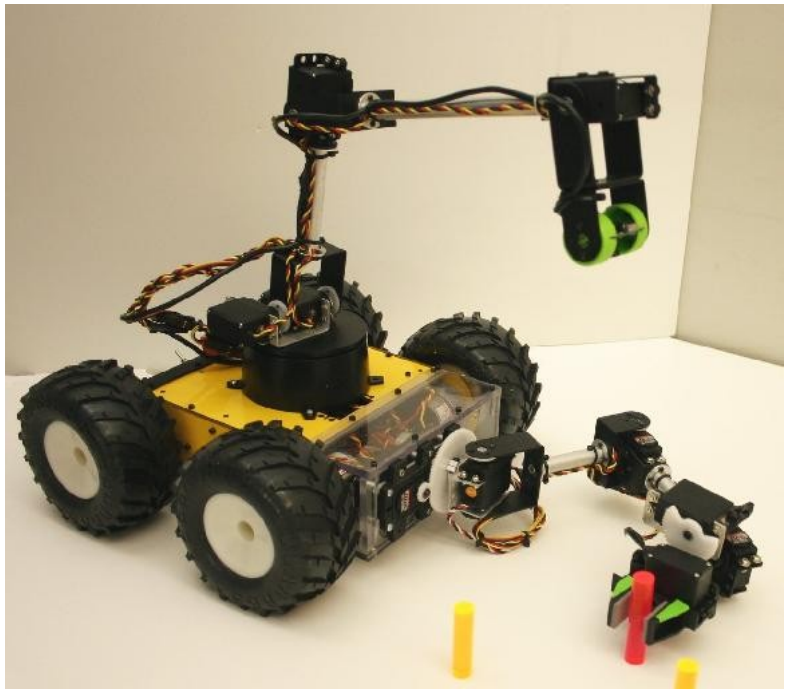
  - Ptgt is the target point to move to (in base frame coordinates)
  - link is the index of some effector on the body, e.g., LFrLegOrder+PawFrameOffset
  - Peff is a point on the effector that is to be moved to Ptgt, in the reference fame of that effector.

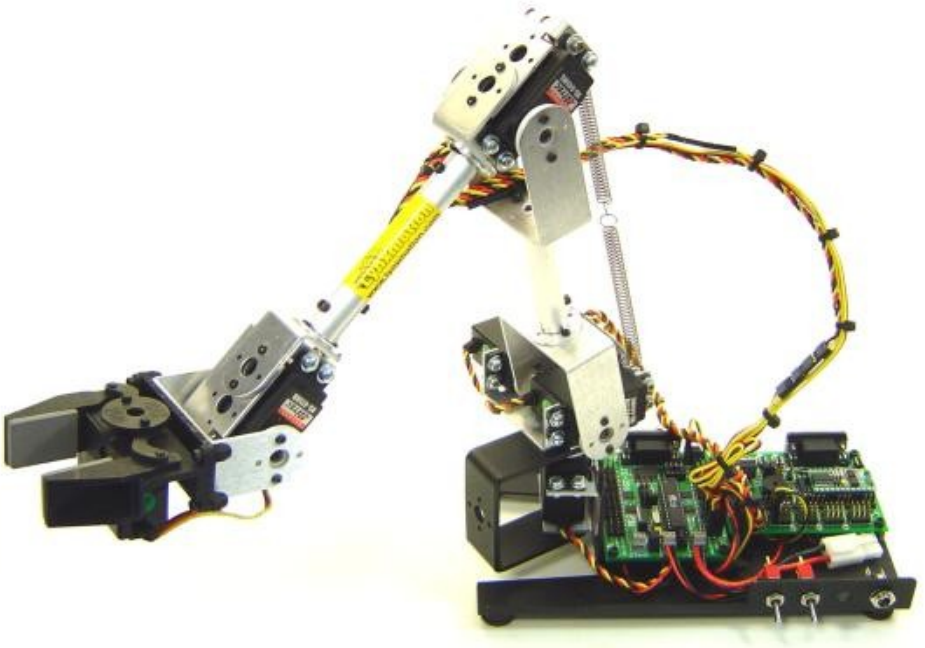
- Returns true if a solution was found. False if no solution exists (e.g., joint limits exceeded, distance too far, etc.)
- Solution is stored in the PostureEngine as joint values.

Classic Kinematics Application: Arm Control

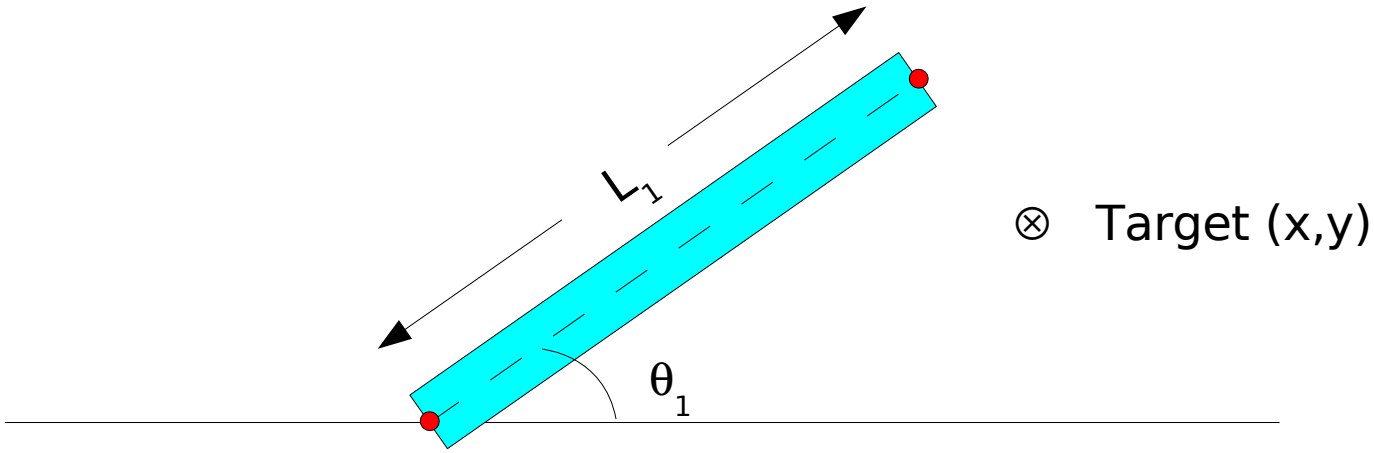
Tekkotsu includes limited support for Lynx Motion arms.

Better support coming soon.





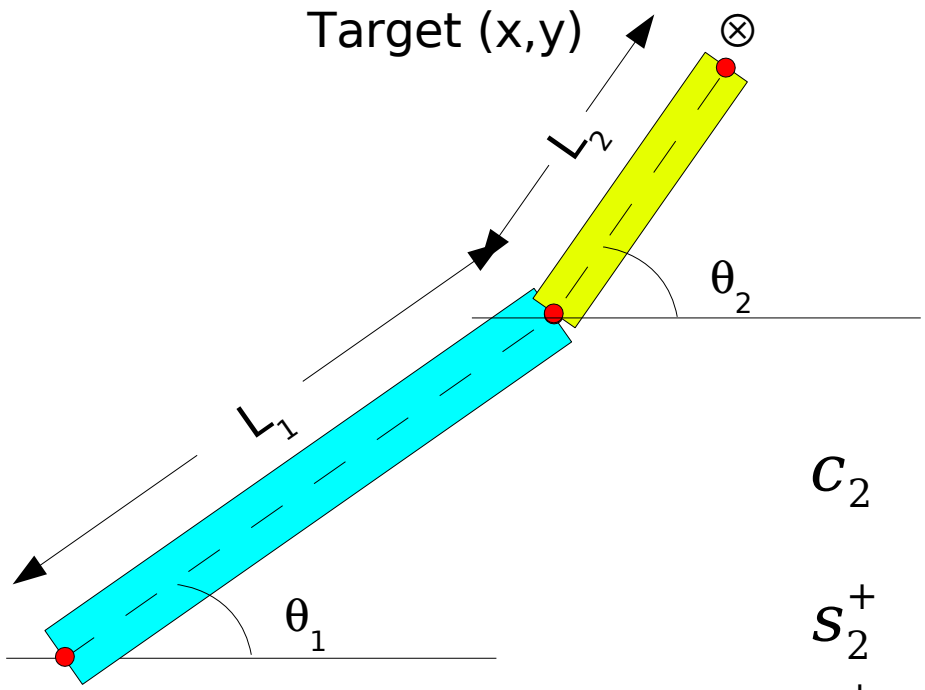
Solving the 1-Link Arm



Reachable if: $L_1 = \sqrt{x^2 + y^2}$

Solution: $\theta_1 = \text{atan2}(y, x)$

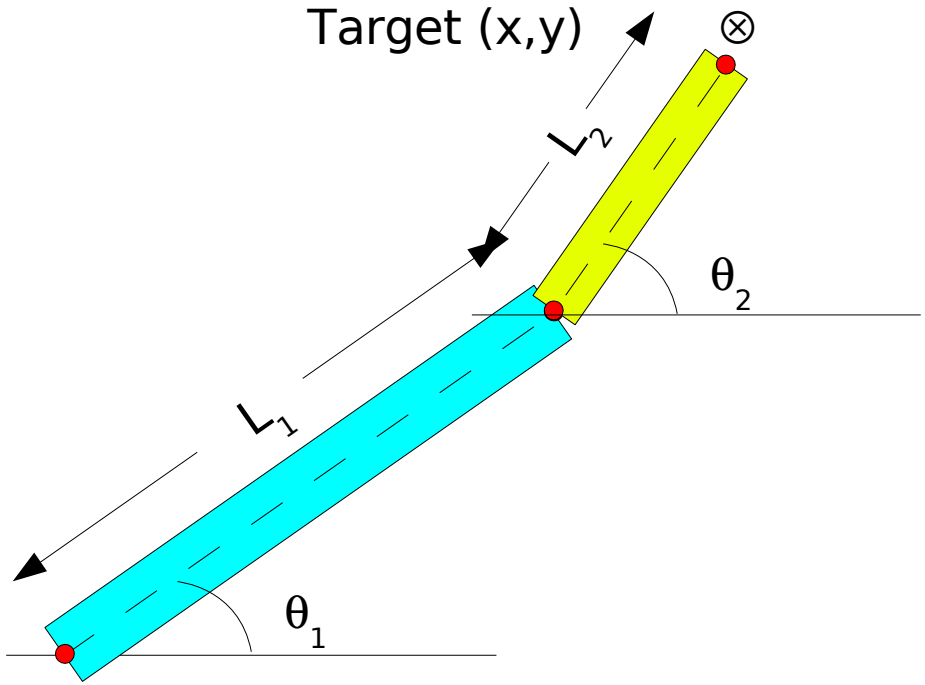
Solving the 2-Link Arm

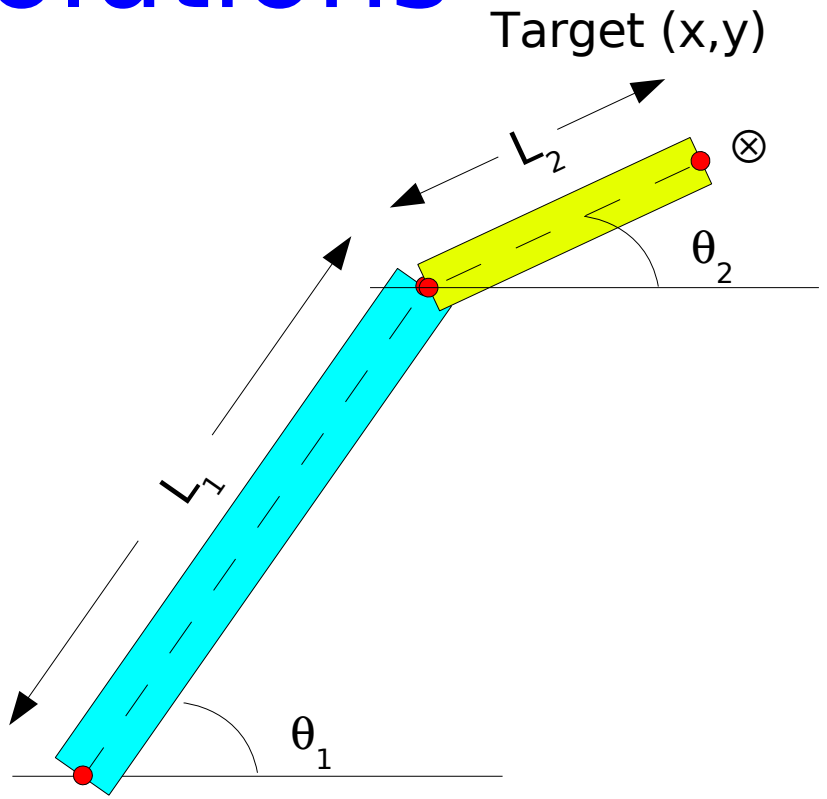


$$
\begin{aligned}
c_2 &= \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2} \\
s_2^+ &= \sqrt{1-c_2^2} \\
\theta_2^+ &= \mathrm{atan2}(s_2^+, c_2) \\
K_1 &= L_1 + c_2 L_2 \\
K_2 &= s_2^+ L_2 \\
\theta_1 &= \mathrm{atan2}(y, x) - \mathrm{atan2}(K2, K1)
\end{aligned}
$$

Reachable if: $c_2^2 \leq 1$

Two Possible Solutions





$$s_2^+ = \sqrt{1-c_2^2}$$
$$\theta_2^+ = \text{atan2}(s_2^+, c_2)$$

$$s_2^- = -\sqrt{1-c_2^2}$$
$$\theta_2^- = \text{atan2}(s_2^-, c_2)$$