Homework 1

15-462: Computer Graphics

February 3, 2009

1. For what values of α is $\mathbf{a}$ orthogonal to $\mathbf{b} - \alpha\mathbf{a}$? What about the special case where $||\mathbf{a}|| = 1$? The case where $||\mathbf{a}|| = 0$?

2. Are these a pure rotation matrices? Why or why not?

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & -\sqrt{2} & 0 \end{bmatrix} \qquad \frac{1}{3\sqrt{2}}\begin{bmatrix} 2 & 3 & -4 \\ 4 & 0 & 4 \\ 2 & -3 & -4 \end{bmatrix}$$

3. Please sketch the 2D parametric curve for the function $f : [0, 1) \to \mathbf{R}^2$ where

$$f = \begin{cases} \left[\ 0, \quad 3t\ \right]^T & \text{if } 0 \le t < \frac{1}{3} \\ \left[\ \frac{1}{3}\sin\left(3\pi(t - \frac{1}{3})\right), \quad \frac{3}{4} + \frac{1}{4}\cos\left(3\pi(t - \frac{1}{3})\right)\ \right]^T & \text{if } \frac{1}{3} \le t < \frac{2}{3} \\ \left[\ t - \frac{2}{3}, \quad \frac{1}{2} - \frac{3}{2}(t - \frac{2}{3})\ \right]^T & \text{if } \frac{2}{3} \le t < 1 \end{cases}$$

   Also please label the 2D coordinates of the endpoints. What is the normal at $\left[0, \frac{1}{3}\right]$? How about at $t = \frac{1}{2}$? (*Hint:* You will Recognize this shape.)

4. Please sketch the 2D implicit curve for the function $f : \mathbf{R}^2 \to \mathbf{R}$ where:

$$f(x, y) = |x| + |y| - 1$$

   Please label 2D coordinates as appropriate. What is the normal at $\left[\frac{1}{2}, \frac{1}{2}, \right]$? Does this function satisfy the inside/outside convention for implicit surfaces? Why?

5. Given parallel lines $\mathbf{f}_1(t) = [x_1, y_1, z_1]^T + t\,[u, v, w]^T$ and $\mathbf{f}_2(t) = [x_2, y_2, z_2]^T + t\,[u, v, w]^T$, what is their common 2D vanishing point under the perspective projection $p(x, y, z) = \left[\frac{x}{z}, \frac{y}{z}\right]^T$?

6. Given a continuous parametric function $f : \mathbf{R}^2 \to \mathbf{R}^3$ specifying a 2D surface in 3D space, define a continuous implicit function $g : \mathbf{R}^3 \to \mathbf{R}$ corresponding to the same surface. *Notes:*

   - You do not need to prove the continuity of your function. (But you can for extra credit!)
   - You'll likely want to use the *infimum* function.[1]
   - You can ignore the inside/outside convention: g can be everywhere nonnegative.

[1] http://en.wikipedia.org/wiki/Infimum