15-456 Computational Geometry, Spring 2013

Homework 3 Due: Mon 25 Feb 2013

Guidelines: Please justify all answers in a succienct (yet complete) manner. In particular, when presenting an algorithm the code if any should be presented at a high level. A full algorithm will contain the input, the output, and any loop invariants.

Question	Points	Score
1	15	
2	25	
3	15	
Total:	55	

(15) 1. **Linear Programming** ($10 = 5 + 10$)

(a) What is the worst case runtime for the 2DLP? Describe an ordering of n lines that attains that has an $\Omega(n^2)$ runtime.

(b) Is there always a permutation of n halfplane constraints (in general position) such that the algorithm requires at least $\Omega(n^2)$ time? If so, explain how to construct an ordering given any fixed set of n halfplanes. If not, give an example.

We have discussed two versions of the Seidel algorithm. For simplicity assume we are analyzing affine one where we are given a bounding box. You may assume that you, as the adversary, get to pick the bounding box.

(25) 2. **Line segment Stabbing** ($20 = 5+10+10$ points) In this problem the goal is to quickly find a line that intersects(stabs) a collection of n line segments or report that it is not possible,

(a) Start by showing how to solve the problem in linear expected time when the segments are a collection of vertical rays, i.e. $\{(a_i, y) \mid y \geq b_i\}$ or $\{(a_i, y) \mid y \leq b_i\}$.

(b) Extend your algorithm to handle the case when the segments are not necessarily vertical. Does to matter if they intersect?

(c) Modify your your algorithm so that when the algorithm reports back that no such line exists, it gives a constant size/time proof that it is not possible to find such a line.

(15) 3. **Line segment Stabbing: the Dual Problem** ($15 = 5 + 10$ points) [Note: This problem is partially repeated from Assignment 1.] Recall the duality of Oriented Projective Geometry $\langle w, x, y\rangle^* = [w, x, y]$.

(a) Find the plane P in $\mathbb{R}^3$ in which the points $\langle 1, x, y_1\rangle$ and $\langle 1, x, y_2\rangle$ of the oriented projective plane dualize to parallel lines in the restriction of $\langle 1, x, y_1\rangle^*$ and $\langle 1, x, y_2\rangle^*$ to the plane P. [Hint: Try to look at the plane W=1 as an example.].

(b) Use the observation from (a) to formulate a dual to the following problem: Given n vertical line segments in the plane, we wish to find a line that stabs, i.e., intersects, all segments. For example, in Figure 1, the red line is a line that stabs all of the green vertical line segments.

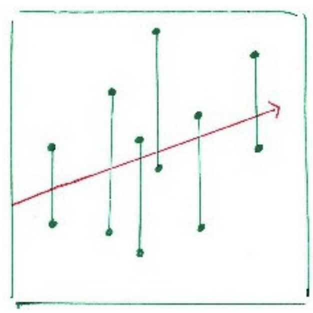

Figure 1: A stabbing line. See qustion 5(b).

(c) How does your answer to question (b) change if the line segments were not all vertical lines?