Computational Geometry: Homework 3
Due: Friday March 5, by 3:30pm

1. Edge-maximal planar graphs.
   - Let e be an edge in an edge-maximal graph embedded in the plane such that every triangle containing e is a face of the embedding. Prove that an edge-maximal graph is still edge maximal after contracting an edge.
   - Prove that plane triangulations with at least 4 vertices are 3 connected.

2. Here is a slightly simplified version of a real problem that arises in wireless communication between vehicles.

   Suppose we have n cars in the plane. Each car will be represented by a unit disk. The antenna is located in the center of the disk. The disks are all disjoint. Two cars have an unobstructed line of sight if the line between their antennae does not intersect any other cars. We want to know which are visible from which. Give an $O(n \log n)$ algorithm for finding all lines of sight from a given car. This will yield an $O(n^2 \log n)$ time algorithm for all of the cars.

   Present your algorithm in pseudocode and prove that it is correct. Also give a proper runtime analysis.

3. **The non-planarity of K_5 and $K_{3,3}$.** In class, we showed that if a planar graph is 3-connected, then the faces are just the non-separating cycles. Use this Theorem and Euler's formula to show that K_5 and $K_{3,3}$ are not planar.

4. Recall that the history DAG data structure for the incremental Delaunay triangulation algorithm works by keeping around the different triangulations created throughout the process. There is an edge between triangles that overlap and are contained in triangulations that differ by only one point. If a triangle has only one outgoing edge, we contract it.

   We gave an analysis of this structure in the case where the input order is random. In this problem, you'll give another analysis under an assumption that the Delaunay triangulation the divides the points nicely. This assumption is not exactly true, but it's close.

Imagine it were the case that the expected number of points in a Delaunay ball (i.e. the circumball of a Delaunay triangle) of a subset of r points is $O(n/r)$. Prove that the history DAG can do point location in expected $O(n \log n)$ time under this assumption.