Computational Geometry: Homework 2
Due: Friday Feb 5.

1. Prove that the LineSweep algorithm presented in class for computing the intersections of n line segments requires $O((n + |I|) \log n)$ time where I is the set of intersections.

2. Recall that Euler's formula tells us that for embedded planar graphs, $V - E + F = 2$, where V, E, and F are the number of vertices, edges, and faces (including the infinite face) respectively. Use this formula to compute the maximum number of triangles that may appear in any planar triangulation.

3. Suppose we have a set of n points P in the plane such that exactly h of them lie on the convex hull. How many triangles are in any triangulation of P? Give your answer in terms of n and h.

4. Let P be a set of points in the plane. Let x be a point in $CC(P)$, and let $(p_0, \ldots, p_h)$ be a counterclockwise ordering of the vertices of $CH(P)$. If we define p_{h+1} to be p_0, then the following formula gives the area of $CC(P)$.

$$Area(CC(P)) = \frac{1}{2} \sum_{i=0}^{h} \det \begin{bmatrix} p_i - x \\ p_{i+1} - x \end{bmatrix}$$

   - Why is this formula correct?
   - What happens if the p_i's are given in clockwise order?
   - Is it still correct if $x \notin CC(P)$?
   - Why or why not?

5. In class we claimed that the intersection of a plane and a paraboloid in $\mathbb{R}^3$ projects to a circle in the plane. Prove it. If you can come up with a nice proof in $\mathbb{R}^3$ go for it. If you are having trouble, prove it for the general case of a hyperplane and a hyperparaboloid using the following steps. Let $p_1, \ldots, p_{d+1}$ be $d+1$ affinely independent points in R^d and let $p_{i,j}$ denote the jth coordinate of the ith point. Let p_i^+ denote the lifting of point p_i onto the paraboloid, i.e. $p_i^+ = (p_{i,1}, \ldots, p_{i,d}, \sum_{j=1}^{d} p_{i,j}^2)$.

   - Write a formula to describe the points of $\text{aff}(p_1^+, \ldots, p_{d+1}^+)$ in terms of a set of affine coefficients $\alpha_1, \ldots, \alpha_{d+1}$.

- Prove that the intersection of $\text{aff}(p_1^+, \ldots, p_{d+1}^+)$ projects to the circumsphere of $p_1, \ldots, p_{d+1}$ for the special case when the center of this sphere is the origin.
- Prove that the location of the origin does not matter by showing that moving all the points by the same amount along one coordinate axis, does not affect the projected shape.