The Line Segment Intersection Prob

CG
1/22/10

Input: a set of n line segments
Output: Report all intersections I intersection

Naive: $O(n^2)$
Goal: $O(n \log n + |I|)$ Today $O((n+|I|) \log n)$
Worst case: $|I| = \Omega(n^2)$

Motivation: Map Overlay

Algorithm: sweep line
Optional: Incremental

Map overlay prob

<u>Segments:</u> $S = \{s_1, \dots, s_n\}$ (no vertical seg)

<u>Output:</u> Break all segments into subseg s.t.

Two subseg intersect only at endpoints

Sweep Line Alg

Let P = endpoints of seg.

ℓ = horizontal line disjoint from $P \cup I$.

ℓ linearly orders S.

The order only changes at $P \cup I$

Store order in Balanced BST.

Events $\equiv P \cup I$

Idea: sweep ℓ top to bottom

stoping at events.

But we do not know I!

Compute events in I just-in-time.

<u>Claim</u> If next event is the intersection of s & s' then s & s' are neighbors.

Priority Queue Q_ℓ of events

Inductively: Q_ℓ contains

1) P

2) Neighboring inter below ℓ.

Handling Events

$\mathcal{U}(p)$ = subseg with upper end point p
$L(p)$ = " " lower " "
$C(p)$ = " " intersection p.

Procedure HandleEvent(p point; T tree; Q queue)

0) $\forall s \in C(p)$ form new sub seg add to $\mathcal{U}(p)$ & $L(p)$.
1) $\forall s \in L(p)$ delete(s, T)
2) $\forall s \in \mathcal{U}(p)$ insert(s, T)
3) $\forall$ new neighs add intersection to Q

Let m = # sub segs

Alg runs in $O(m \log n)$ Time

there are at most m delete/inserts into T & Q

How many Subseg?

To show: $\#\text{subseg} = O(n + |I|)$

Embedded Planar Graph $(G = (V, E), \varphi : G \to R^2)$

$\varphi(e) = \text{path}$

$\varphi(e) \cap \varphi(e') = \text{only endpoints}$

Eulers Formula (G, φ)

$n_v = \#\text{ vertices}$

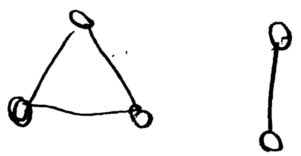
$n_e = \#\text{ edges}$

$n_f = \#\text{ connected boundaries (faces)}$

$c = \#\text{ connected components}$

then $n_f - n_e + n_v = 2c$

eg



$n_v = 5$

$n_e = 4$

$n_f = 3$

$c = 2$

$5 - 4 + 3 = 2 \cdot 2 = 4$

Claim $3n_v \geq n_e$

pf 1) add edge until G is connected

2) " " each face size is 3.

3) no parallel edges.

$3n_f \leq 2n_e$ n

$n_f \leq 2/3 n_e$

$2/3 n_e - n_e + n_v \geq 2$

$-1/3 n_e + n_v \geq 2$

$n_v \geq 2 + 1/3 n_e$

$3n_v > n_e$

Sweep Line is $O((n+I) \log n)$ time

Wrong! We sorted the intersection!