15-451 Algorithms, Spring 2019
Practice Problems

Using Flows: Squares and Stars You are given an $n \times n$ chessboard where some of the squares are colored red on them. You can place stars on the board, so that each star is placed on a red square, each row has at most one star, and each column has at most one star. Find the maximum number of stars you can place.

Using Flows: BaseBall Elimination. We are trying to figure out if the Pittsburgh Pirates can make it to the playoffs. There are n teams. We know, for each team i, the number of games W_i they have already won. And for each pair of teams (i, j), we know how many games G_{ij} they still have to play. We want to find out if there are some outcomes possible for all the remaining games so that the Pirates (which we shall say is team #1) has the maximum number of final wins (possibly tied with someone else). Show how to solve this using max-flows. (Go Bucs!)

Rock-Paper-Scissors with a Twist Suppose we have a non-standard game of rock-paper-scissors, which is still zero-sum, but with the following payoffs for the row player (Alice):

Bob plays

Alice plays		r	p	s
	r	0	-1	2
	p	1	0.5	-1
	s	-1	2	-1

If Alice decides to play $\mathbf{p} = (p_1, p_2, 1 - p_1 - p_2)$ as her strategy, what should Bob play to minimize the payoff to Alice? What is Alice's payoff if he does this.

Practice with Zero-Sum Games. Consider a zero-sum game with payoffs:

```
(-1/2, 1/2) (3/4, -3/4)
(1, -1)     (-3/2, 3/2)
```

Show the minimax-optimal strategies are $\mathbf{p} = (2/3, 1/3), \mathbf{q} = (3/5, 2/5)$ and value of the game is 0.

Now consider the game with payoffs:

```
(-1/2, 1/2) (3/4, -3/4)
(1, -1)     (-2/3, 2/3)
```

Show that minimax-optimal strategies are $\mathbf{p} = (\frac{4}{7}, \frac{3}{7}), \mathbf{q} = (\frac{17}{35}, \frac{18}{35})$ and value of the game is $\frac{1}{7}$.

Now consider the game with payoffs:

```
(-1/2, 1/2)    (-1, 1)
(1, -1)        (2/3, -2/3)
```

Show that minimax-optimal strategies are $\mathbf{p} = (0, 1), \mathbf{q} = (0, 1)$ and value of game is $\frac{2}{3}$.