Dataflow Analysis (part 2)

15-411/15-611 Compiler Design

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September 24, 2020

Today

- Dataflow Analysis
  - reaching definitions
  - liveness
  - available expressions
  - very busy expressions
- Framework

A sample program

```
int fib10(void) {
   int n = 10;
   int older = 0;
   int old = 1;
   int result = 0;
   int i;

   if (n <= 1) return n;
   for (i = 2; i<n; i++) {
       result = old + older;
       older = old;
       old = result;
   }
   return result;
}
```

```
1:    n <- 10
2:    older <- 0
3:    old <- 1
4:    result <- 0
5:    if n <= 1 goto 14
6:    i <- 2
7:    if i > n goto 13

8:    result <- old + older
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10:   old <- result
11:   i <- i + 1
12:   JUMP 7
13:   return result
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```

Simple Constant Propagation

- Can we do SCP?
- How do we recognize it?
- What aren't we doing?
- Metanote:
  - keep opts simple!
  - Use combined power

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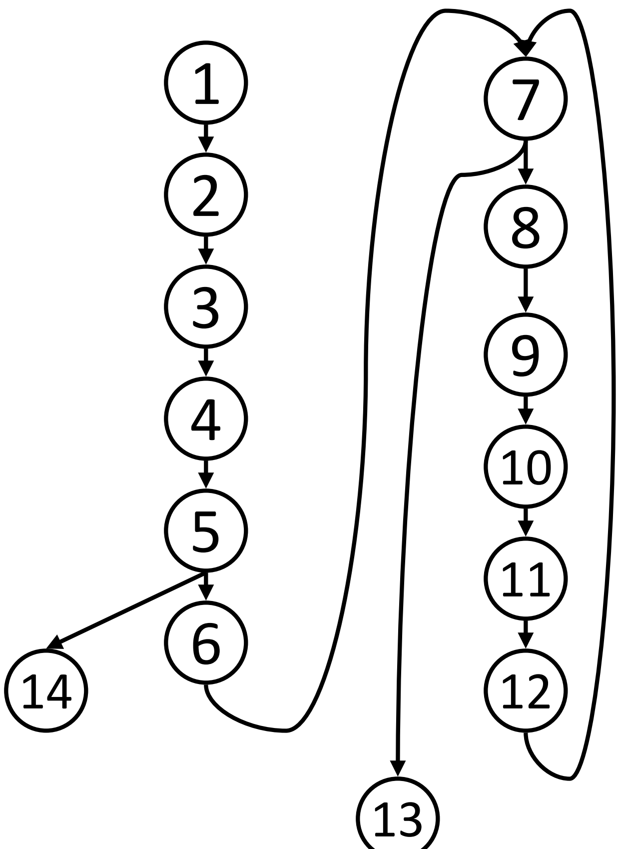
Reaching Definitions

A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.

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Reaching Definitions (ex)

- 1 reaches 5, 7, and 14

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```

14, Really?

Meta-notes:

- (almost) always conservative
- only know what we know
- Keep it simple:
  - What opt(s), if run before this would help
  - What about:

```
1: x <- 0
2: if (false) x<-1
3: … x …
```

    - Does 1 reach 3?
    - What opt changes this?

Calculating Reaching Definitions

A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.

- Build up RD stmt by stmt
- Stmt s, “d: v <- x op y”, generates d
- Stmt s, “d: v <- x op y”, kills all other defs(v)

Or,

- Gen[s] = { d }
- Kill[s] = defs(v) – { d }

Gen and kill for each stmt

Stmt	Gen	kill
`1: n <- 10`	1	
`2: older <- 0`	2	9
`3: old <- 1`	3	10
`4: result <- 0`	4	8
`5: if n <= 1 goto 14`		
`6: i <- 2`	6	11
`7: if i > n goto 13`		
`8: result <- old + older`	8	4
`9: older <- old`	9	2
`10: old <- result`	10	3
`11: i <- i + 1`	11	6
`12: JUMP 7`		
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we determine the defs that reach a node by using:

- control flow information
- gen and kill info

Computing in[n] and out[n]

- In[n]: the set of defs that reach the beginning of node n
- Out[n]: the set of defs that reach the end of node n

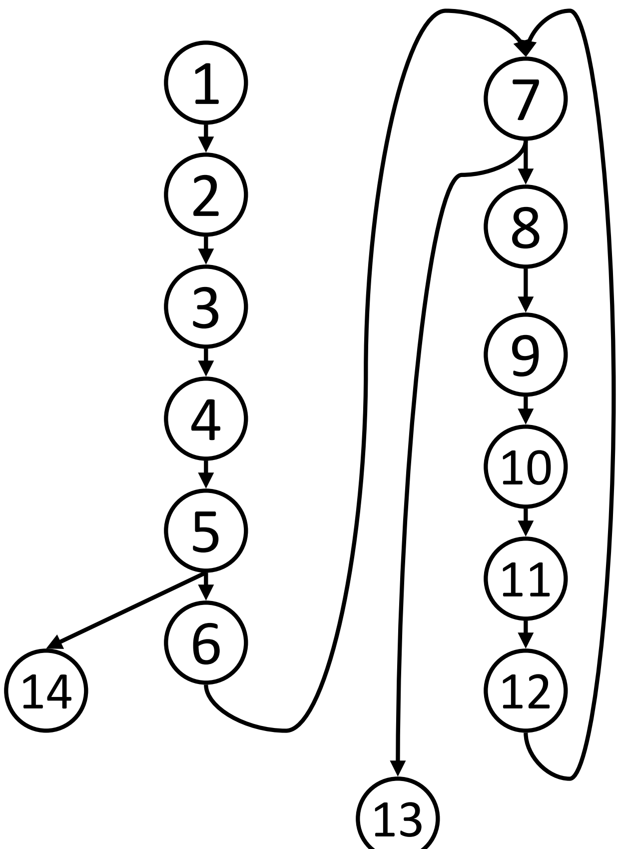
$$\text{in[n]} = \bigcup_{p \in pred[n]} out[p]$$

$$\text{out[n]} = gen[n] \cup (in[n] - kill[n])$$

- Initialize in[n]=out[n]={} for all n
- Solve iteratively

pred[n]?

- Pred[n] are all nodes that can directly reach n in the control flow graph.
- E.g., pred[7] = { 6, 12 }

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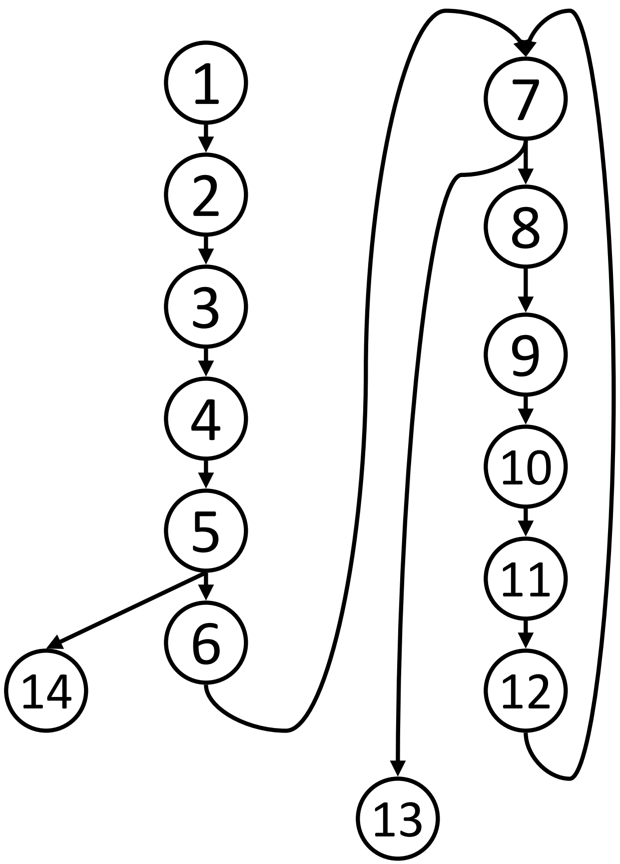
What order to eval nodes?

- Does it matter?
- Lets do: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

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Example:

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

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Example (pass 2)

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Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	9	2	1-3,6,8	1,3,6,8,9
`10: old <- result`	10	3	1,3,6,8,9	1,6,8-10
`11: i <- i + 1`	11	6	1,6,8-10	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6,8-11	1-4,6,8-11
`14: return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	9	2	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	10	3	1,3,6,8,9	1,6,8-10
`11: i <- i + 1`	11	6	1,6,8-10	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6,8-11	1-4,6,8-11
`14: return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	9	2	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	10	3	1,3,6,8-11	1,6,8-11
`11: i <- i + 1`	11	6	1,6,8-10	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6,8-11	1-4,6,8-11
`14: return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	9	2	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	10	3	1,3,6,8-11	1,6,8-11
`11: i <- i + 1`	11	6	1,6,8-11	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6,8-11	1-4,6,8-11
`14: return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \bigcup (in[n] - kill[n])$$

	Gen	kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8	1-3	1-4
5: if n <= 1 goto 14			1-4	1-4
6: i <- 2	6	11	1-4	1-4,6
7: if i > n goto 13			1-4,6,8-11	1-4,6,8-11
8: result <- old + older	8	4	1-4,6,8-11	1-3,6,8-11
9: older <- old	9	2	1-3,6,8-11	1,3,6,8-11
10: old <- result	10	3	1,3,6,8-11	1,6,8-11
11: i <- i + 1	11	6	1,6,8-11	1,8-11
12: JUMP 7			1,8-11	1,8-11
13: return result			1-4,6,8-11	1-4,6,8-11
14: return n			1-4	1-4

An Improvement: Basic Blocks

- No need to compute this one stmt at a time
- For straight line code:
  - In[s1; s2] = in[s1]
  - Out[s1; s2] = out[s2]
- Combine the gen and kill sets into one per BB.

- Gen[BB]={2,3,4,5}
- Kill[BB]={1,8,11}

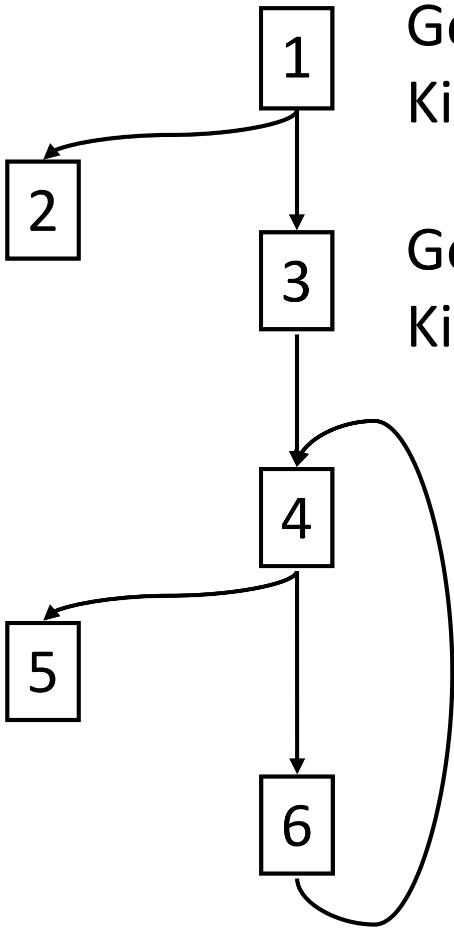
	Gen	kill
`1: i <- 1`	1	8,4
`2: j <- 2`	2	
`3: k <- 3 + i`	3	11
`4: i <- j`	4	1,8
`5: m <- i + k`	5	

BB sets

		Gen	kill		
	`1: n <- 10`	1			
	`2: older <- 0`	2	9		
1	`3: old <- 1`	3	10		
	`4: result <- 0`	4	8		
	`5: if n <= 1 goto 14`			1,2,3,4	8,9,10
3	`6: i <- 2`	6	11	6	11
4	`7: if i > n goto 13`				
	`8: result <- old + older`	8	4		
	`9: older <- old`	9	2		
6	`10: old <- result`	10	3		
	`11: i <- i + 1`	11	6		
	`12: JUMP 7`			8-11	2-4,6
5	`13: return result`				
2	`14: return n`				

BB sets

In out



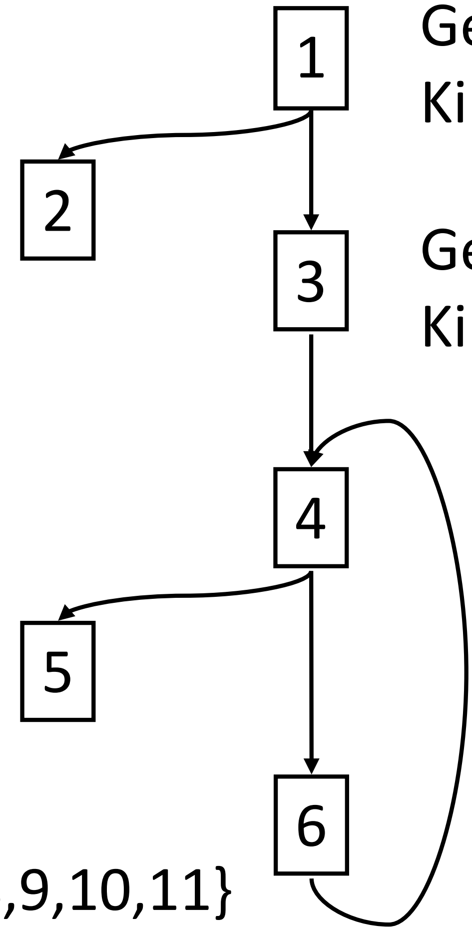
Gen={1,2,3,4}
Kill={8,9,10}

1,2,3,4

Gen={6}
Kill={11}

Gen={8,9,10,11}
Kill={2,3,4,6}

BB sets



Gen={1,2,3,4}
Kill={8,9,10}

Gen={6}
Kill={11}

Gen={8,9,10,11}
Kill={2,3,4,6}

In	out
	1,2,3,4
1,2,3,4	1,2,3,4,6
1-4,6,8-11	1-4,6,8-11
1-4,6,8-11	1,8-11

Forward Dataflow

- Reaching definitions is a forward dataflow problem:
  It propagates information from the predecessors of a node to the node
- Defined by:
  - Basic attributes: (gen and kill)
  - Transfer function: F_{bb} $out[n] = gen[n] \bigcup (in[n] - kill[n])$
  - Meet operator: union $in[n] = \bigcup_{p \in pred[n]} out[p]$
  - Set of values (a lattice, in this case powerset of program points)
  - Initial values for each node b
- Solve for fixed point solution

How to implement?

- Values?
- Gen?
- Kill?
- F_{bb}?
- Order to visit nodes?
- When are we done?
  - In fact, do we know we terminate?

Implementing RD

- Values: bits in a bit vector
- Gen: 1 in each position generated, otherwise 0
- Kill: 0 in each position killed, otherwise 1
- F_{bb}: out[b] = gen[b] | (in[b] & kill[b])
- Init in[b]=out[b]=0

- When are we done?
- What order to visit nodes? Does it matter?

RD Worklist algorithm

Initialize: in[B] = out[b] = $\varnothing$

Initialize: in[entry] = $\varnothing$

Work queue, W = all Blocks in topological order

while (|W| != 0) {

remove b from W

old = out[b]

in[b] = {over all pred(p) $\in$ b} $\cup$ out[p]

out[b] = gen[b] $\cup$ (in[b] - kill[b])

if (old != out[b]) W = W $\cup$ succ(b)

}

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

`1: n <- 10`	1	
`2: older <- 0`	1	1,2
`3: old <- 1`	1,2	1,2,3
`4: result <- 0`	1-3	1-4
`5: if n <= 1 goto 14`	1-4	1-4
`6: i <- 2`	1-4	1-4,6
`7: if i > n goto 13`	1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	1,3,6,8-11	1,6,8-11
`11: i <- i + 1`	1,6,8-11	1,8-11
`12: JUMP 7`	1,8-11	1,8-11
`13: return result`	1-4,6	1-4,6
`14: return n`	1-4	1-4

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

```
1:  n <- 10                   1
2:  older <- 0                1             1,2
3:  old <- 1                  1,2           1,2,3
4:  result <- 0               1-3           1-4
5:  if n <= 1 goto 14         1-4           1-4
6:  i <- 2                    1-4           1-4,6
7:  if i > n goto 13          1-4,6,8-11    1-4,6,8-11
8:  result <- old + older     1-4,6,8-11    1-3,6,8-11
9:  older <- old              1-3,6,8-11    1,3,6,8-11
10: old <- result             1,3,6,8-11    1,6,8-11
11: i <- i + 1                1,6,8-11      1,8-11
12: JUMP 7                    1,8-11        1,8-11
13: return result             1-4,6         1-4,6
14: return n                  1-4           1-4
```

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

1:	n <- 10	1	
2:	older <- 0	1	1,2
3:	old <- 1	1,2	1,2,3
4:	result <- 0	1-3	1-4
5:	if n <= 1 goto 14	1-4	1-4
6:	i <- 2	1-4	1-4,6
7:	if i > n goto 13	1-4,6,8-11	1-4,6,8-11
8:	result <- old + older	1-4,6,8-11	1-3,6,8-11
9:	older <- old	1-3,6,8-11	1,3,6,8-11
10:	old <- result	1,3,6,8-11	1,6,8-11
11:	i <- i + 1	1,6,8-11	1,8-11
12:	JUMP 7	1,8-11	1,8-11
13:	return result	1-4,6	1-4,6
14:	return n	1-4	1-4

Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

`1: n <- 10`	1	
`2: older <- 0`	1	1,2
`3: old <- 1`	1,2	1,2,3
`4: result <- 0`	1-3	1-4
`5: if n <= 1 goto 14`	1-4	1-4
`6: i <- 2`	1-4	1-4,6
`7: if i > n goto 13`	1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	1,3,6,8-11	1,6,8-11
`11: i <- i + 1`	1,6,8-11	1,8-11
`12: JUMP 7`	1,8-11	1,8-11
`13: return result`	1-4,6	1-4,6
`14: return n`	1-4	1-4

Constant Folding + DCE

~~1: n <- 10~~	~~1~~	
2: older <- 0	1	1,2
3: old <- 1	1,2	1,2,3
4: result <- 0	1-3	1-4
~~5: if 10 <= 1 goto 14~~	~~1-4~~	~~1-4~~
6: i <- 2	1-4	1-4,6
7: if i > 10 goto 13	1-4,6,8-11	1-4,6,8-11
8: result <- old + older	1-4,6,8-11	1-3,6,8-11
9: older <- old	1-3,6,8-11	1,3,6,8-11
10: old <- result	1,3,6,8-11	1,6,8-11
11: i <- i + 1	1,6,8-11	1,8-11
12: JUMP 7	1,8-11	1,8-11
13: return result	1-4,6	1-4,6
~~14: return 10~~	~~1-4~~	~~1-4~~

Better Constant Propagation

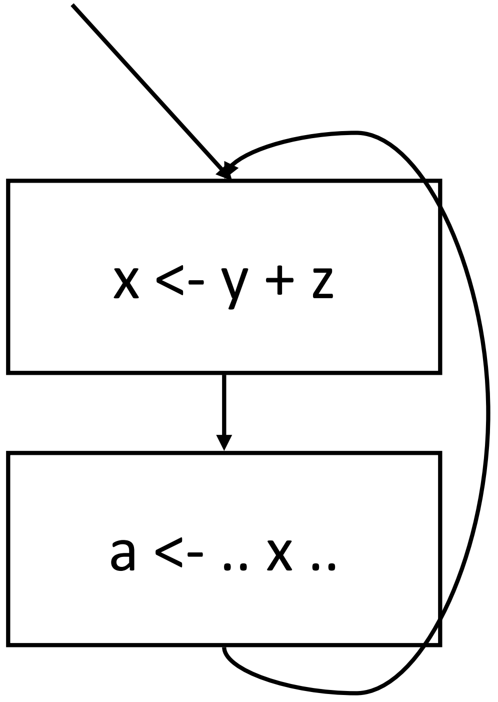
- What about:

```
x <- 1
if (y > z)
    x <- 1
a <- x
```

- How might you solve this with SSA?

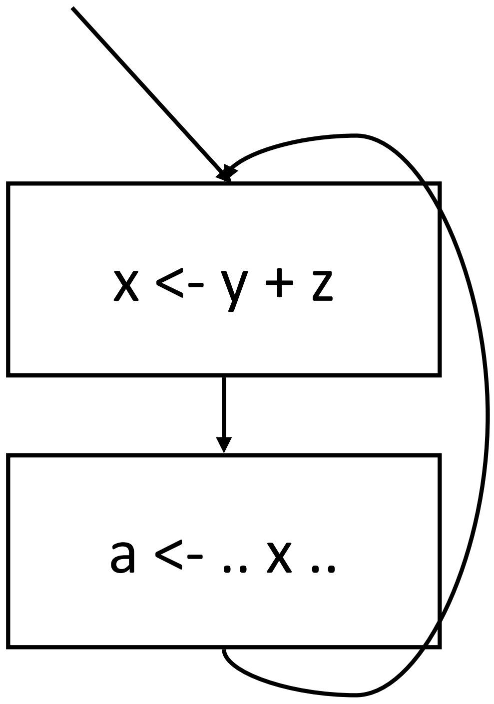
Loop Invariant Code Motion

- When can expression be moved out of a loop?



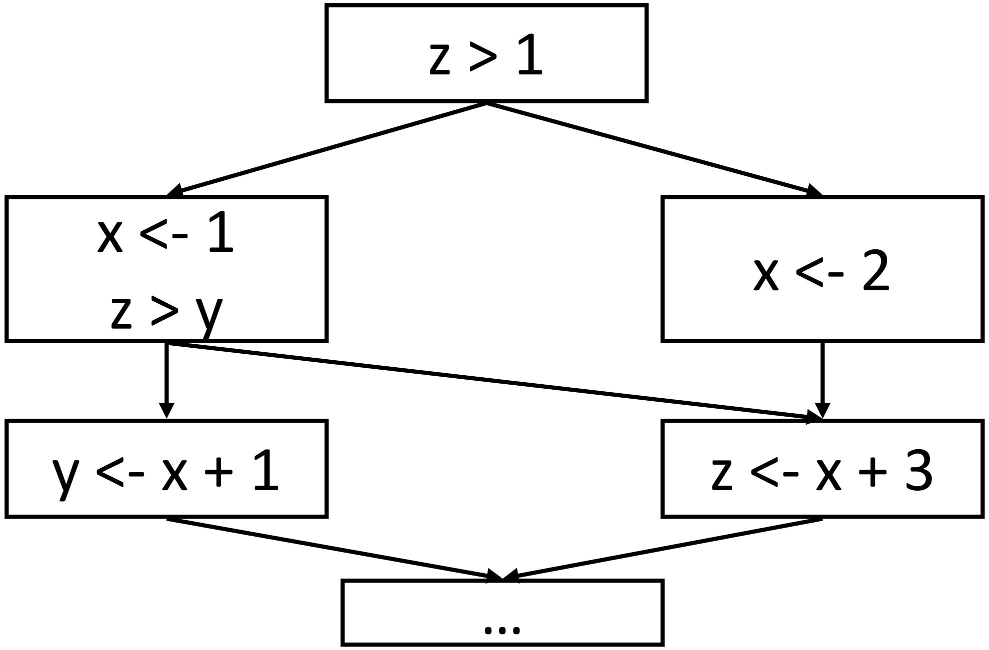
Loop Invariant Code Motion

- When can expression be moved out of a loop?
- When all reaching definitions of operands are outside of loop, expression is loop invariant
- Use ud-chains to detect



Def-use chains are valuable too

- Def-use chain: for each definition of var x, a list of all uses of that definition
- Computed from liveness analysis, a backward dataflow problem
- Def-use is NOT symmetric to use-def



Liveness (def-use chains)

- A variable x is live-out of a stmt s if x can be used along some path starting a s, otherwise x is dead.

Liveness as a dataflow problem

- This is a backwards analysis
  - A variable is live out if used by a successor
  - Gen: For a use: indicate it is live coming into s
  - Kill: Defining a variable v in s makes it dead before s (unless s uses v to define v)
  - Lattice is just live (top) and dead (bottom)
- Values are variables
- In[n] = variables live before n

  $$= (out[n] - kill[n]) \cup gen[n]$$

- Out[n] = variables live after n

  $$= \bigcup_{s \in succ(n)} In[s]$$

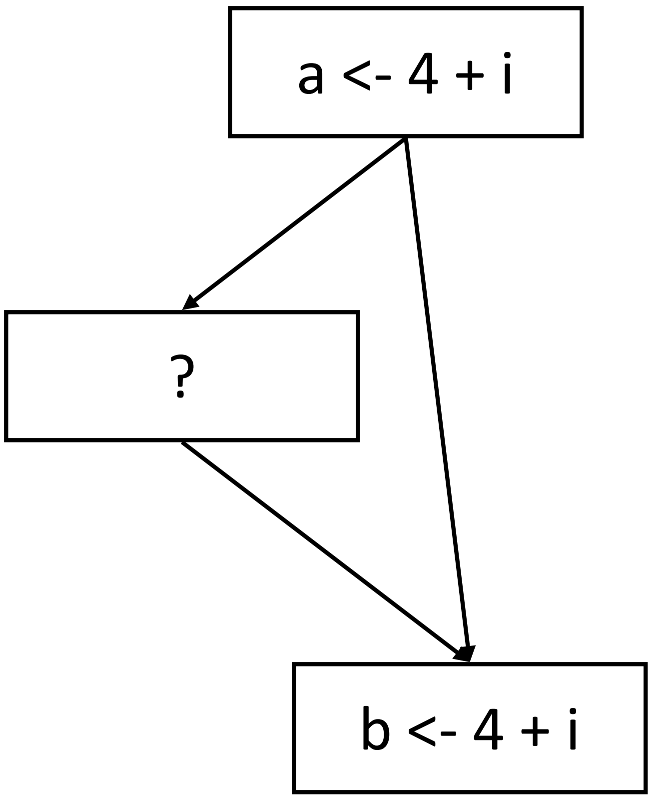
Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?

Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?
  - When the definition is dead, and
  - When the instruction has no side effects
- So:
  - run liveness
  - Construct def-use chains
  - Any instruction which has no users and has no side effects can be eliminated

When can we do CSE?



Available Expressions

- X+Y is "available" at statement S if
  - x+y is computed along every path from the start to S AND
  - neither x nor y is modified after the last evaluation of x+y

```
a <- b+c

b <- a-d

c <- b+c

d <- a-d
```

Available Expressions

- X+Y is "available" at statement S if
  - x+y is computed along every path from the start to S AND
  - neither x nor y is modified after the last evaluation of x+y

a <- b+c

b <- a-d

c <- b+c ← b+c Not available, since b redefined

d <- a-d ← a-d is available

Computing Available Expressions

- Forward or backward?
- Values?
- Lattice?
- gen[b] =
- kill[b] =
- in[b] =
- out[b] =
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x,
  then all exprs using x are killed.
- out[b] = (in[b] – kill[b]) $\cup$ gen[b]
- in[b] = what to do at a join point?
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x, exprs(x) are killed
  out[b] = (in[b] – kill[b]) $\cup$ gen[b]
- in[b] = An expr is avail only if avail on ALL edges, so:
  in[b] = $\cap$ over all p$\in$ pred(b), out[p]
- Initialization
  - All nodes, but entry are set to ALL avail
  - Entry is set to NONE avail

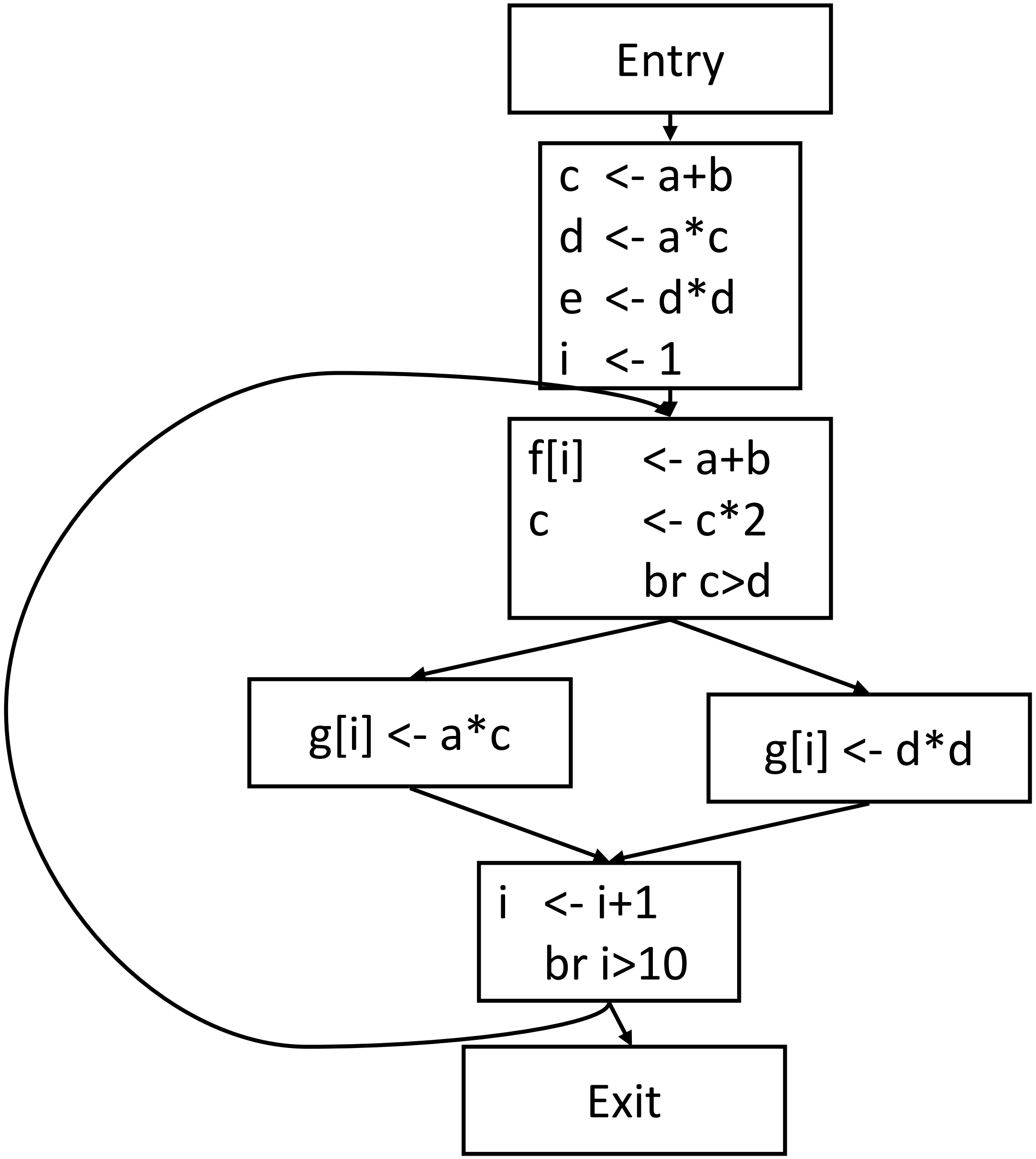
Constructing Gen & Kill

Stmt s	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	
M[a] <- b		
f(a, …)		{M[x] for all x}
t <- f(a,…)		

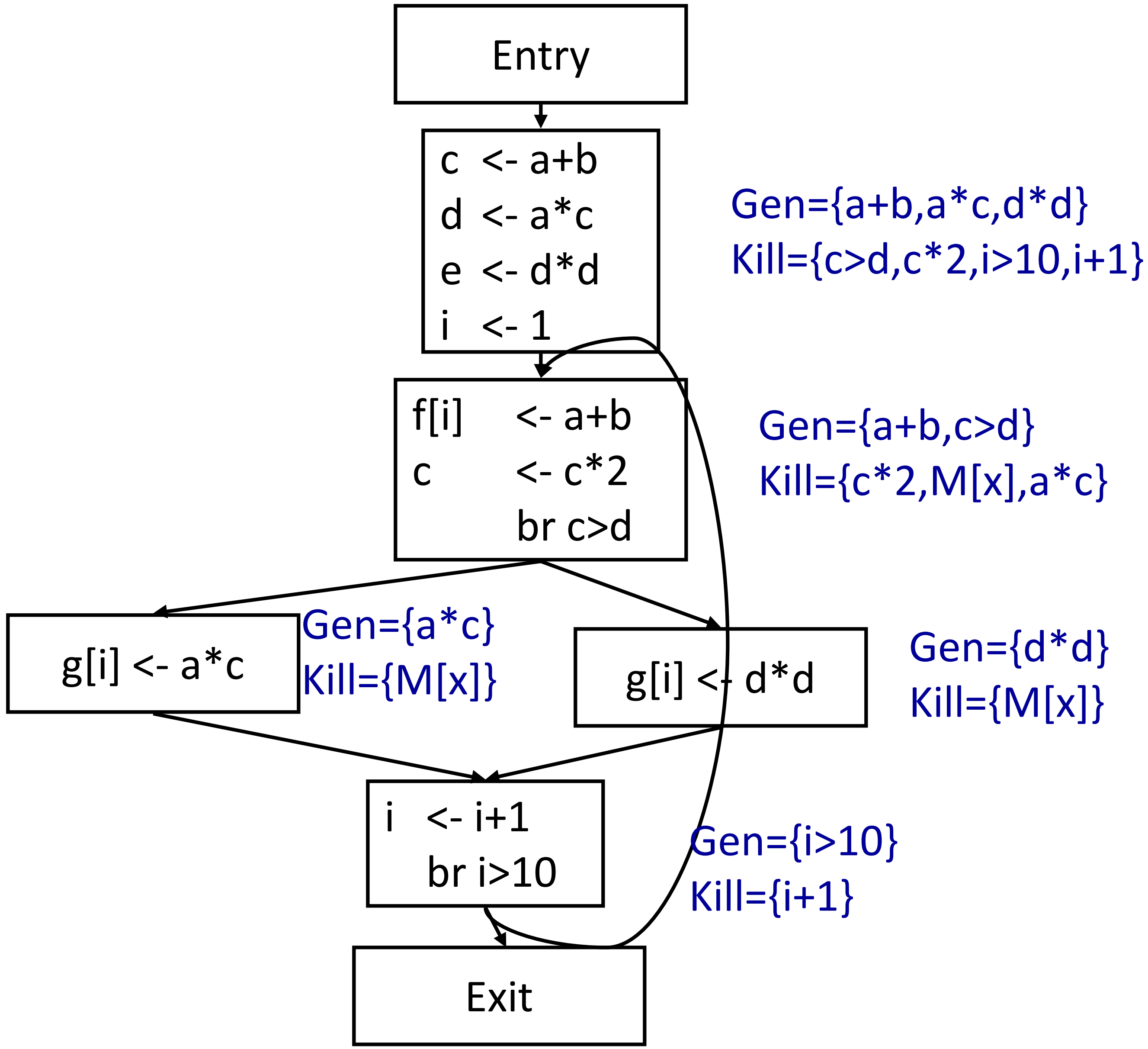
Constructing Gen & Kill

Stmt s	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	{exprs containing t}
M[a] <- b	{}	{for all x, M[x]}
f(a, …)	{}	{for all x, M[x]}
t <- f(a,…)	{}	{exprs containing t for all x, M[x]}

Example

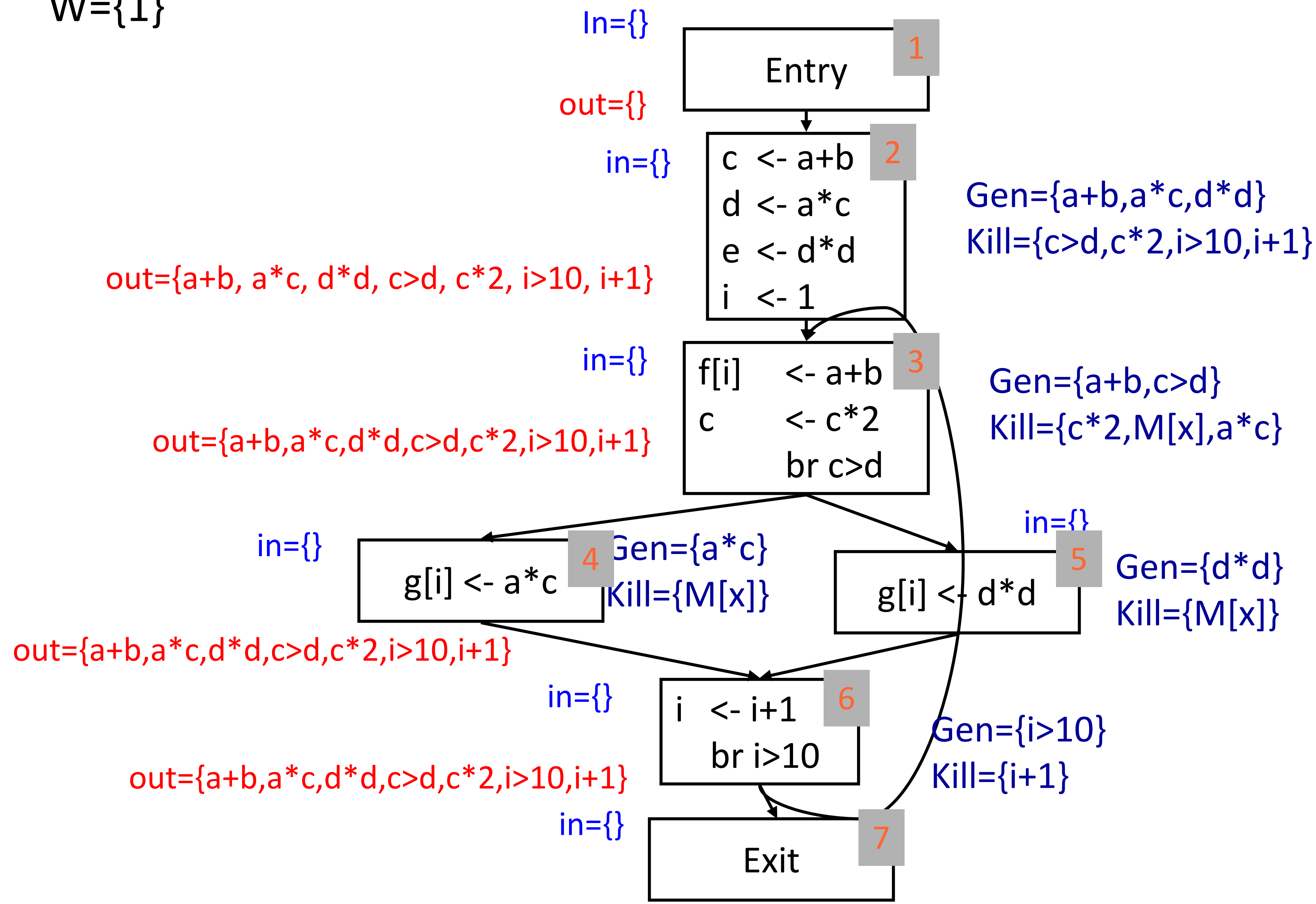


Example


Entry
c <- a+b
d <- a*c
e <- d*d
i <- 1
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
f[i] <- a+b
c <- c*2
br c>d
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
g[i] <- a*c
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
Gen={d*d}
Kill={M[x]}
i <- i+1
br i>10
Gen={i>10}
Kill={i+1}
Exit

Example

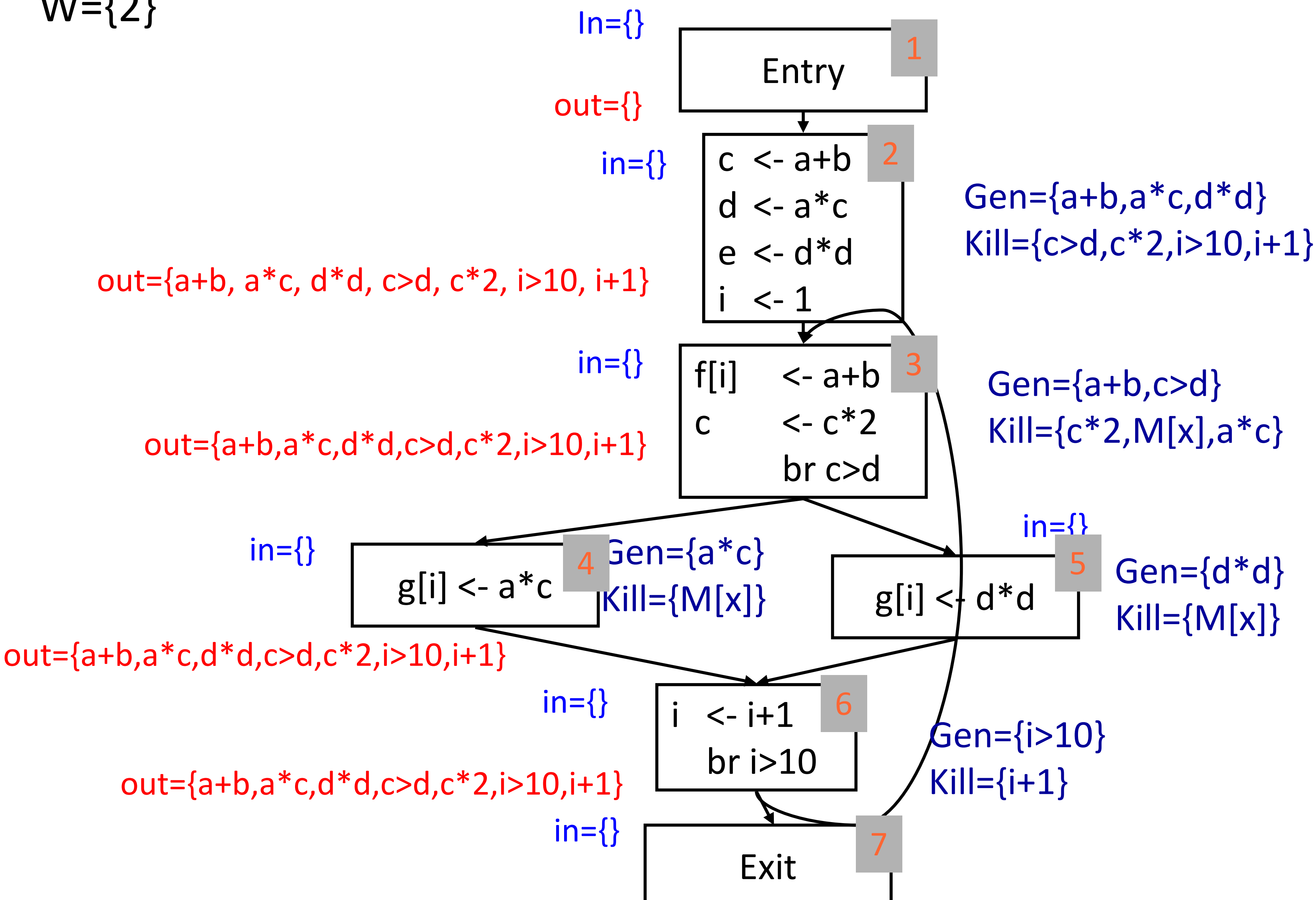
W={1}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d, c>d, c*2, i>10, i+1}
in={}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

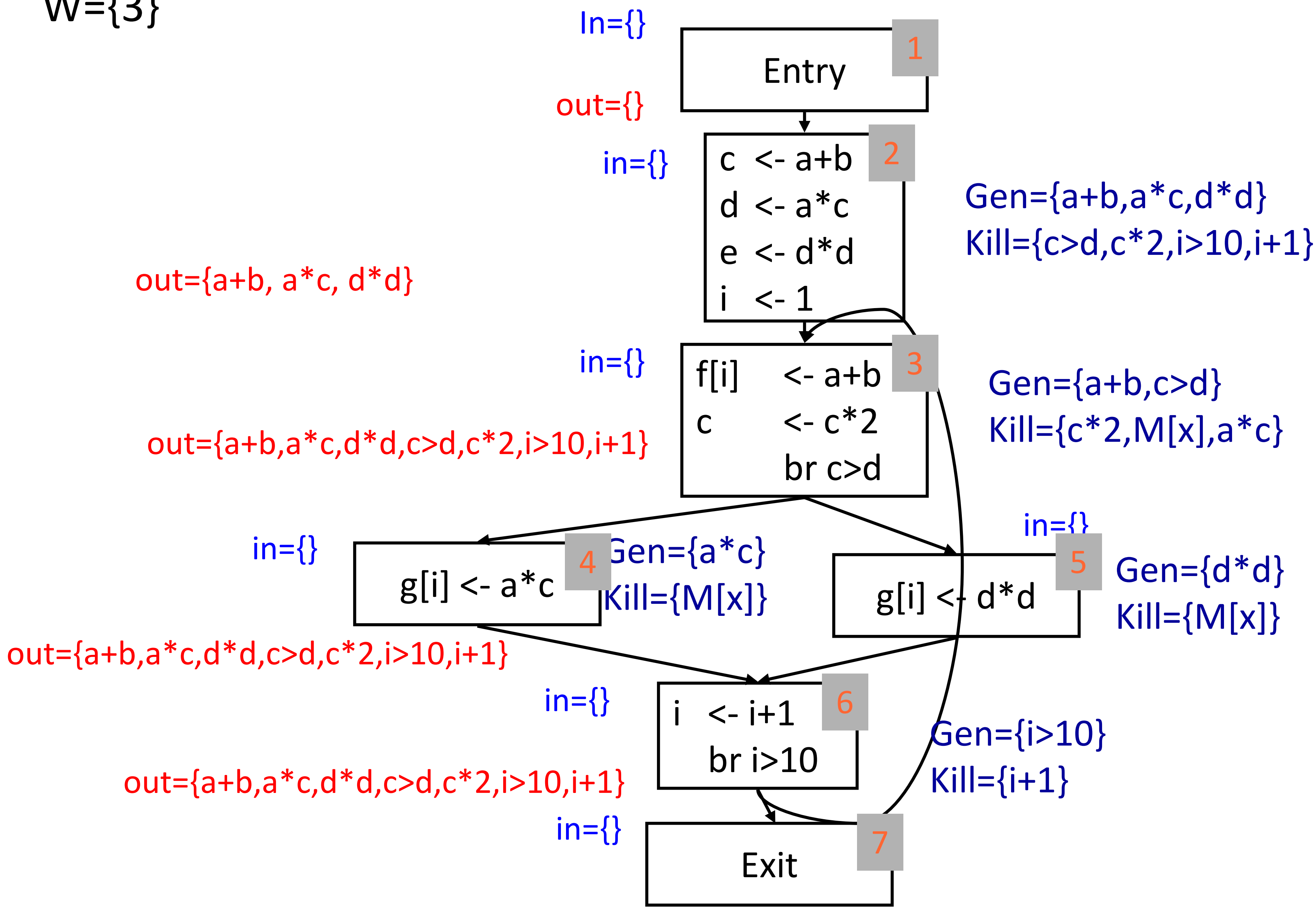
W={2}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d, c>d, c*2, i>10, i+1}
in={}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

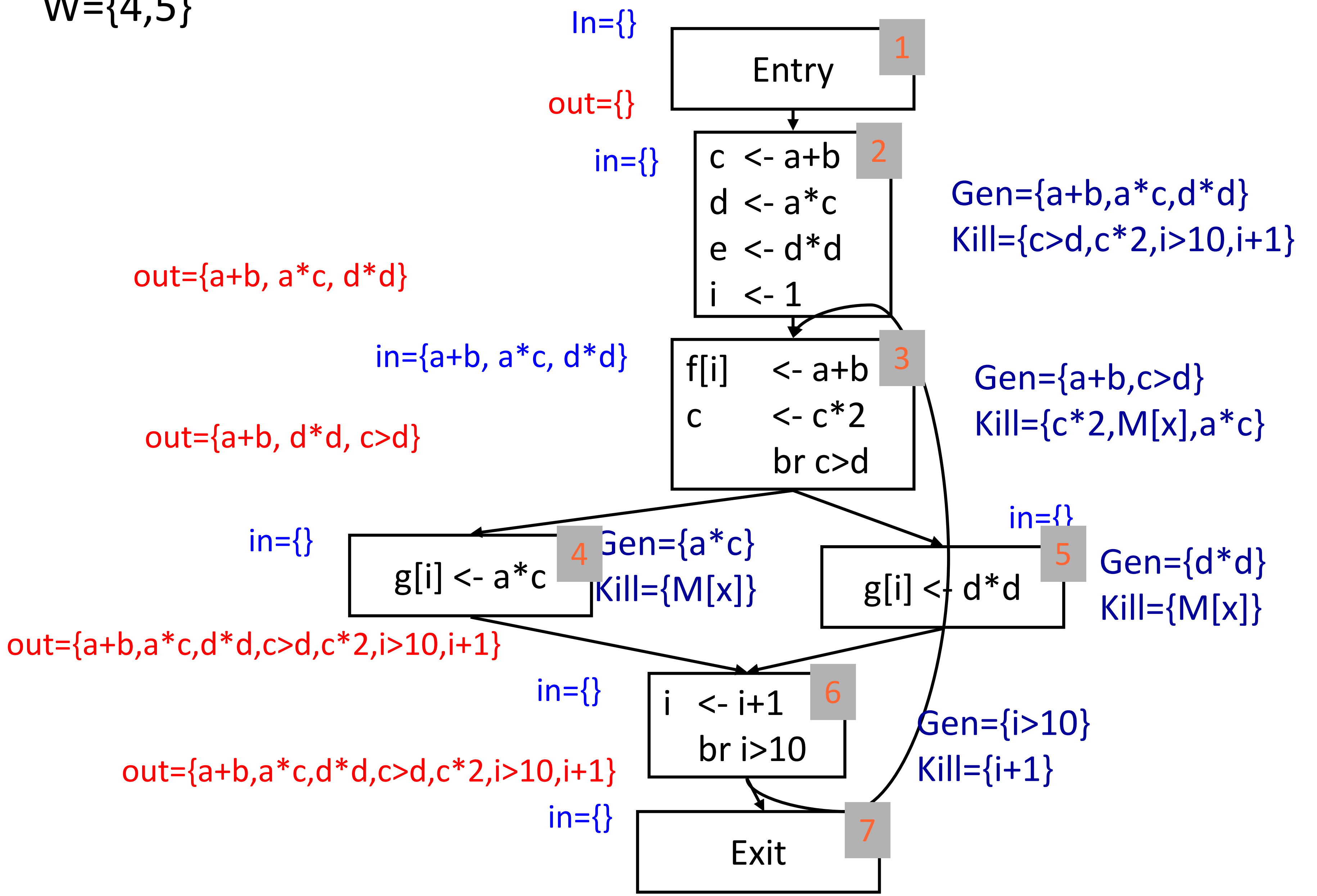
W={3}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

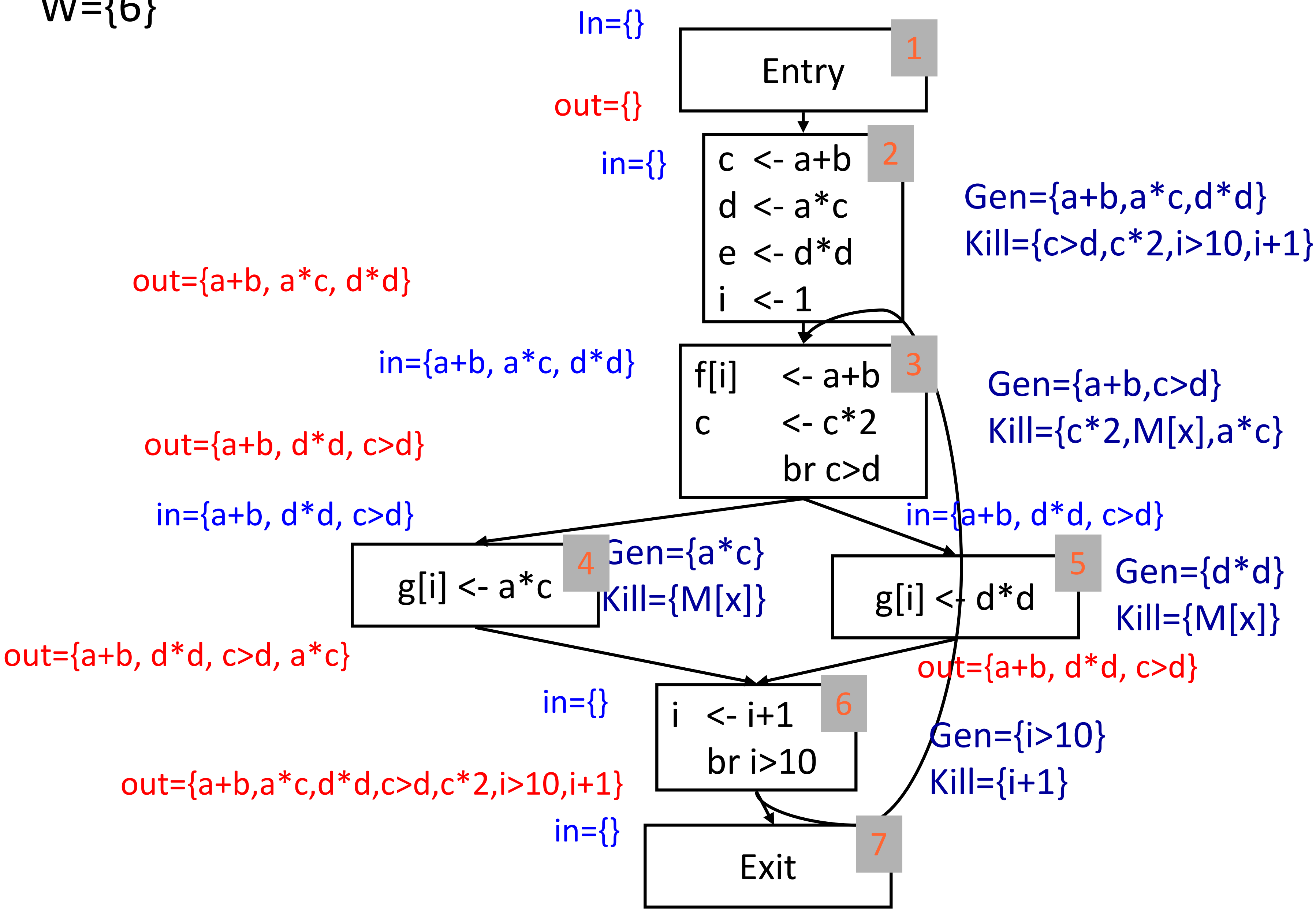
W={4,5}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, a*c, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

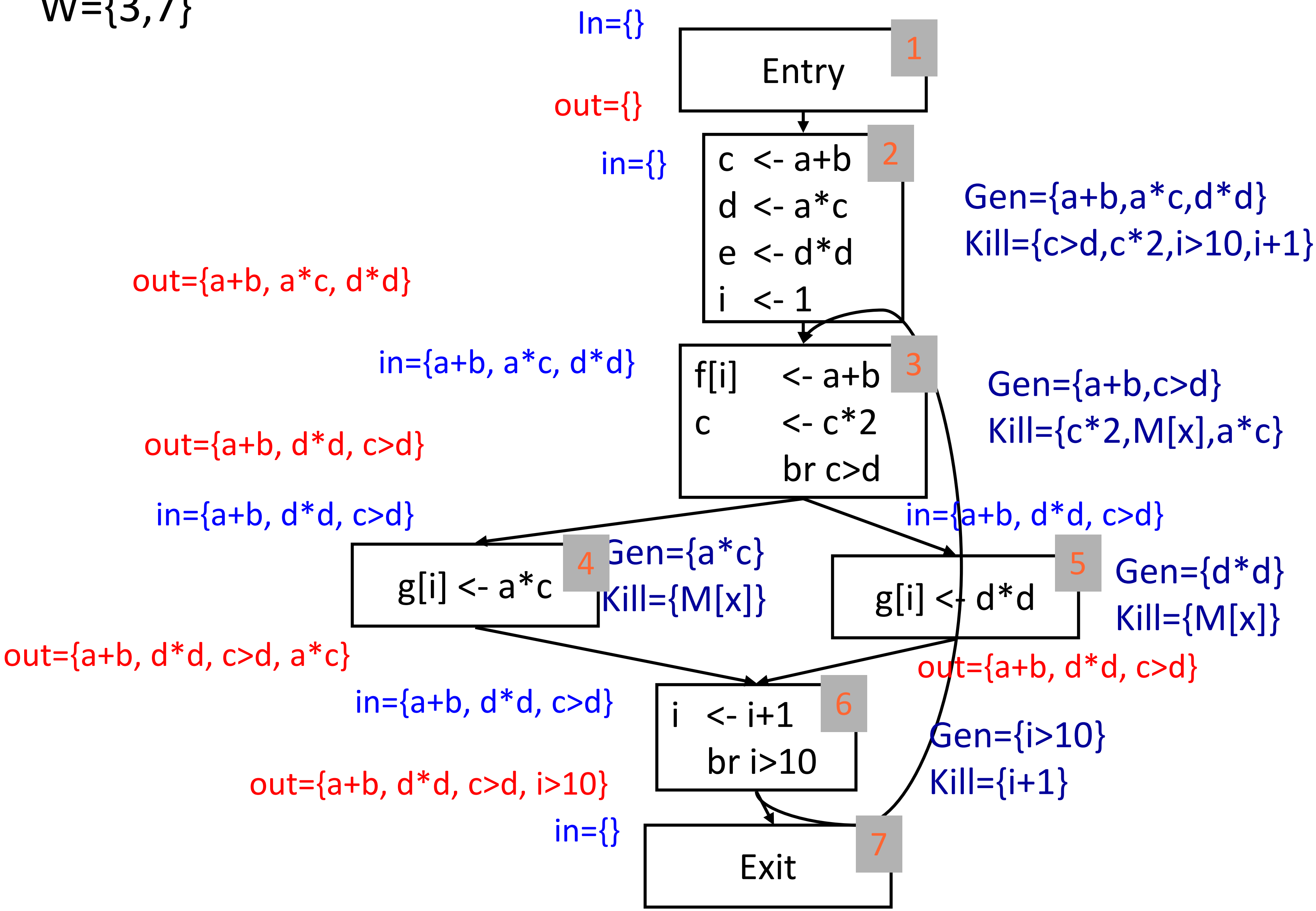
W={6}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, a*c, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
in={a+b, d*d, c>d}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}
in={}
Exit
7

Example

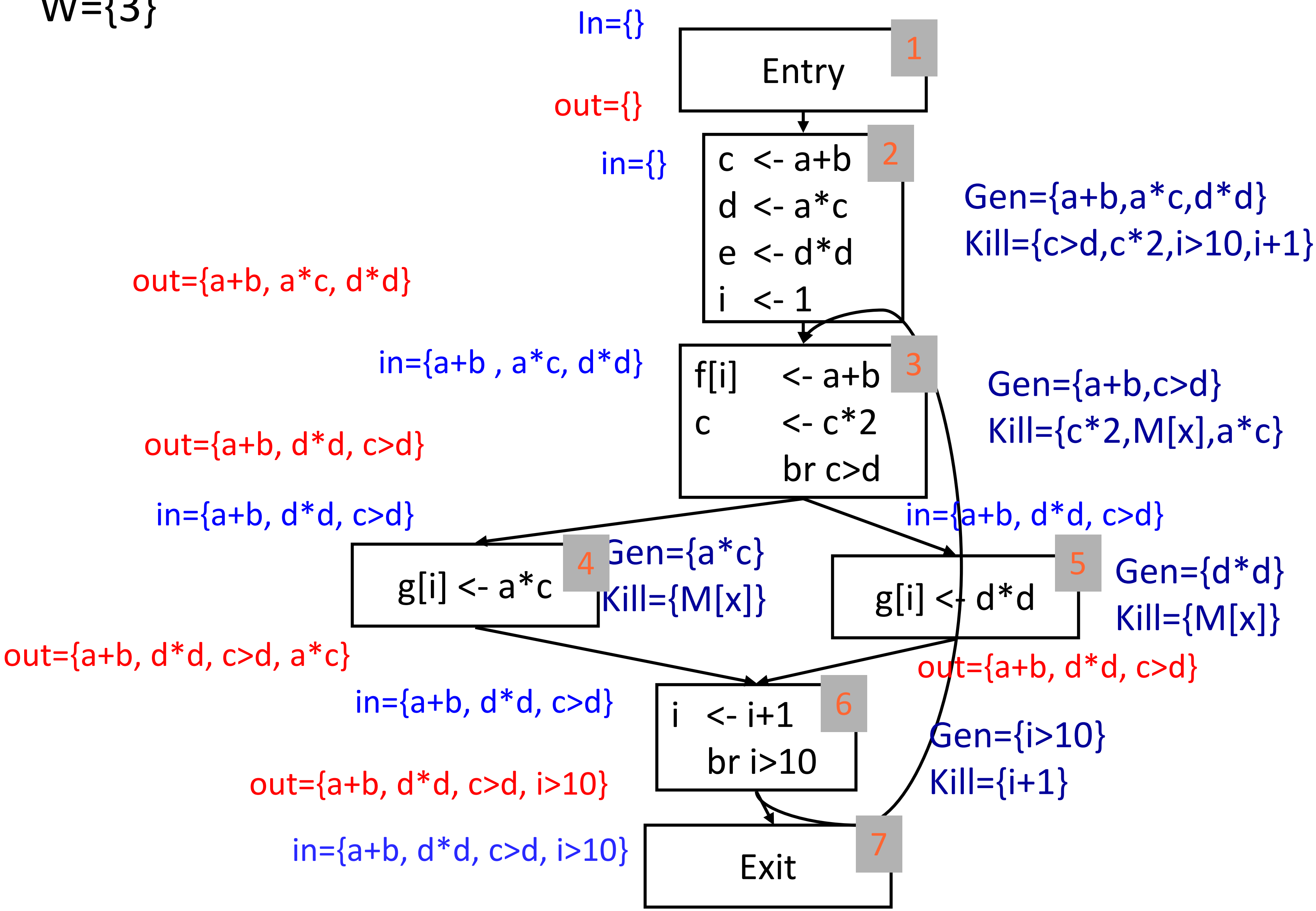
W={3,7}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, a*c, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
in={a+b, d*d, c>d}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}
in={}
Exit
7

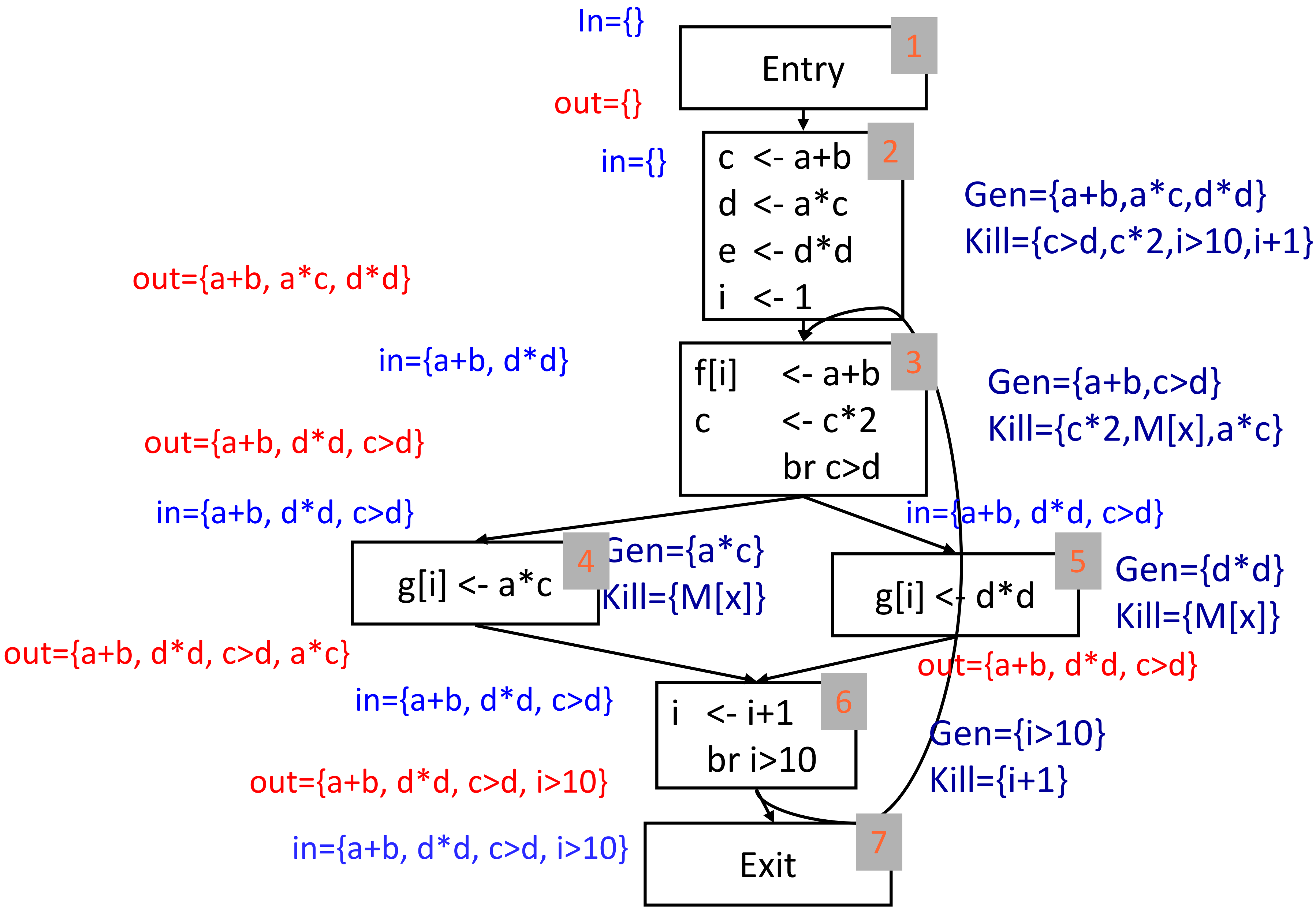
Example

W={3}



In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b , a*c, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
in={a+b, d*d, c>d}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}
in={a+b, d*d, c>d, i>10}
Exit
7

Example


In={}
Entry
1
out={}
in={}
c <- a+b
d <- a*c
e <- d*d
i <- 1
2
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}
in={a+b, d*d}
f[i] <- a+b
c <- c*2
br c>d
3
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
in={a+b, d*d, c>d}
g[i] <- a*c
4
Gen={a*c}
Kill={M[x]}
g[i] <- d*d
5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}
out={a+b, d*d, c>d}
in={a+b, d*d, c>d}
i <- i+1
br i>10
6
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}
in={a+b, d*d, c>d, i>10}
Exit
7

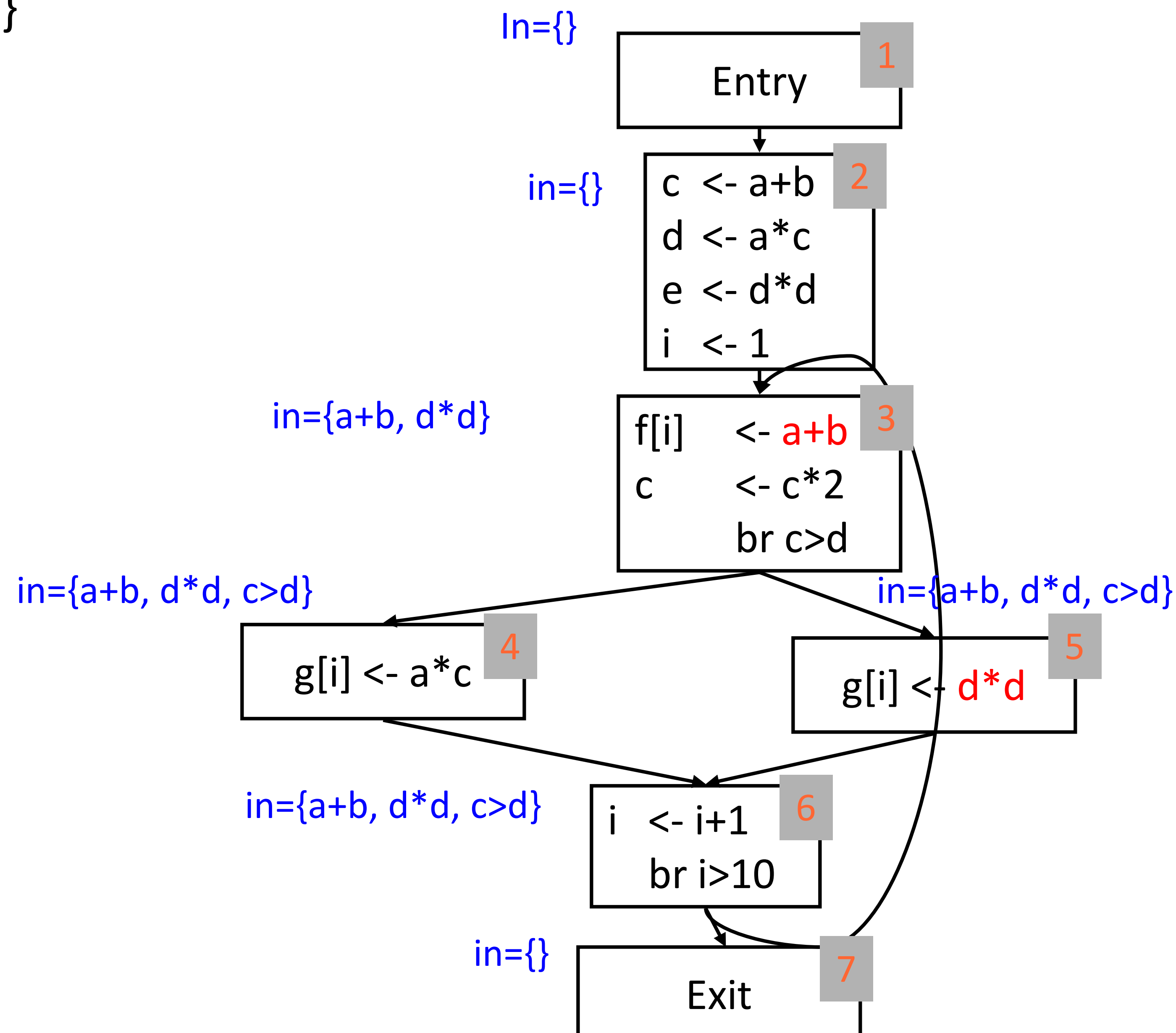
CSE

- Calculate Available expressions
- For every stmt in program

```
If expression, x op y, is available {
    Compute reaching expressions for "x op y"
        at this stmt
    foreach stmt in RE of the form t <- x op y
                rewrite at:   t' <- x op y
                              t <- t'
    }
    replace "x op y" in stmt with t'
}
```

Find x op y available

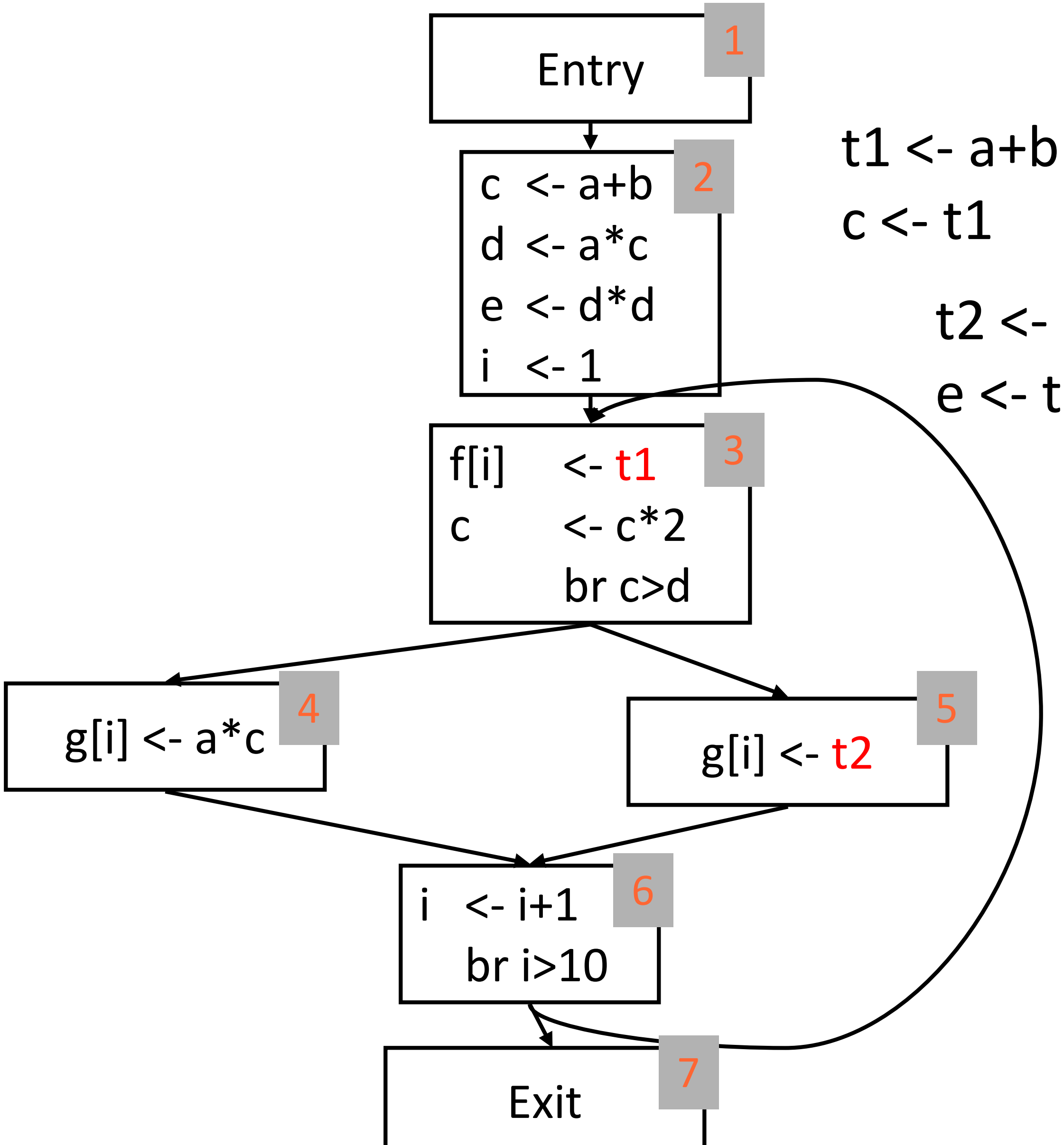
W={3}



Calculating Reaching Expressions

- Could be dataflow problem, but not needed enough, so …
- To find RE for “x op y” at stmt S
  - traverse cfg backward from S until
    - reach t <- x + y (& put into RE)
    - reach definition of x or y

Example



t1 <- a+b

c <- t1

t2 <- d*d

e <- t2

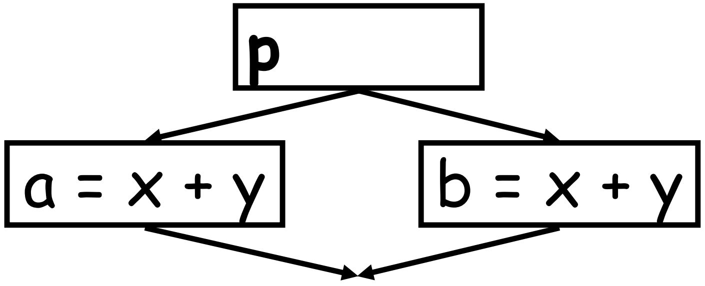
Dataflow Summary

	Union (may)	intersection (must)
Forward	Reaching defs	Available exprs
Backward	Live variables	very busy exprs

Later in course we look at bidirectional dataflow

Very Busy Expressions

- A Backward, Must data flow analysis
- An expression e is *very busy at point p* if On every path from p, e is evaluated before the value of e is changed
- Optimization
  - Can hoist very busy expression computation



Forward Must Data Flow Algorithm

Out(s) = Gen(s) for all statements s

W = {all statements}

Repeat

    Take s from W

    In(s) = $\bigcap_{s' \in pred(s)}$ Out(s')

    Temp = Gen(s) $\cup$ (In(s) – Kill(s))

    If (temp != Out(s)) {

        Out(s) = temp

        W = W $\cup$ succ(s)

    }

Until W = $\varnothing$

Forward May Data Flow Algorithm

Out(s) = Gen(s) for all statements s

W = {all statements}

Repeat

    Take s from W

    In(s) = $\cup_{s' \in pred(s)}$ Out(s')

    Temp = Gen(s) $\cup$ (In(s) – Kill(s))

    If (temp != Out(s)) {

        Out(s) = temp

        W = W $\cup$ succ(s)

    }

Until W = $\varnothing$

Backward May Data Flow Algorithm

In(s) = Gen(s) for all statements s

W = {all statements} (worklist)

Repeat

    Take s from W

    Out(s) = $\cup_{s' \in succ(s)}$ In(s')

    Temp = Gen(s) $\cup$ (Out(s) – Kill(s))

    If (temp != In(s)) {

        In(s) = temp

        W = W $\cup$ pred(s)

    }

Until W = $\varnothing$

Backward Must Data Flow Algorithm

In(s) = Gen(s) for all statements s

W = {all statements} (worklist)

Repeat

    Take s from W

    Out(s) = $\bigcap_{s' \in succ(s)}$ In(s')

    Temp = Gen(s) $\cup$ (Out(s) – Kill(s))

    If (temp != In(s)) {

        In(s) = temp

        W = W $\cup$ pred(s)

    }

Until W = $\emptyset$