SSA

15-411/15-611 Compiler Design

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Today

- Trivial SSA
- ϕ-functions
- Motivation (CCP)
- Dominators
- Placement & Renaming
- Deconstructing SSA

SSA

- Static single assignment is an **IR** where every variable has only ONE definition in the program text
  - single **static** definition
  - (Could be in a loop which is executed dynamically many times.)
- ϕ-functions used at CFG merge points
- Definitions dominate uses

Advantages of SSA

- Makes du-chains explicit
- Makes dataflow optimizations
  - Easier
  - faster
- Improves register allocation
  - Makes building interference graphs easier
  - Easier register allocation algorithm
  - Decoupling of spill, color, and coalesce
- For most programs reduces space/time requirements

One definition for each use

- Key to SSA is single point of definition for each use
- Introduce ϕ-functions to handle joins in CFG

```
x ← …
y ← …
while(x < 100){
  x ← x + 1
  y ← y + 1
}
```

```
   x ← …
   y ← …
   if (x >= 100) goto end
loop:
   x ← x + 1
   y ← y + 1
   if (x < 100) goto loop
end:
```

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One definition for each use

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- Introduce ϕ-functions to handle joins in CFG

$$
\begin{array}{l}
\quad \mathbf{x}_0 \leftarrow \ldots \\
\quad \mathbf{y}_0 \leftarrow \ldots \\
\quad \mathbf{if}\ (\mathbf{x}_0 >= 100)\ \mathbf{goto\ end} \\
\mathbf{loop:} \\
\quad \mathbf{x}_1 \leftarrow \Phi(\mathbf{x}_0, \mathbf{x}_2) \\
\quad \mathbf{y}_1 \leftarrow \Phi(\mathbf{y}_0, \mathbf{y}_2) \\
\quad \mathbf{x}_2 \leftarrow \mathbf{x}_1 + 1 \\
\quad \mathbf{y}_2 \leftarrow \mathbf{y}_1 + 1 \\
\quad \mathbf{if}\ (\mathbf{x}_2 < 100)\ \mathbf{goto\ loop} \\
\mathbf{end:} \\
\quad \mathbf{x}_3 \leftarrow \Phi(\mathbf{x}_0, \mathbf{x}_2) \\
\quad \mathbf{y}_3 \leftarrow \Phi(\mathbf{y}_1, \mathbf{y}_2)
\end{array}
$$

The Φ function

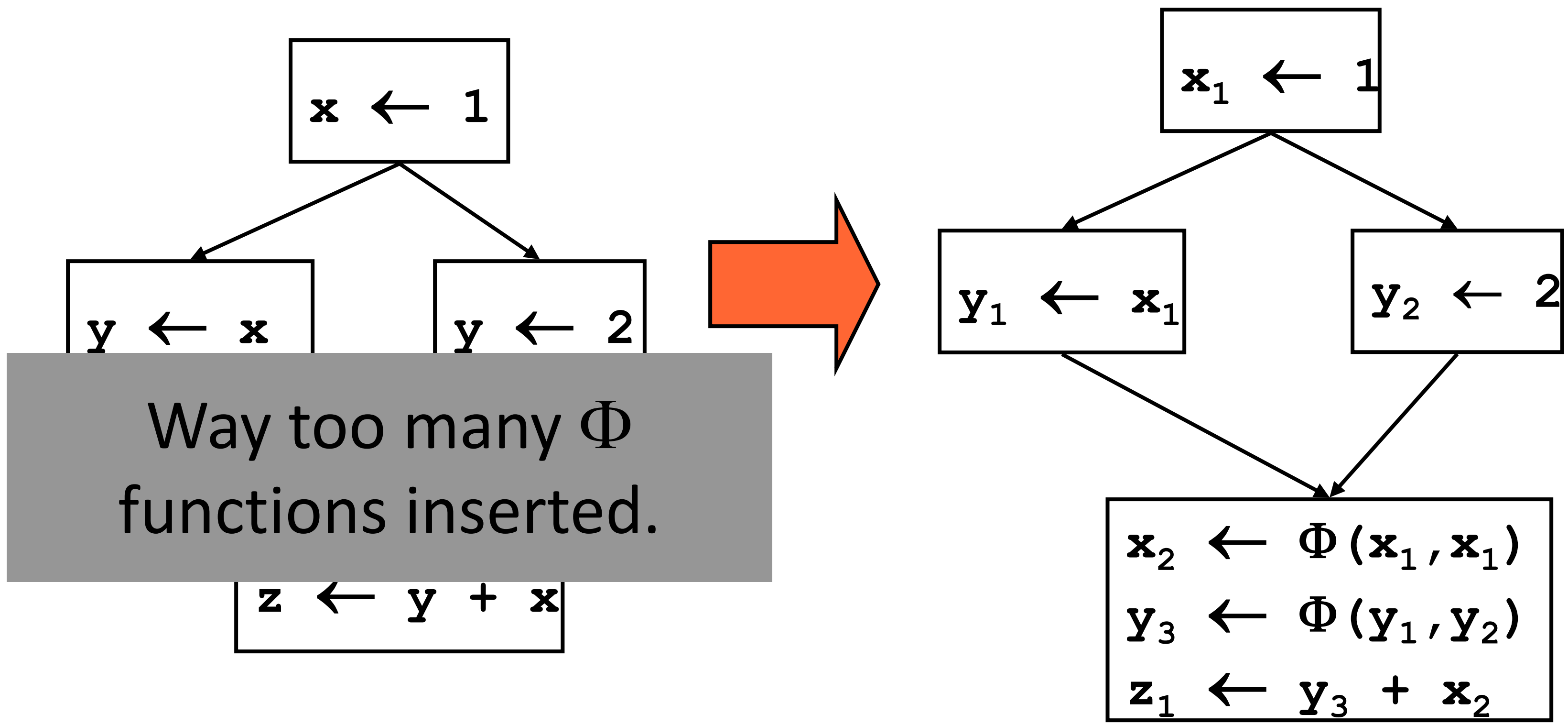
- Φ merges multiple definitions along multiple control paths into a single definition.
- At a BB with p predecessors, there are p arguments to the Φ function.

  $x_{new} \leftarrow \Phi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_p)$

- How do we choose which x_i to use?
  - We don't really care!
  - If we care, use moves on each incoming edge

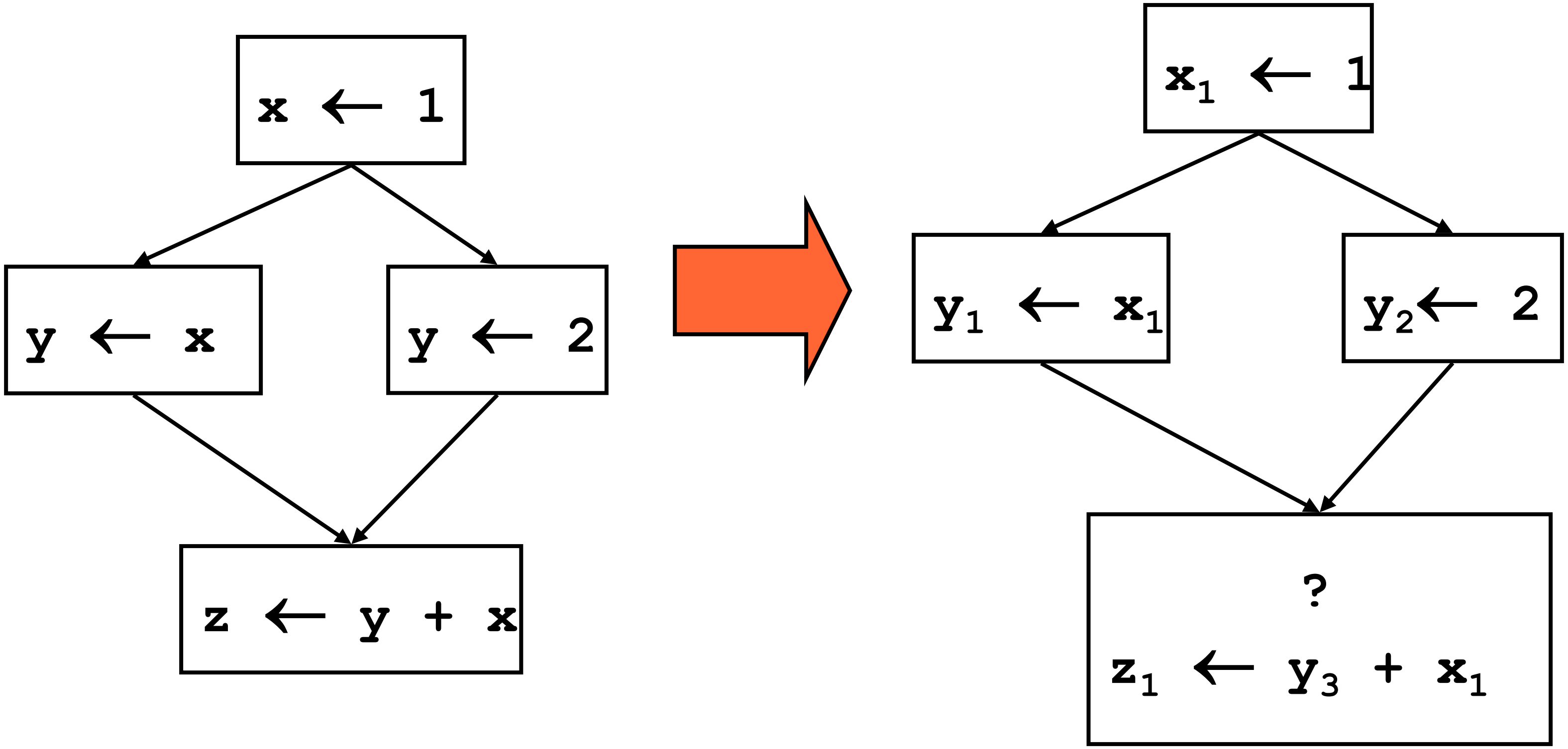
Trivial SSA

- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables.



Minimal SSA

- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all variables with multiple outstanding defs.



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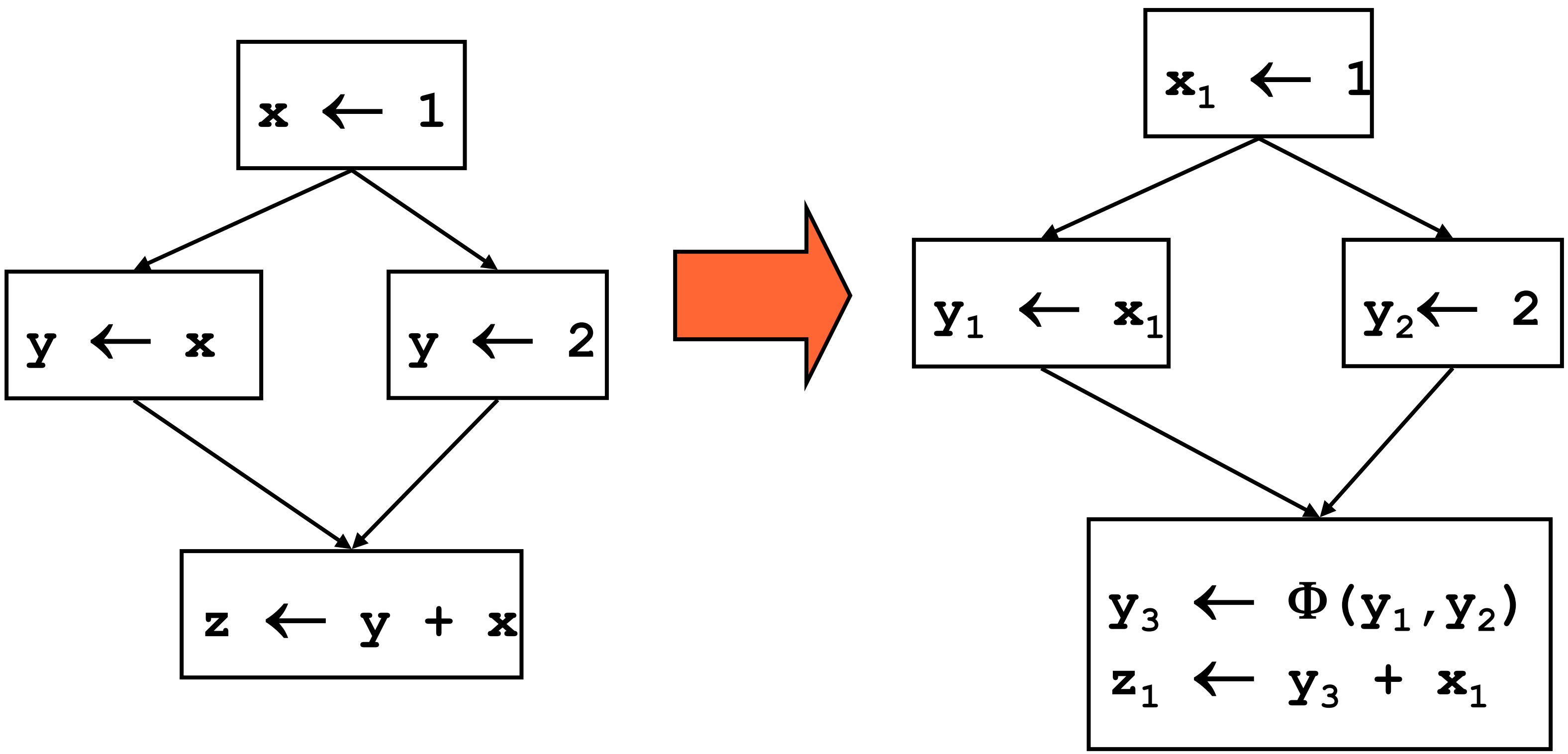
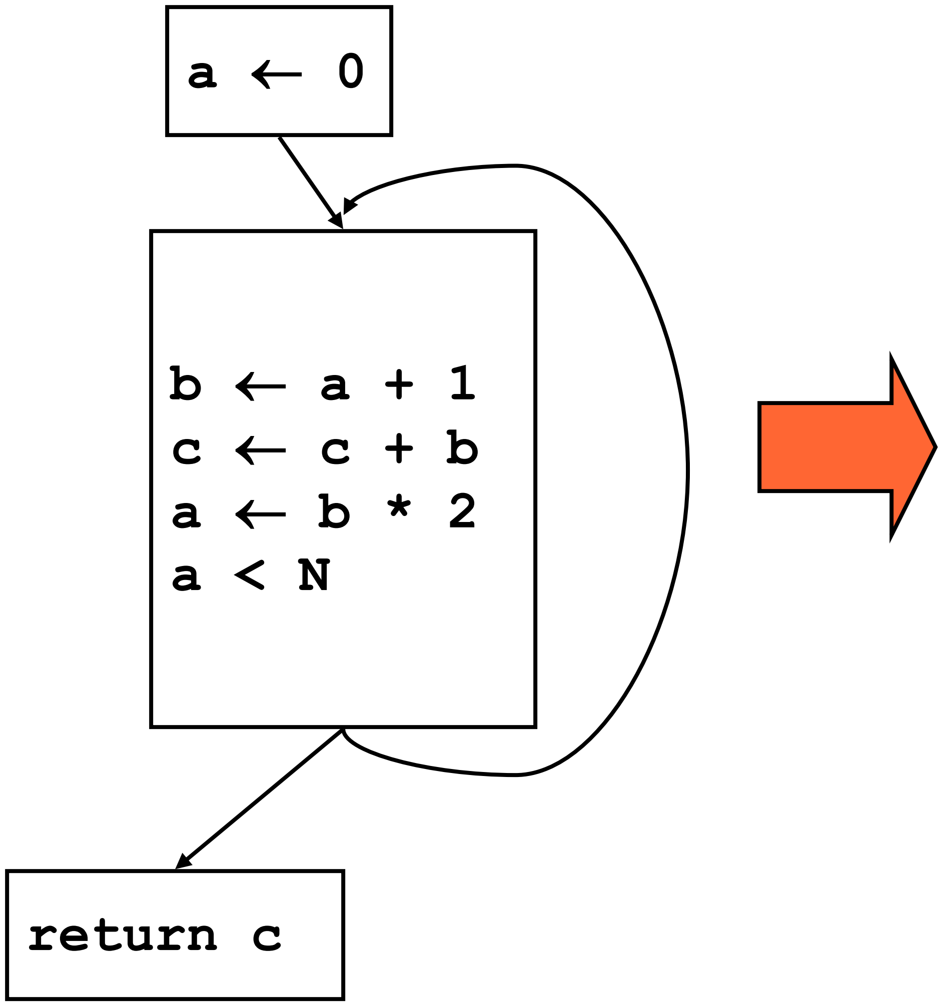


Another Example



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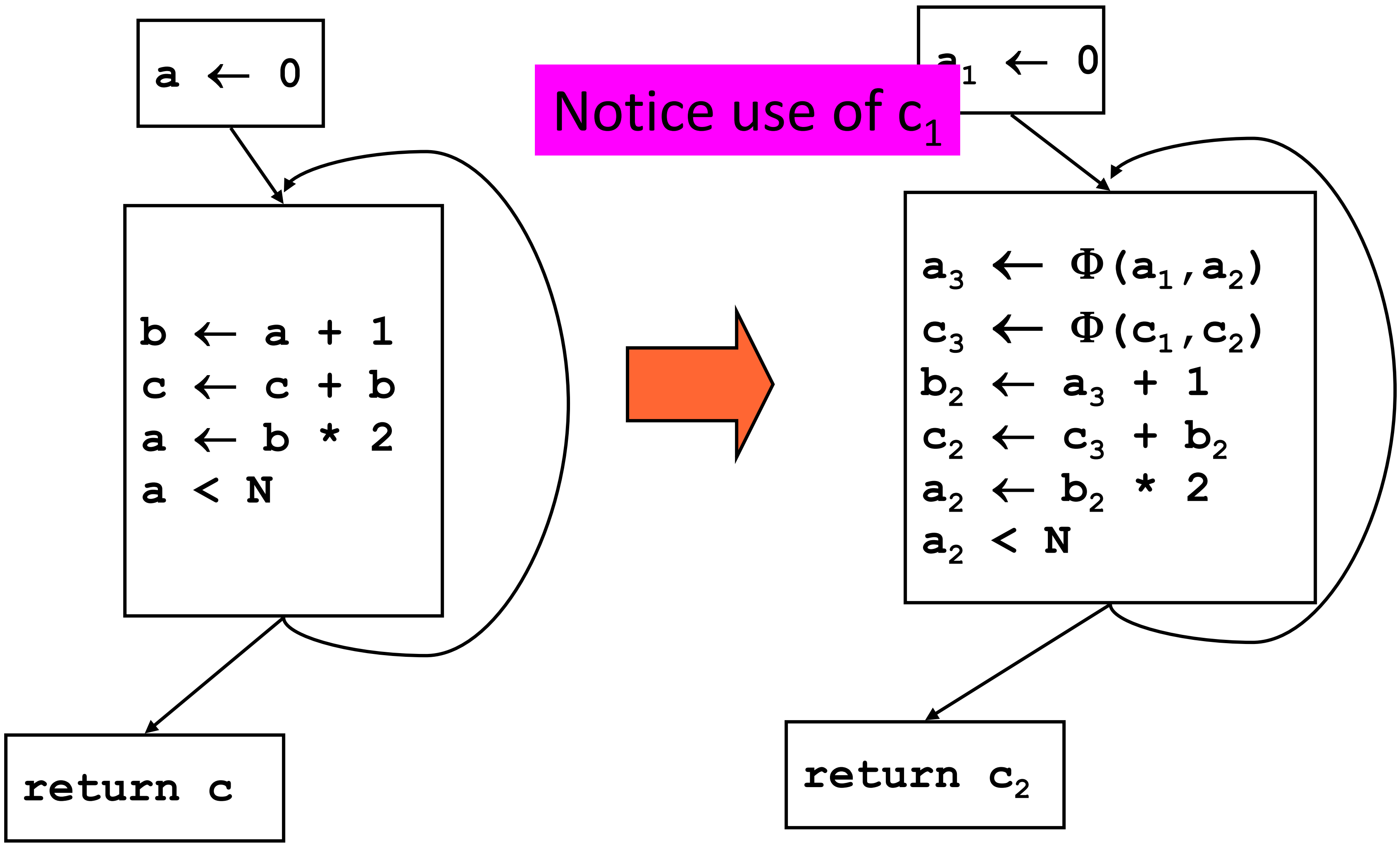
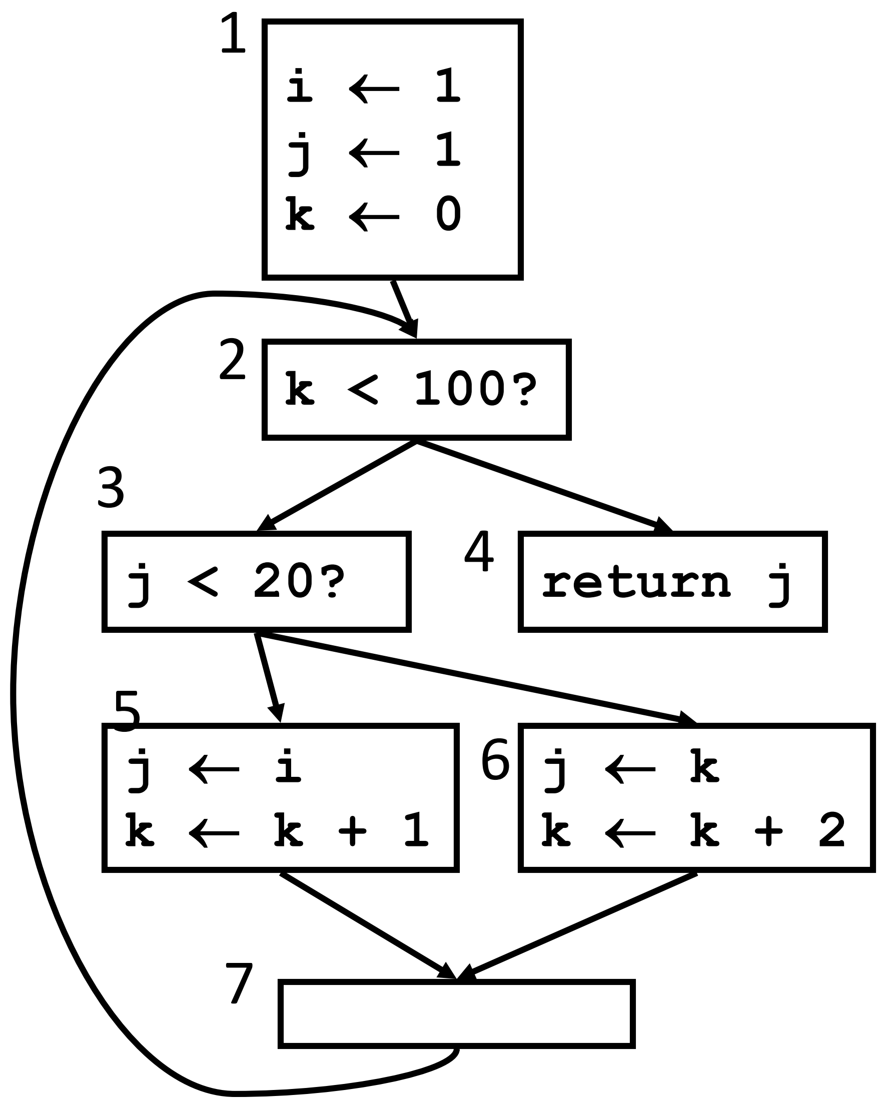
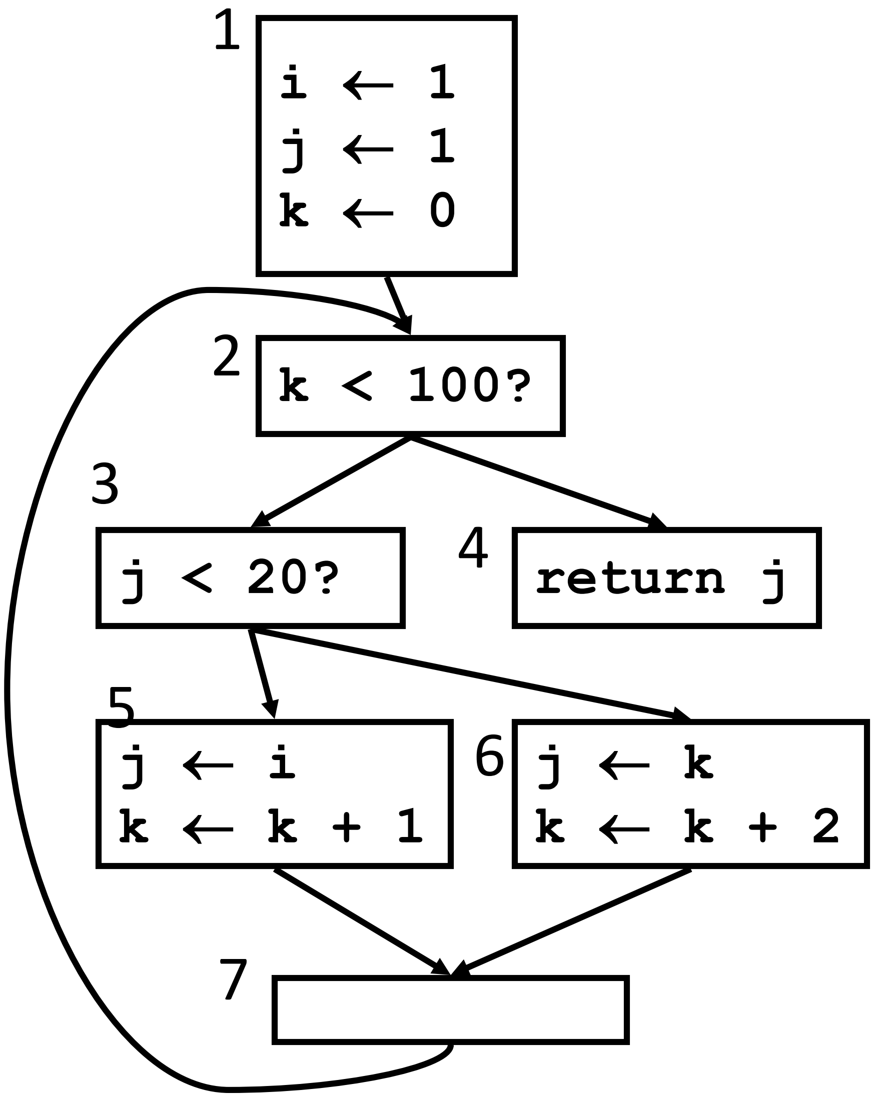


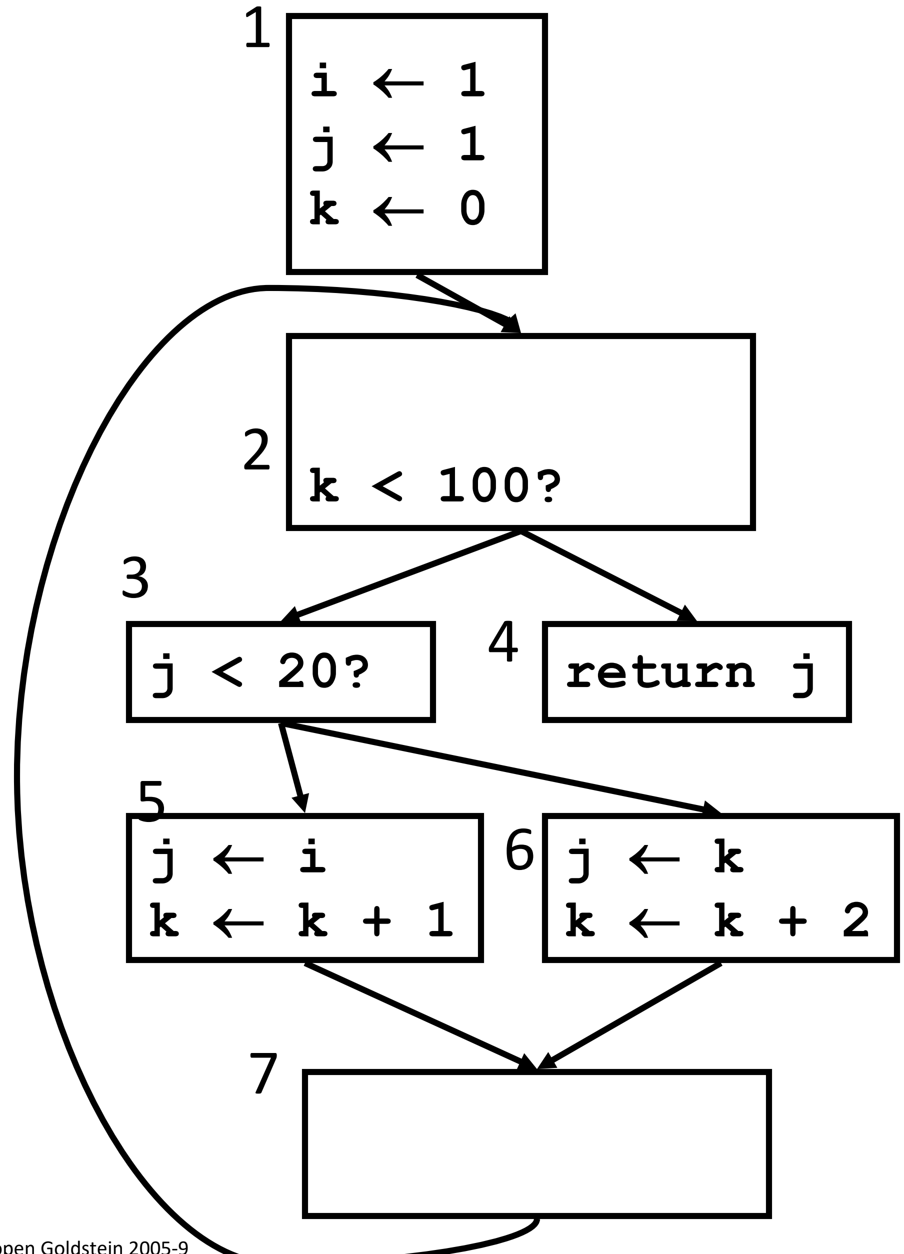
Lets optimize the following:

```
i=1;
j=1;
k=0;
while (k<100) {
  if (j<20) {
      j=i;
      k++;
  } else {
      j=k;
      k+=2;
  }
}
return j;
```

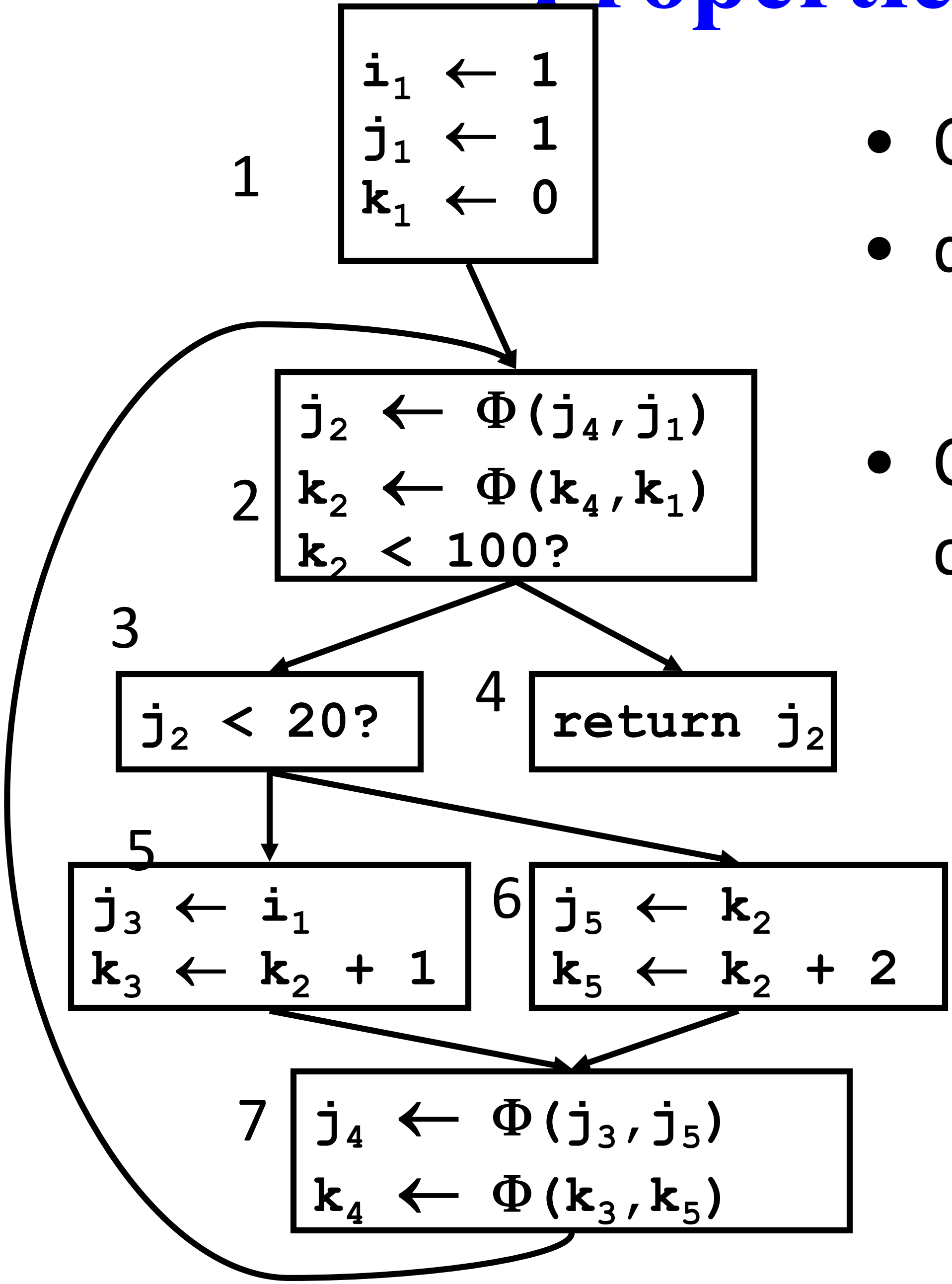


First, turn into SSA





Properties of SSA



- Only 1 assignment per variable
- definitions dominate uses

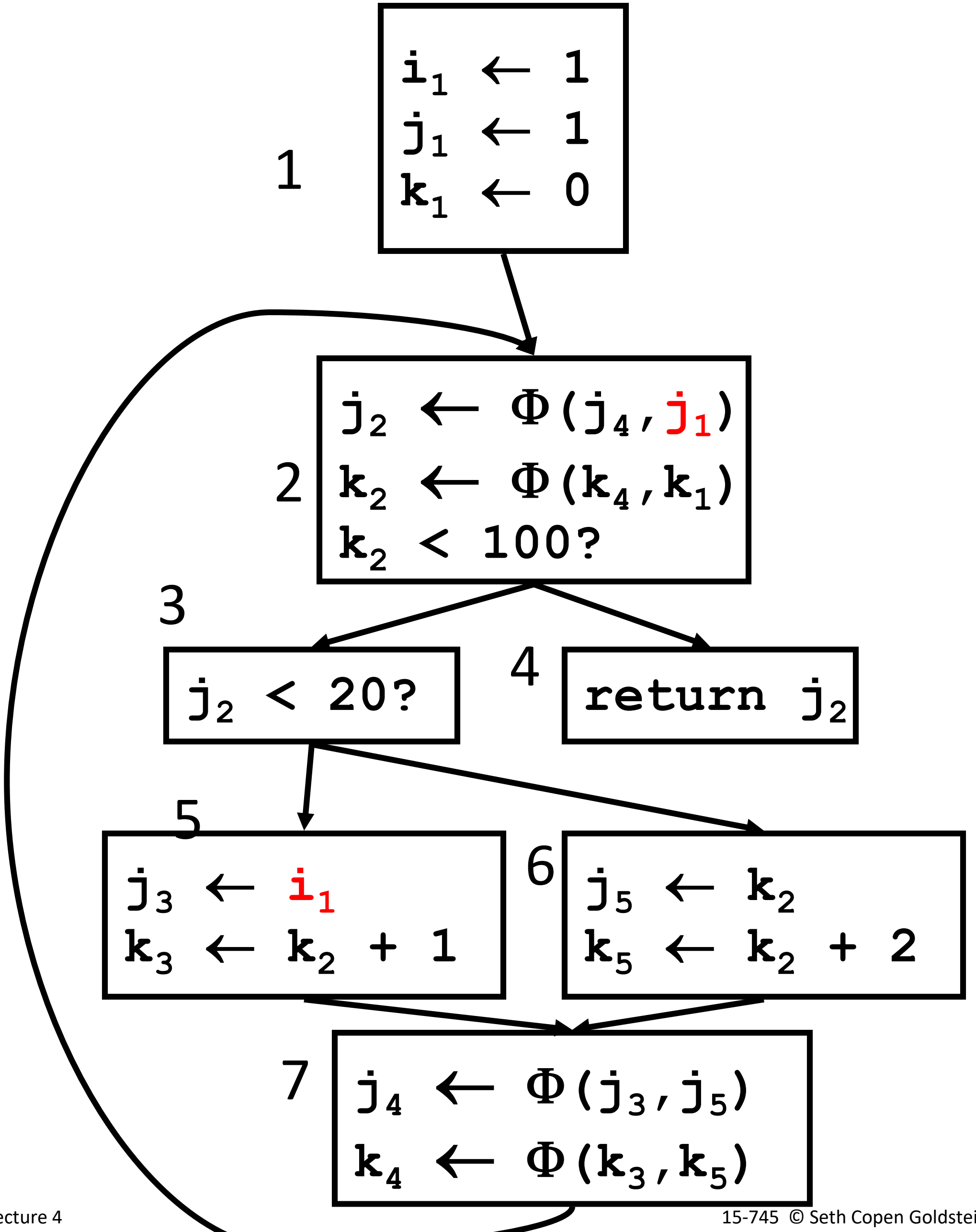
- Can we use this to help with constant propagation?

Constant Propagation

- If "v ← c", replace all uses of v with c
- If "v ← `Φ(c,c,c)`" replace all uses of v with c

```
W <- list of all defs
while !W.isEmpty {
        Stmt S <- W.removeOne
        if S has form "v <- Φ(c,…,c)"
          replace S with V <- c
        if S has form "v <- c" then
          delete S
          foreach stmt U that uses v,
            replace v with c in U
            W.add(U)
}
```

Constant Propagation



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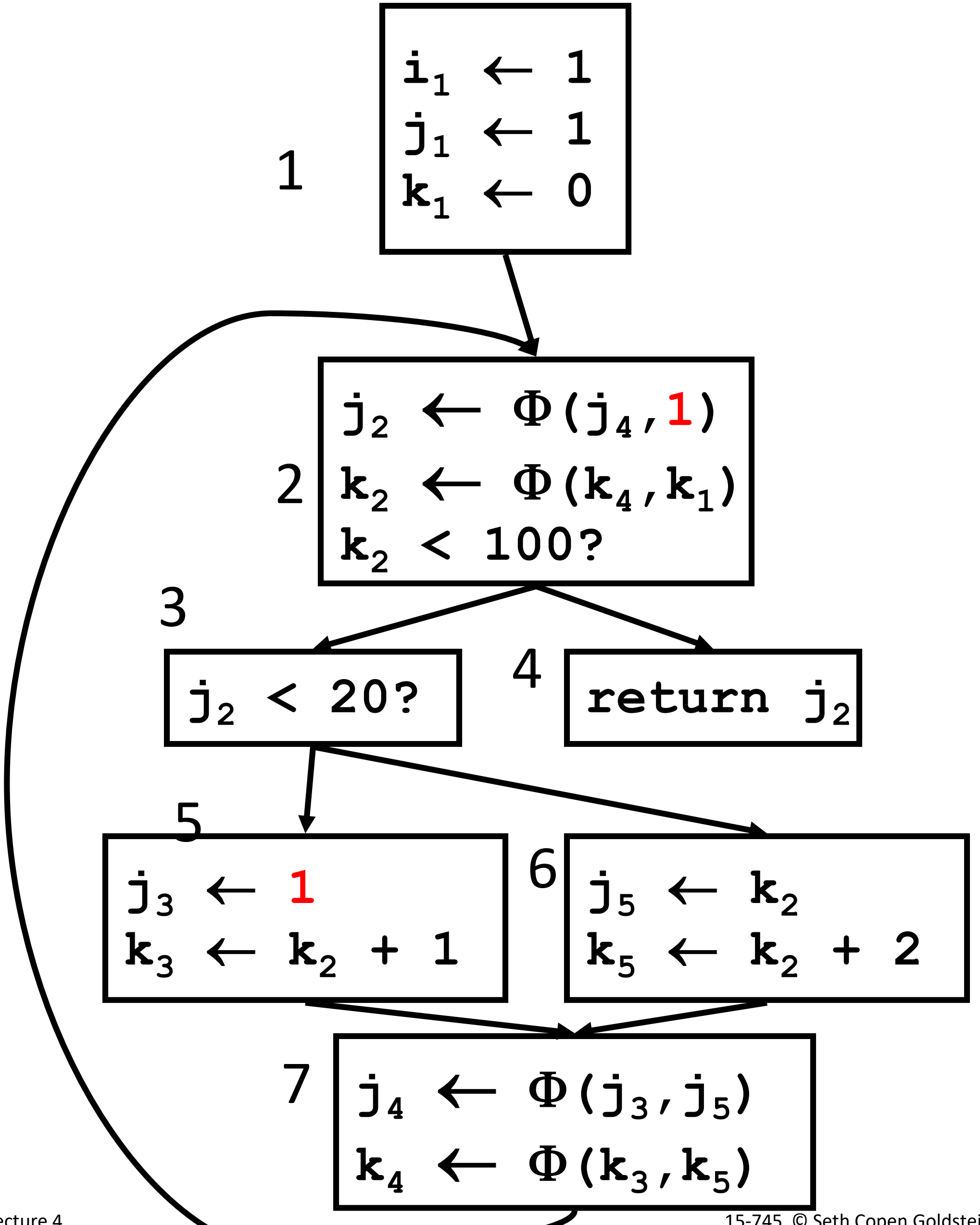

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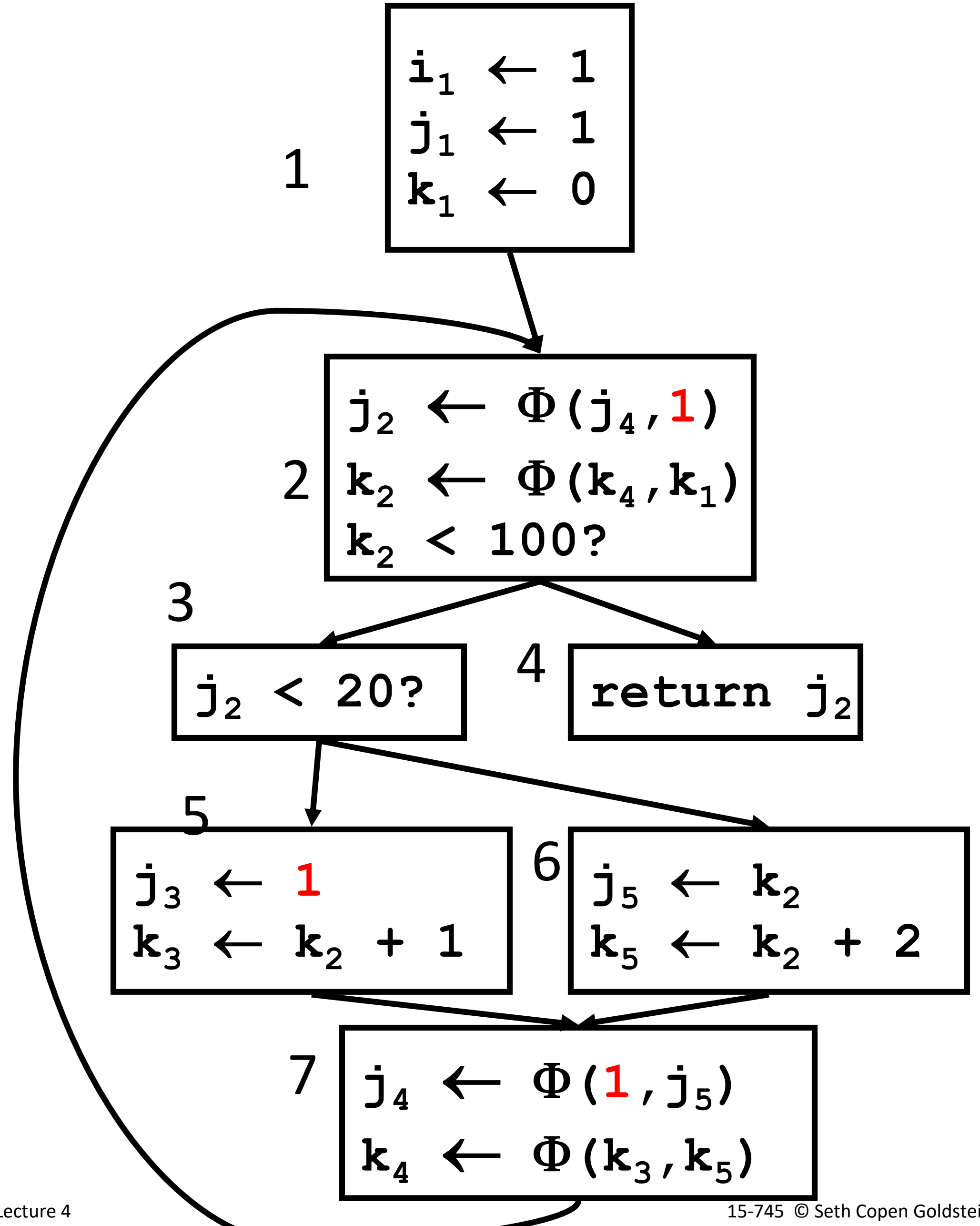

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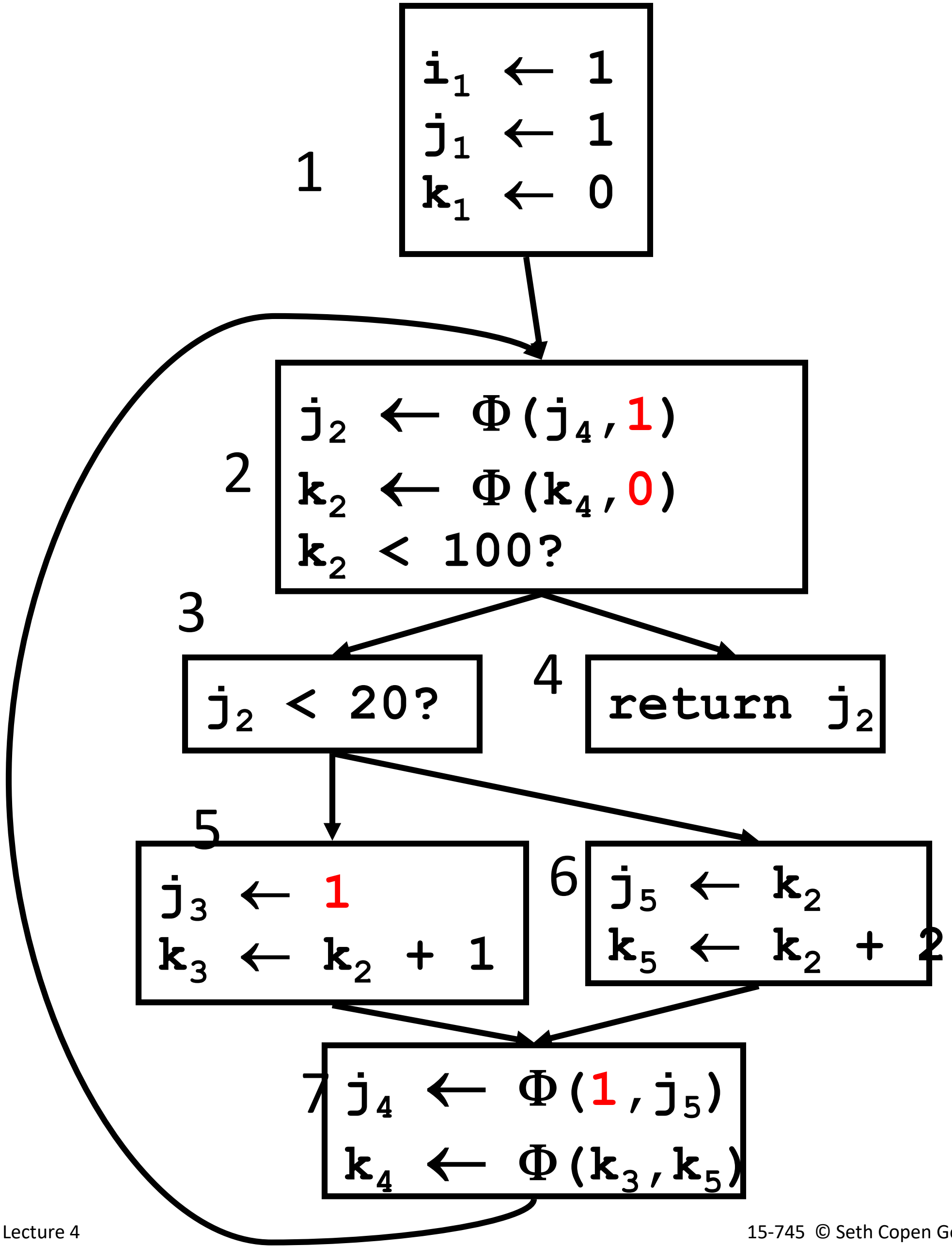
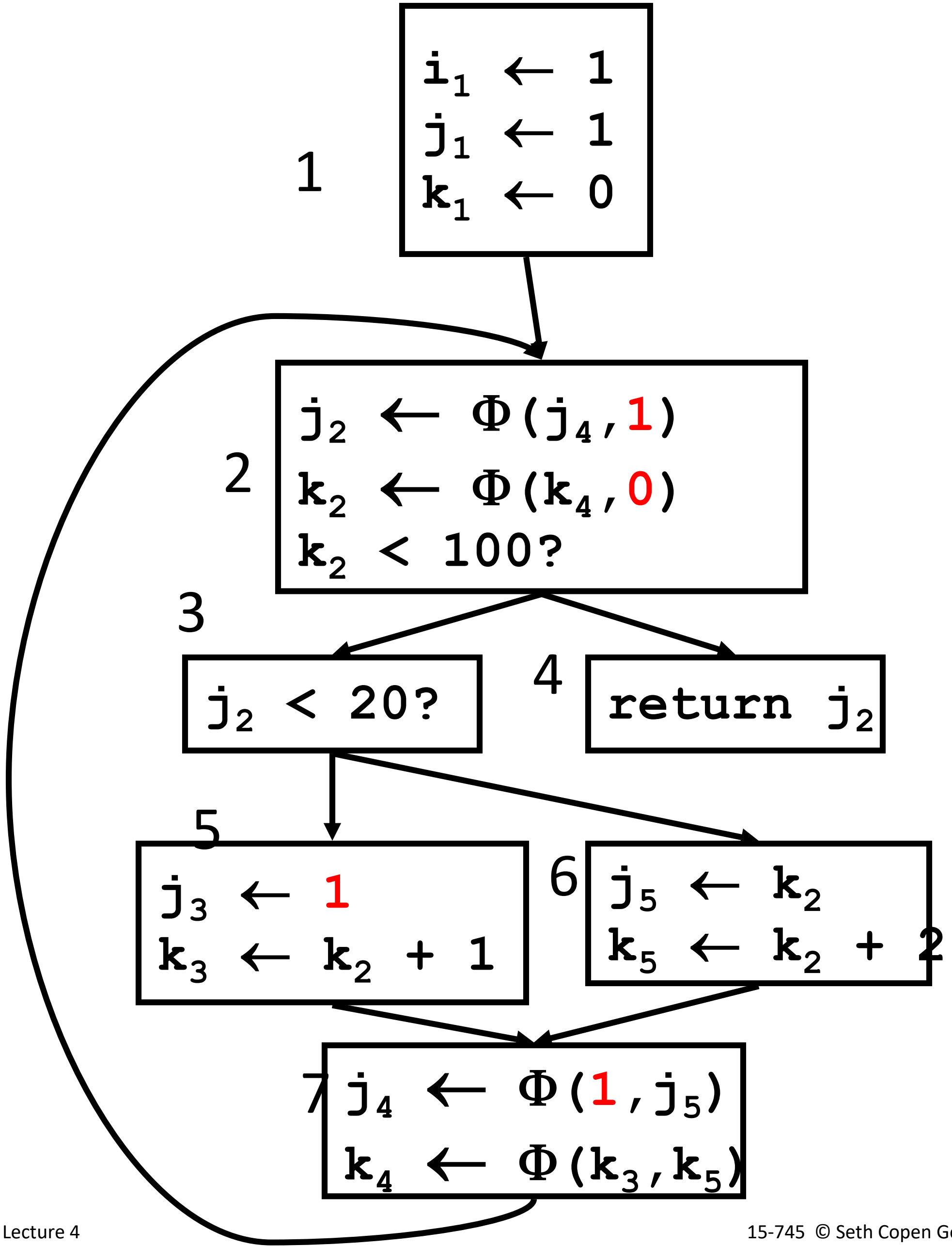


But, so what?

Other stuff we can do?

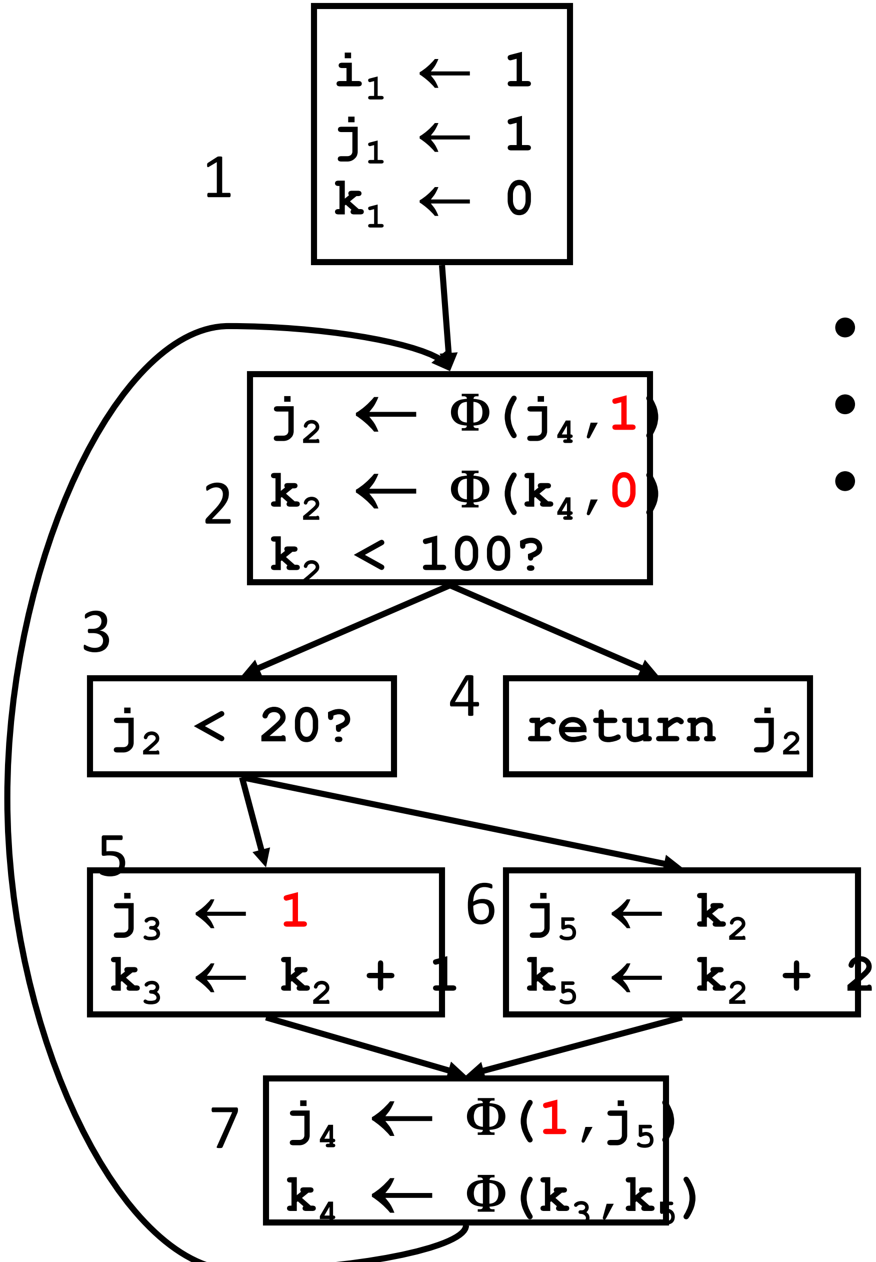
- Copy propagation
  delete “$x \leftarrow \Phi(y)$” and replace all x with y
  delete “$x \leftarrow y$” and replace all x with y
- Constant Folding
  (Also, constant conditions too!)
- Unreachable Code
  Remember to delete all edges from unreachable block

Constant Propagation



But, so what?

Conditional Constant Propagation



- Does block 6 ever execute?
- Simple CP can't tell
- CCP can tell:
  - Assumes blocks don't execute until proven otherwise
  - Assumes Values are constants until proven otherwise

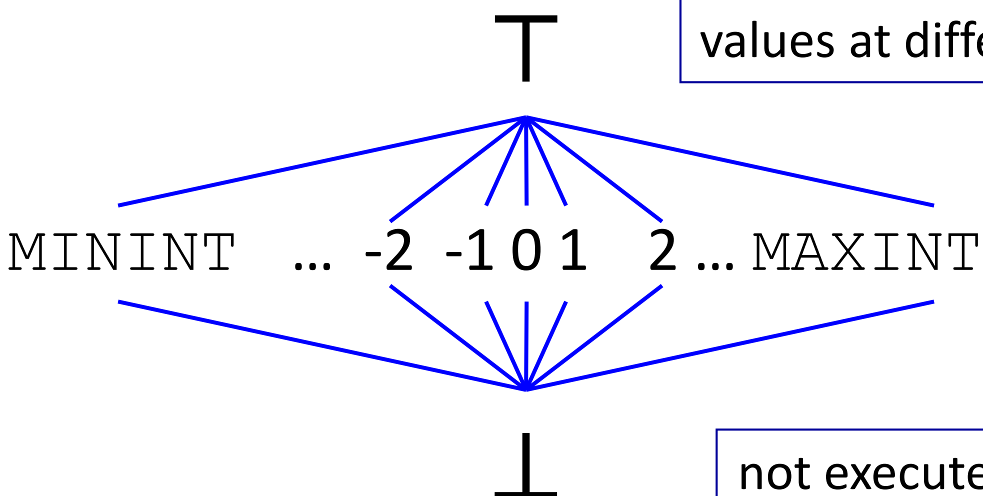
CCP data structures & lattice

Keep track of:

- Blocks (assume unexecuted until proven otherwise)

-Variables (assume not executed, only with proof of assignments of a non-constant value do we assume not constant)

Use a lattice for variables:

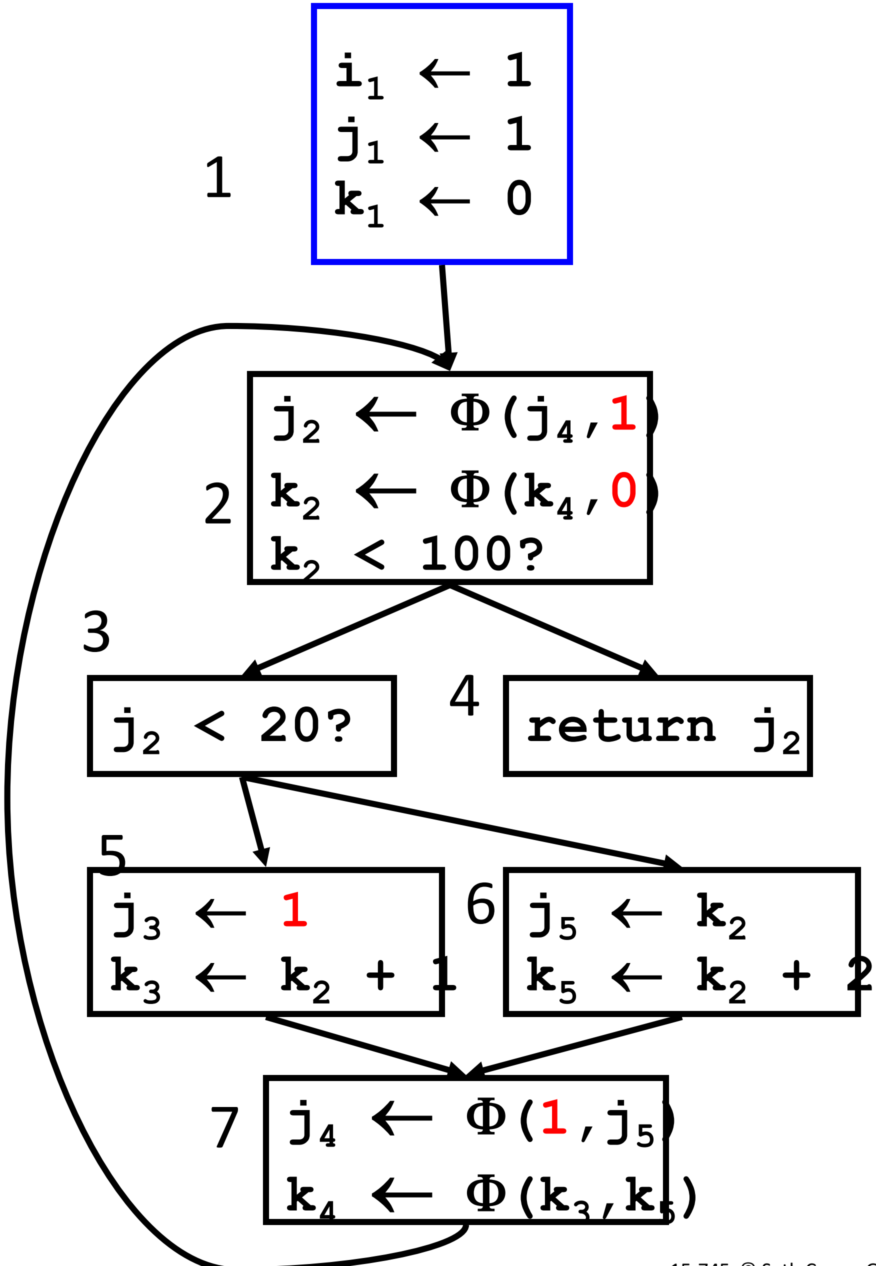


have evidence that variable can hold different values at different times

evidence that the var has been assigned a constant

not executed

Conditional Constant Propagation



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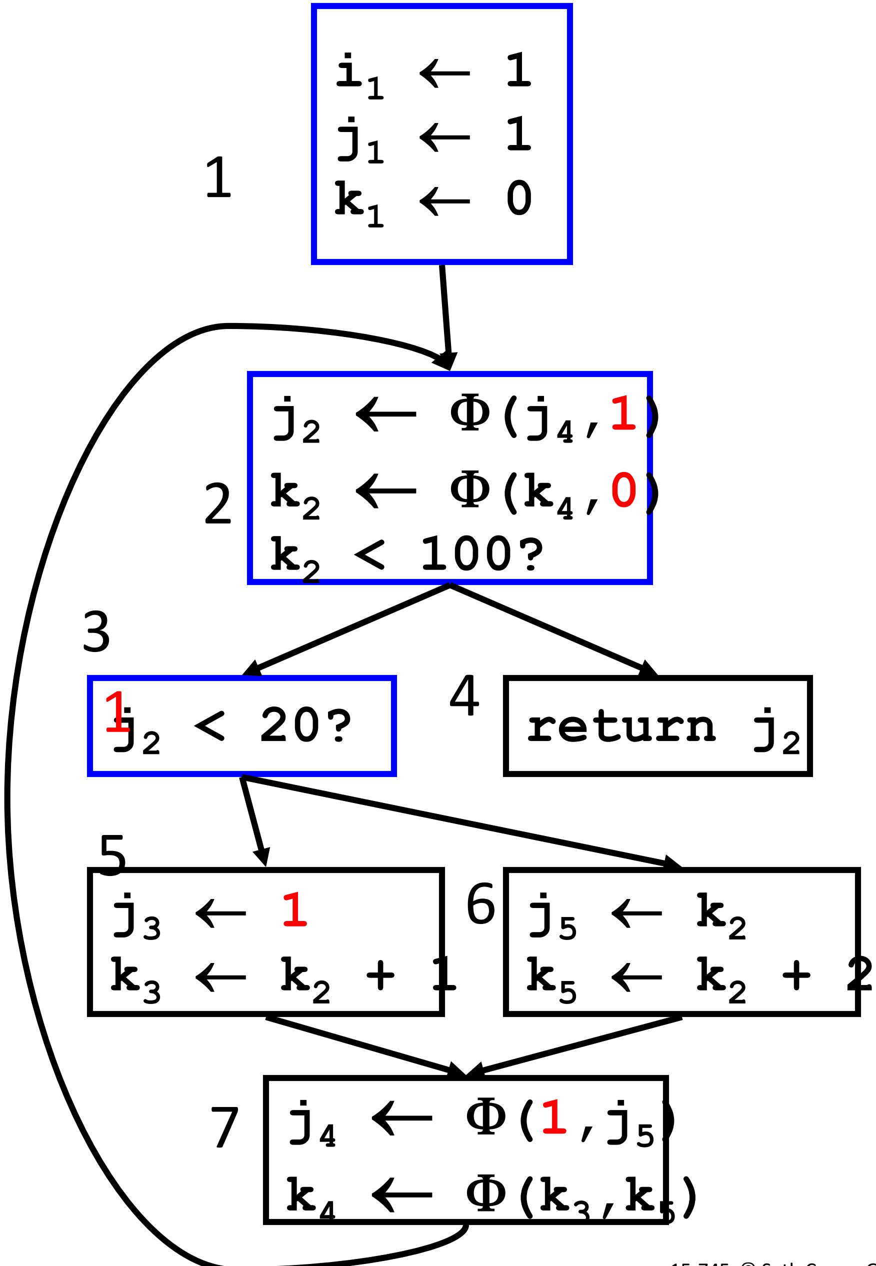

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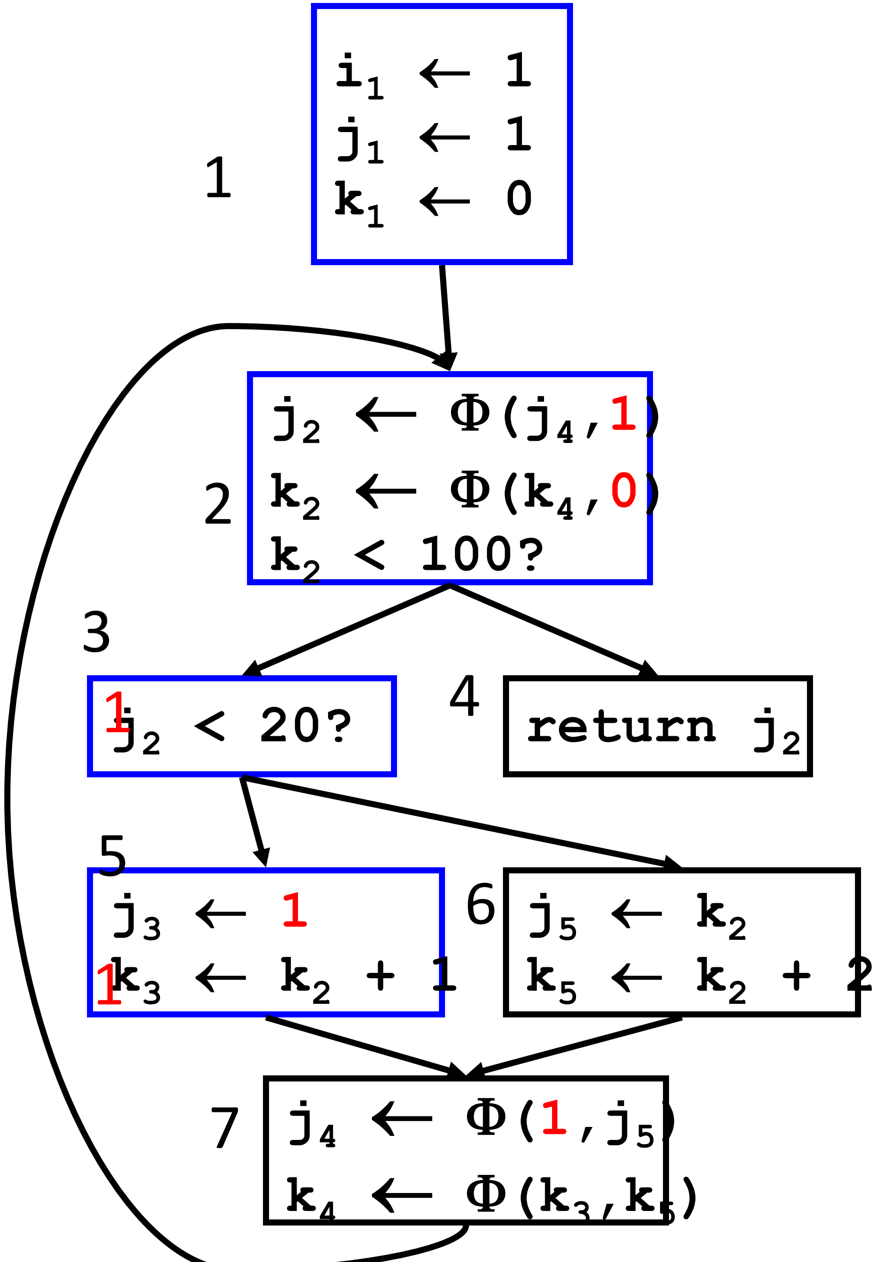

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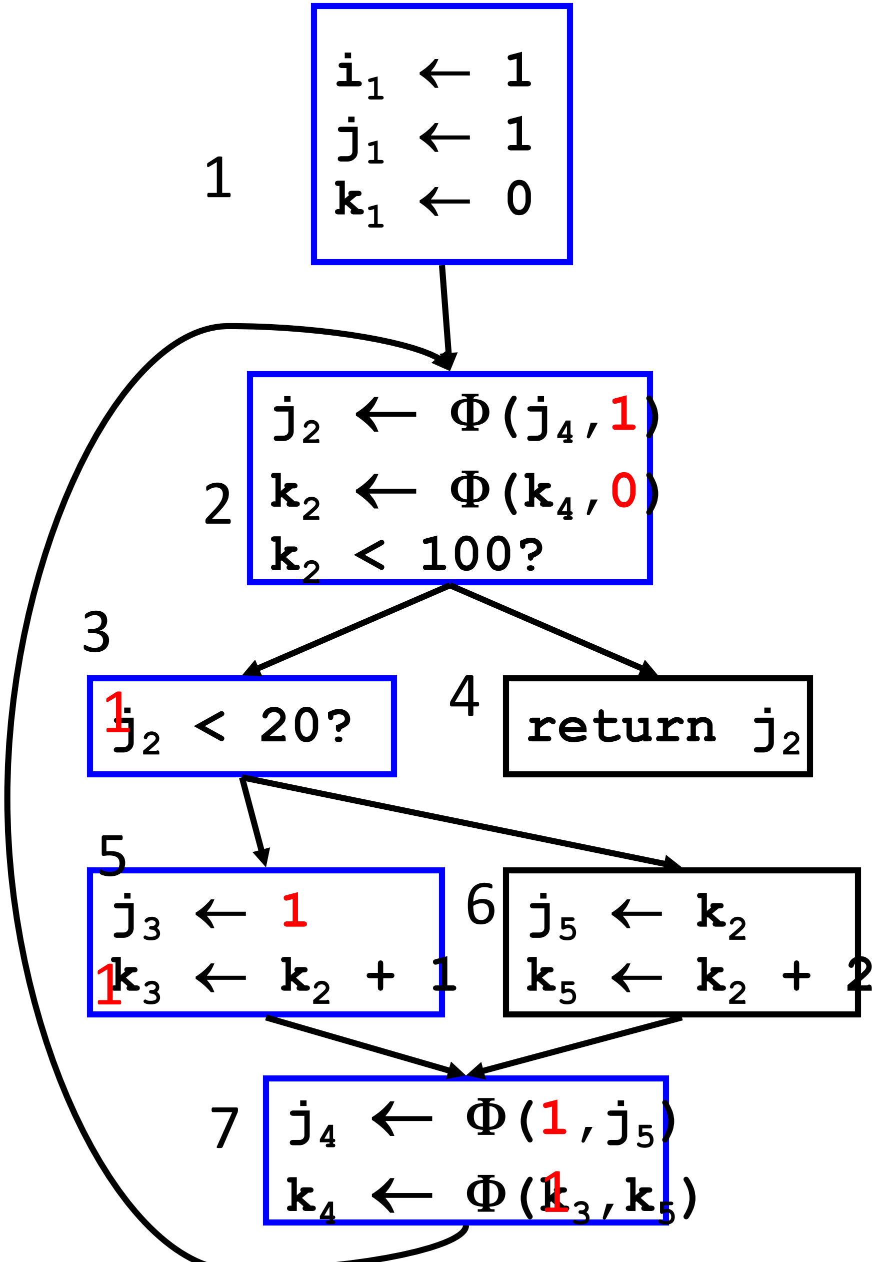

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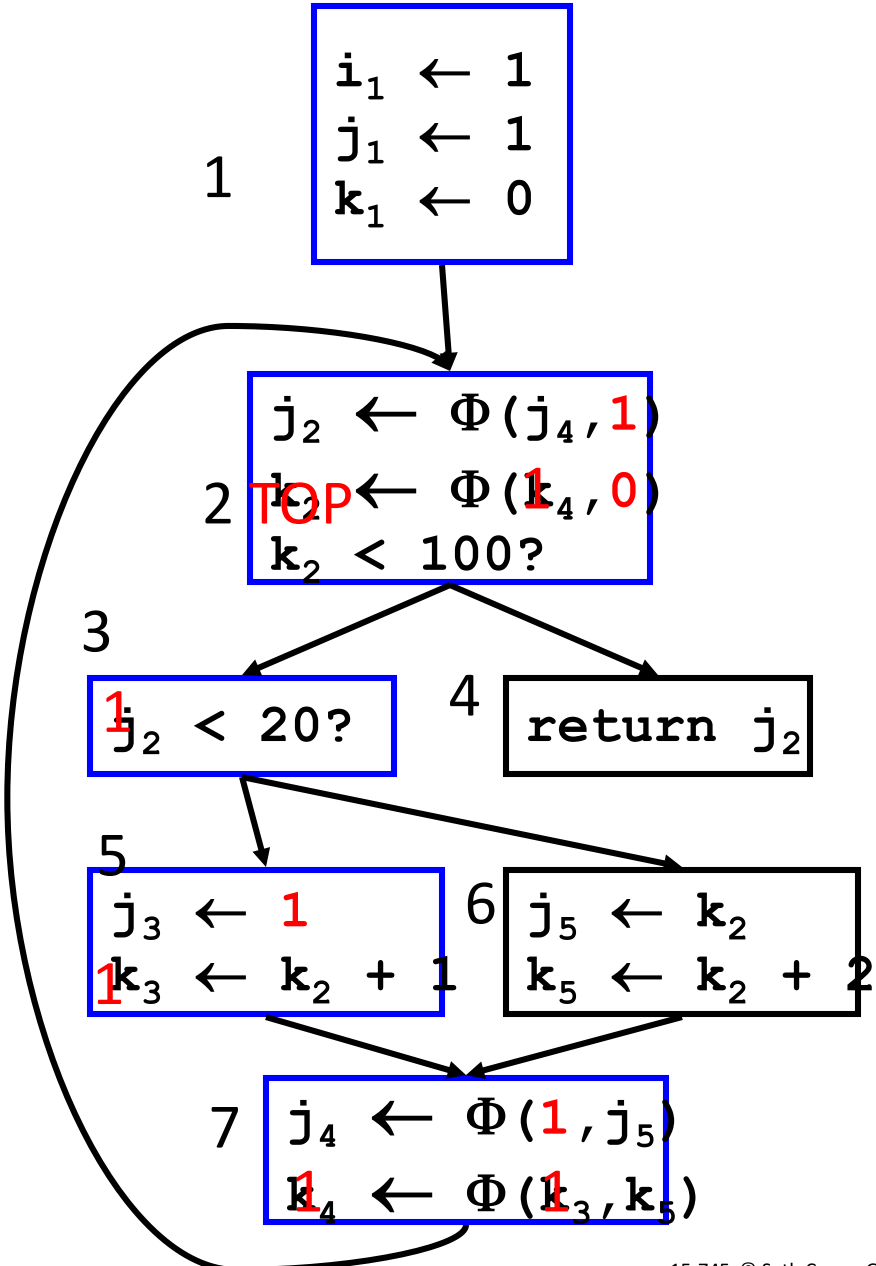

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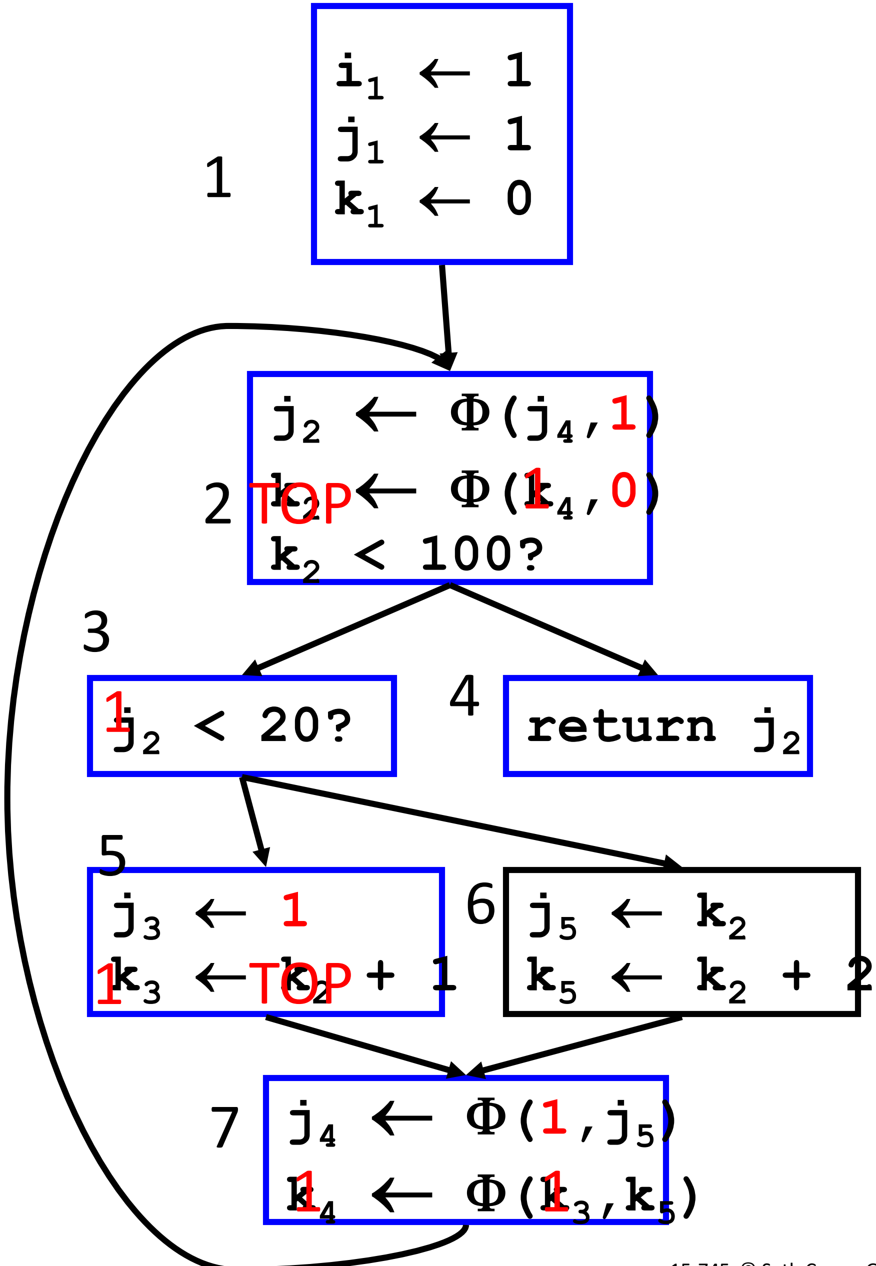

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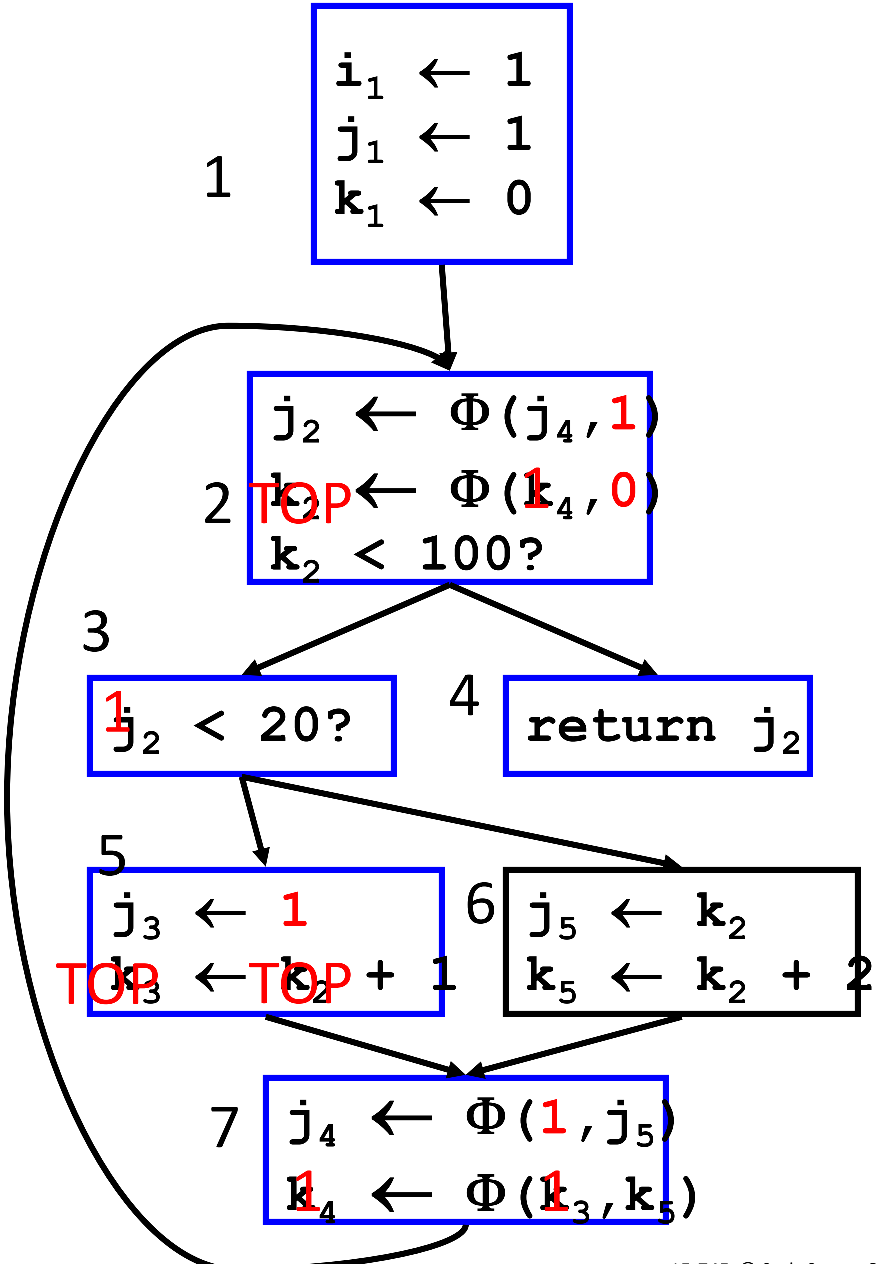

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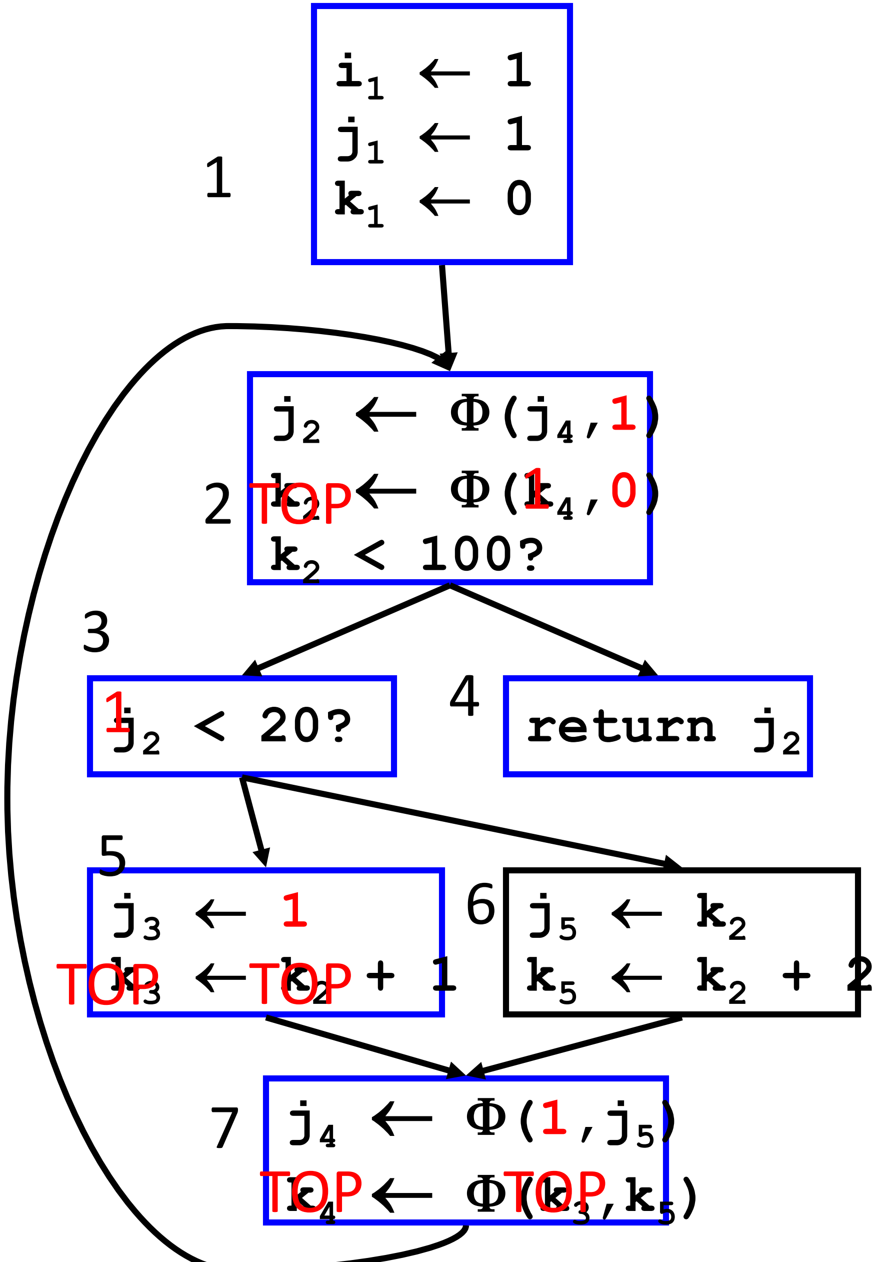

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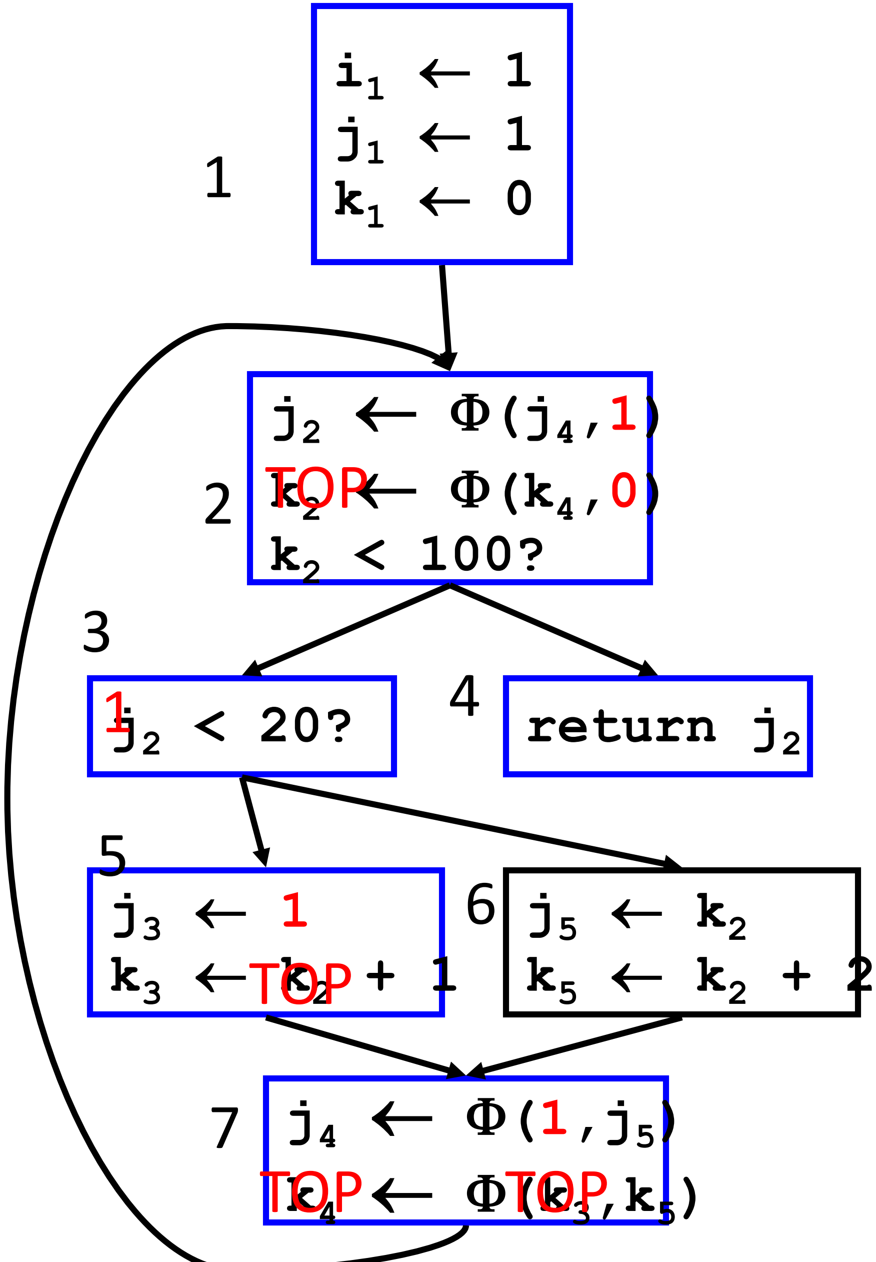

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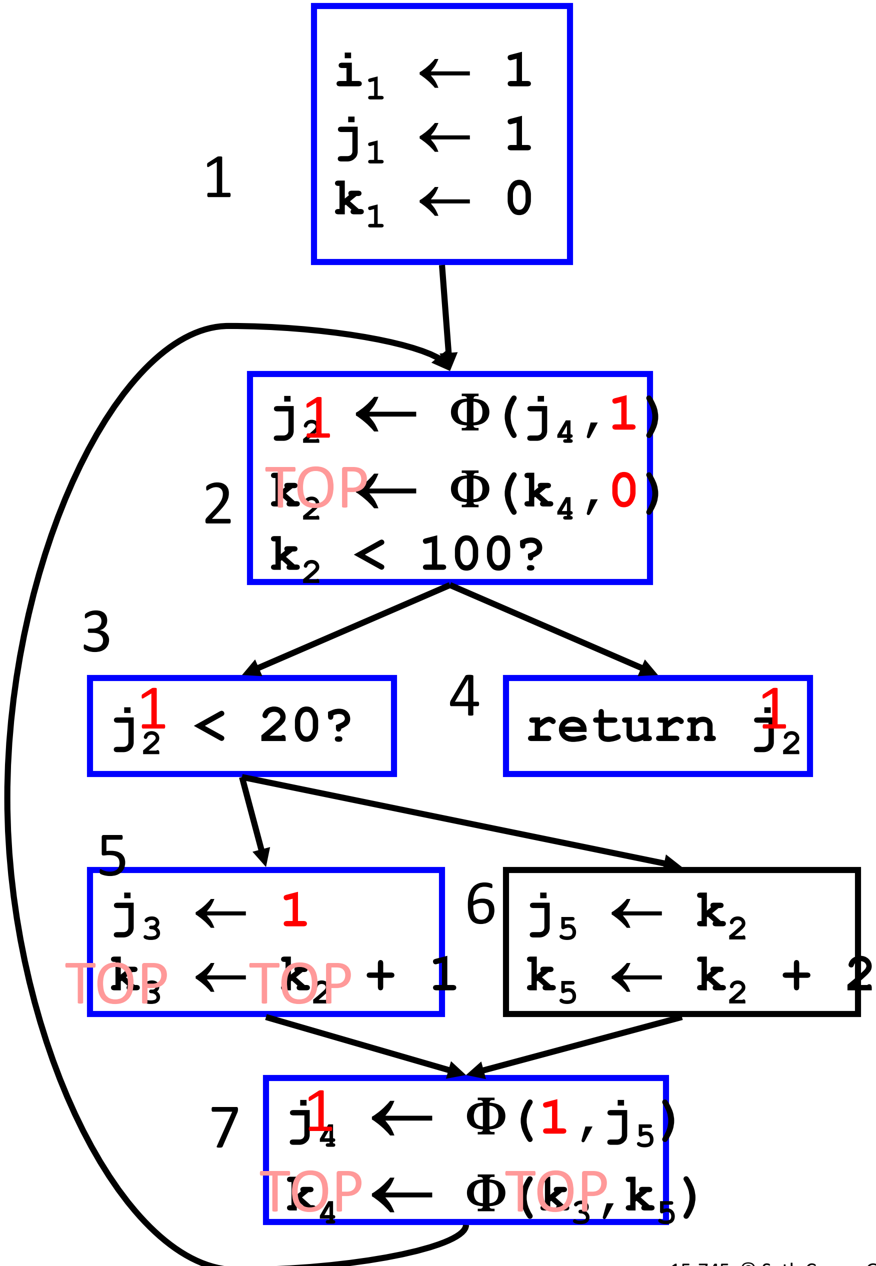
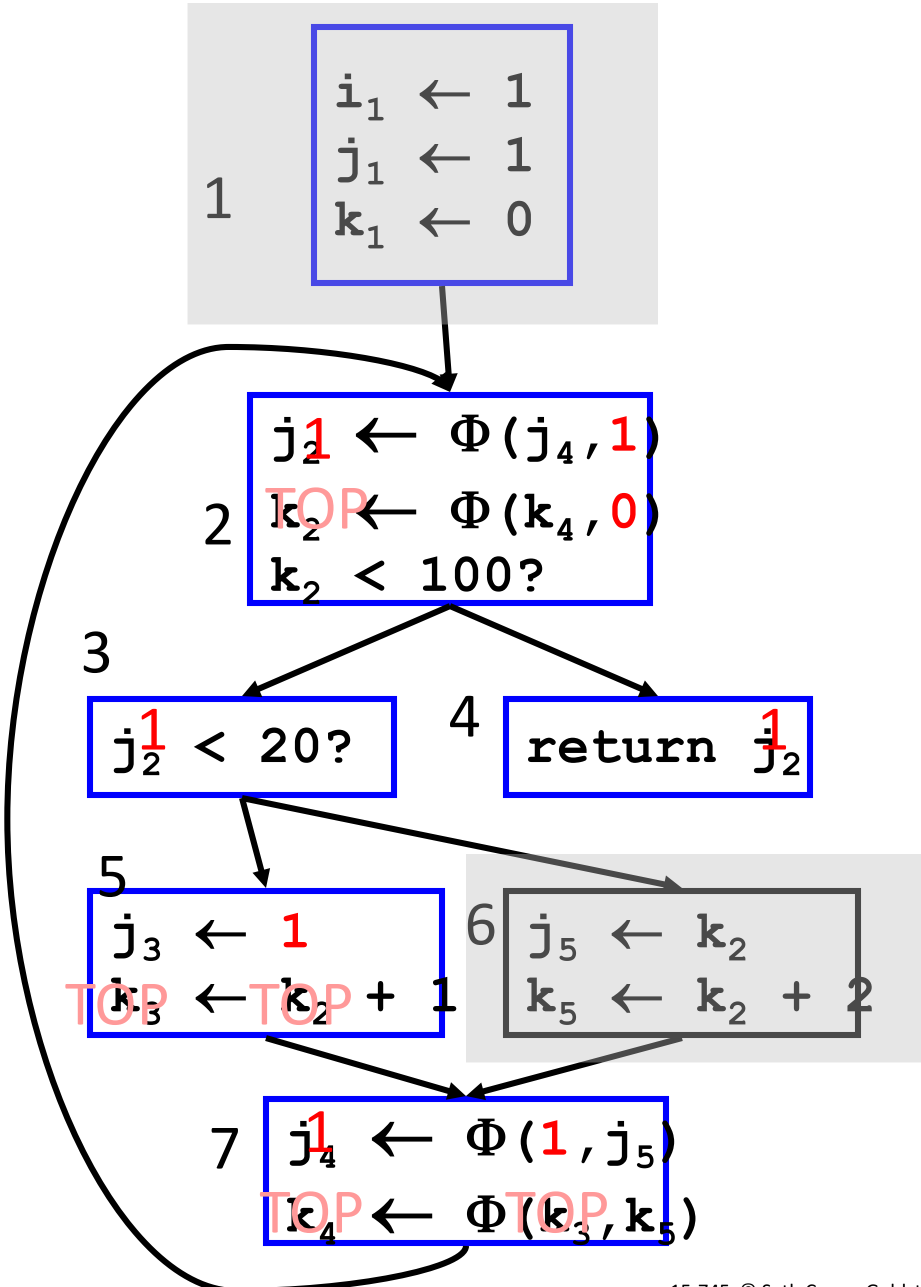


Now What?

Conditional Constant Propagation



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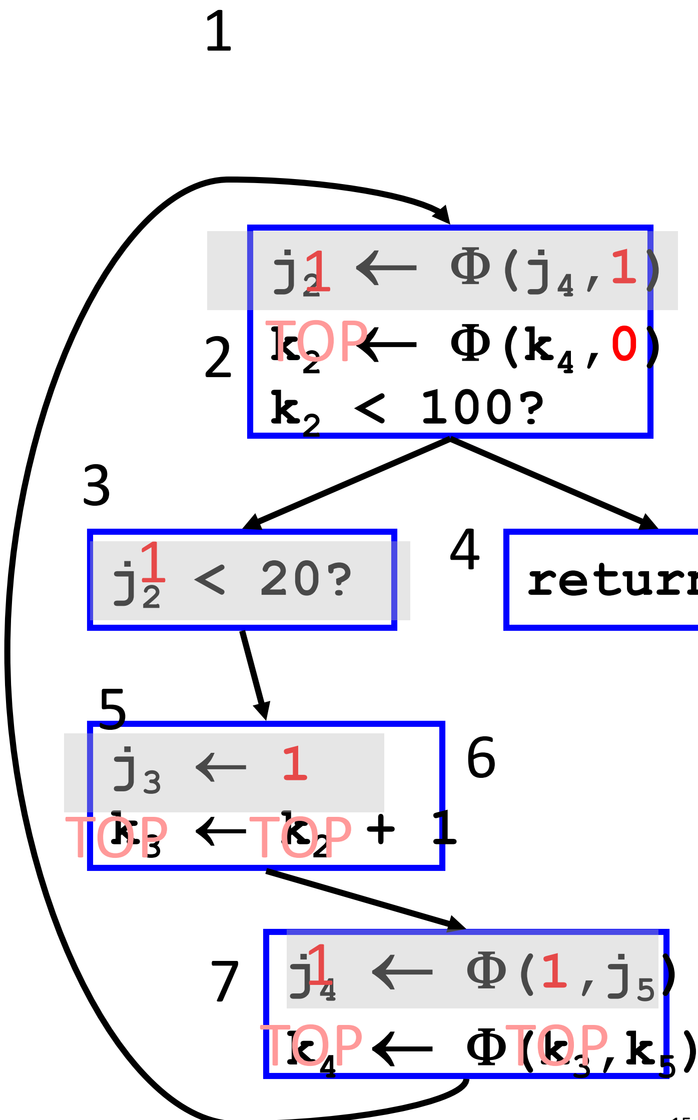

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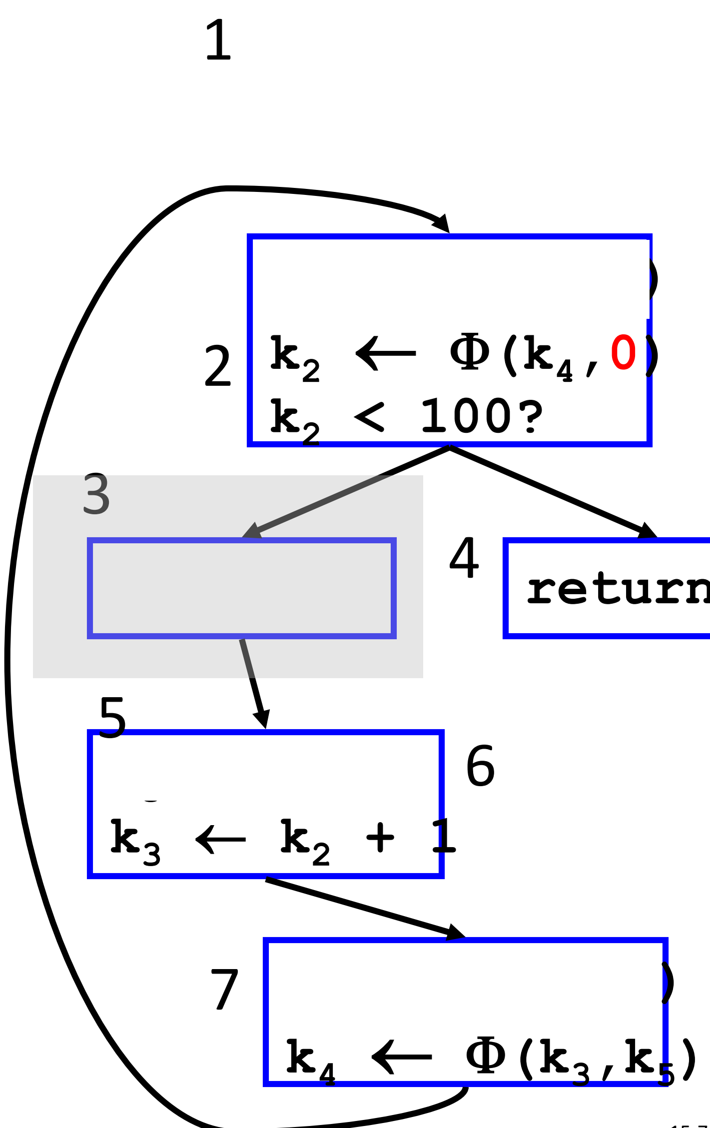
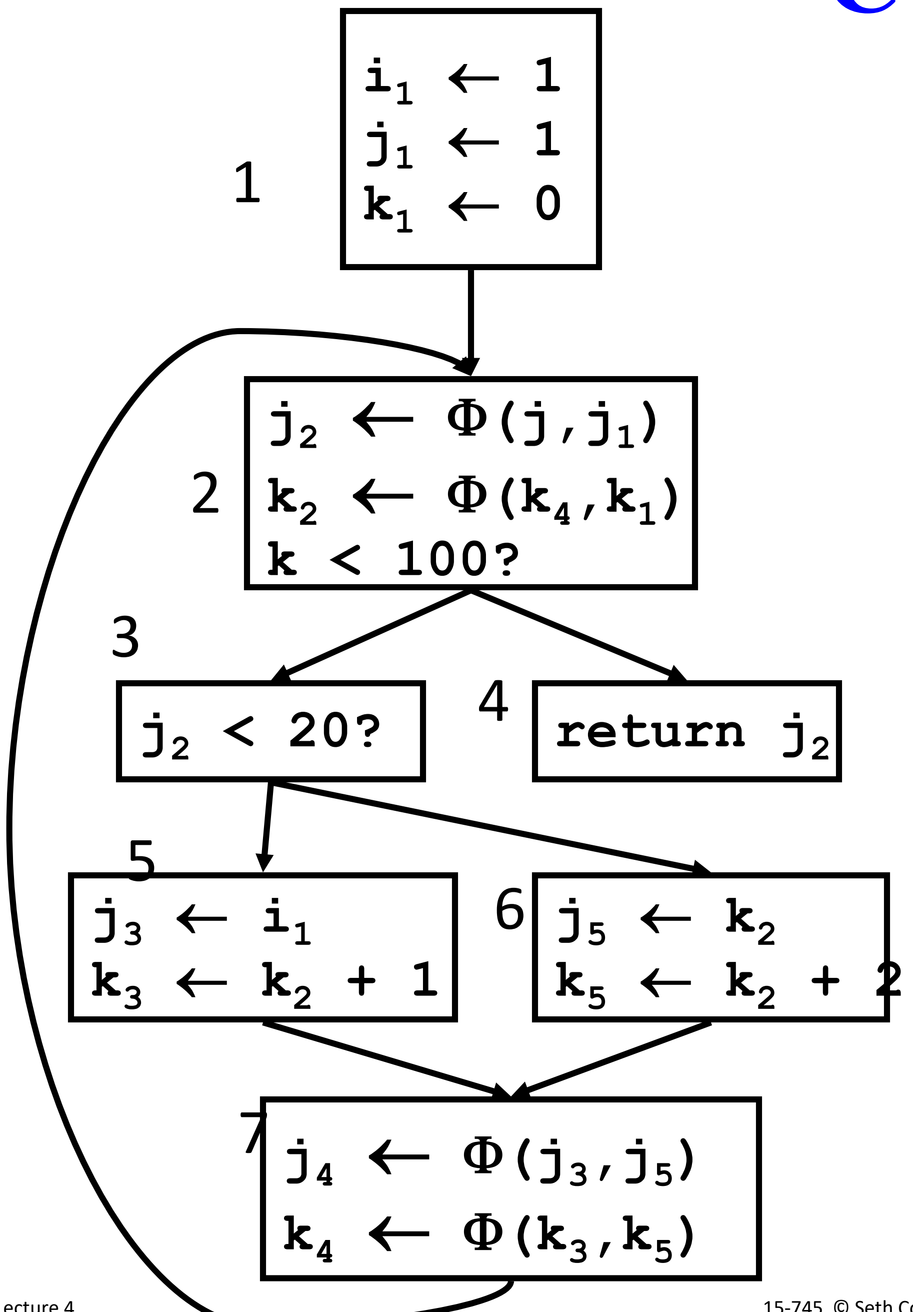
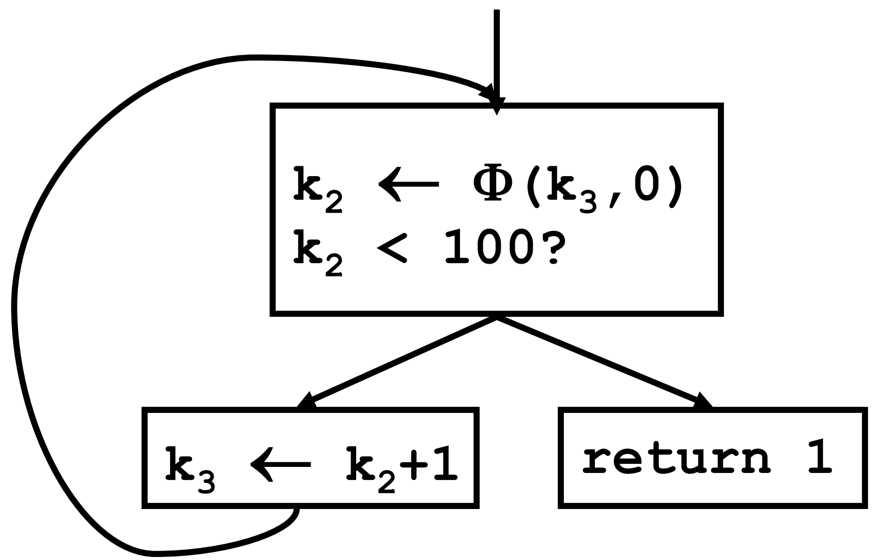


Now What?

CCP





Just a taste of the kinds of optimizations that SSA enables.

Minimal SSA

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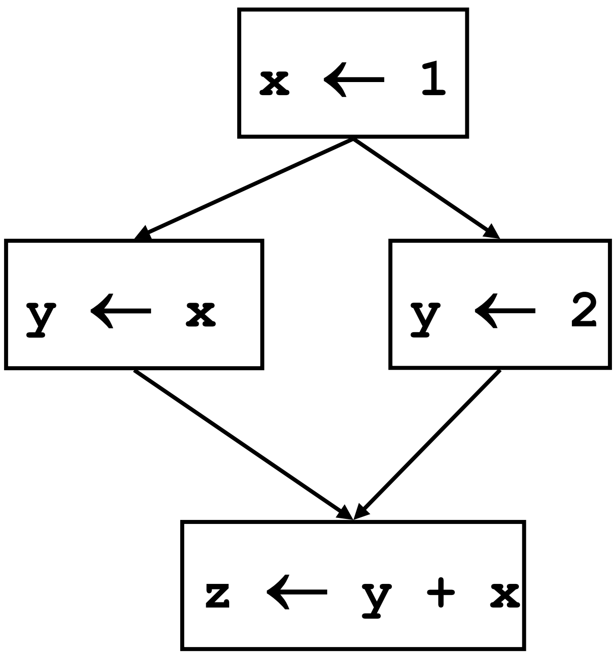
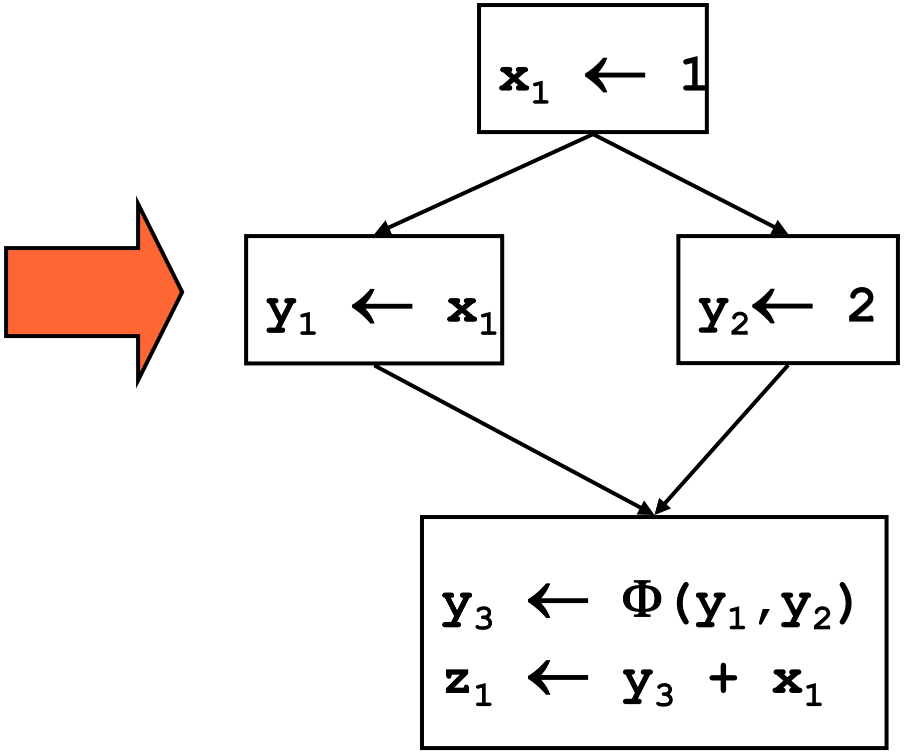
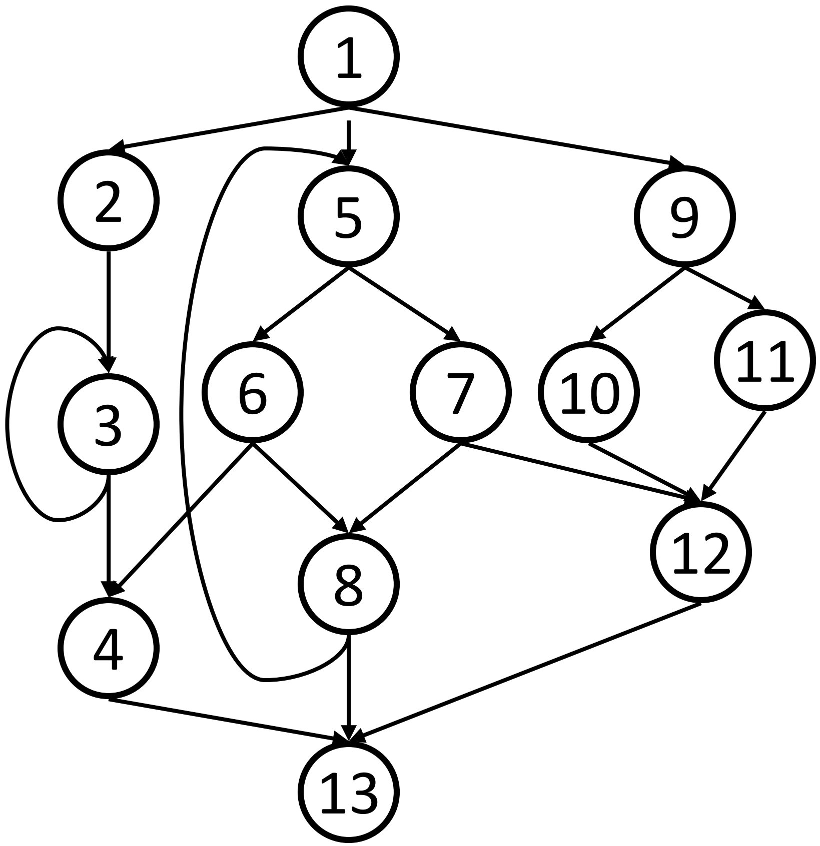




When do we insert Φ?



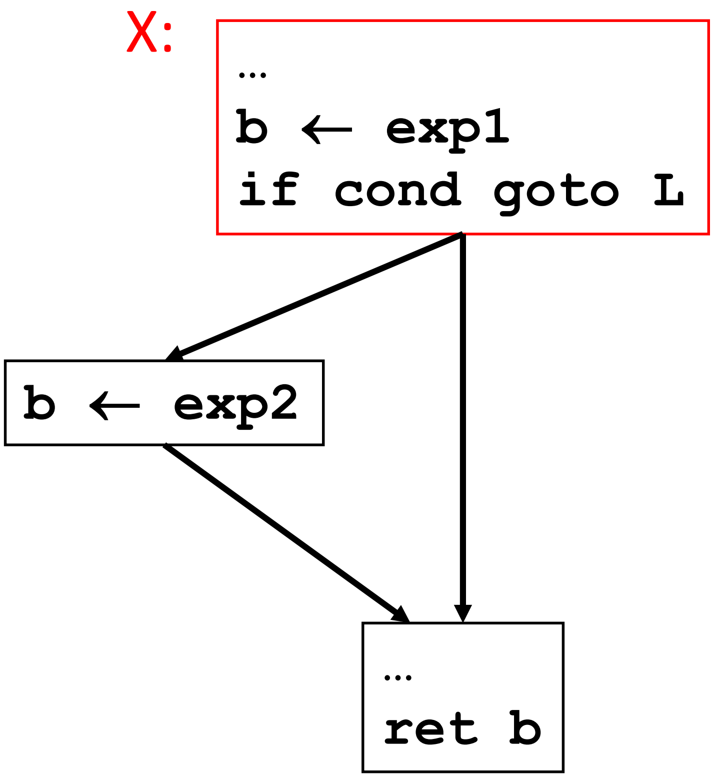
CFG

If there is a def of **a** in block 5, which nodes need a $\Phi()$?

When do we insert Φ?

There should be a Φ-function for variable b at node z of the flow graph exactly when all of the following are true:

- There is a block x containing a def of b
- There is a block y (with y ≠ x) containing a def of b
- There is a nonempty path P_{xz} of edges from x to z
- There is a nonempty path P_{yz} of edges from y to z
- Paths P_{xz} and P_{yz} do not have any node in common other than z, and…
- The node z does not appear within both P_{xz} and P_{yz} prior to the end, though it may appear in one or the other.



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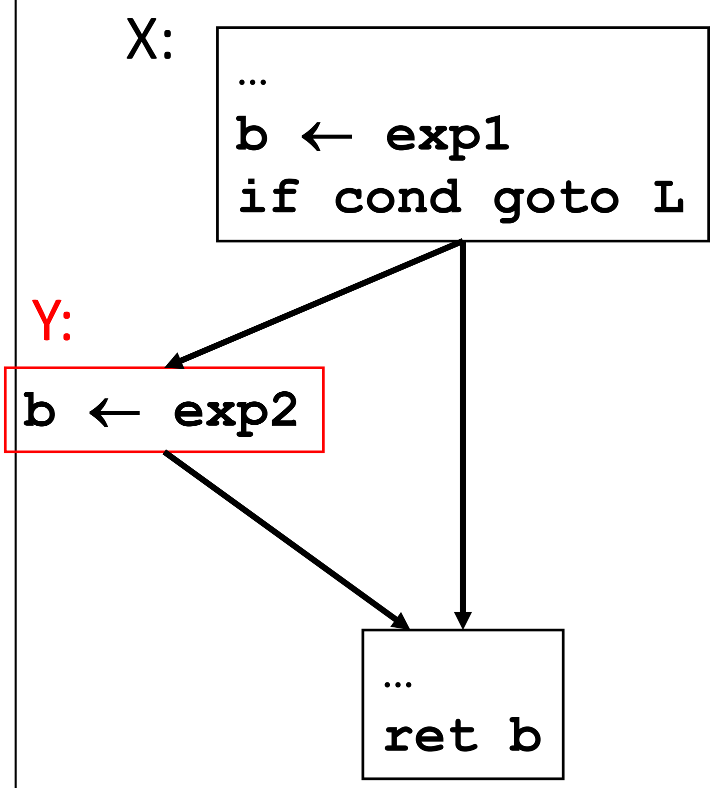

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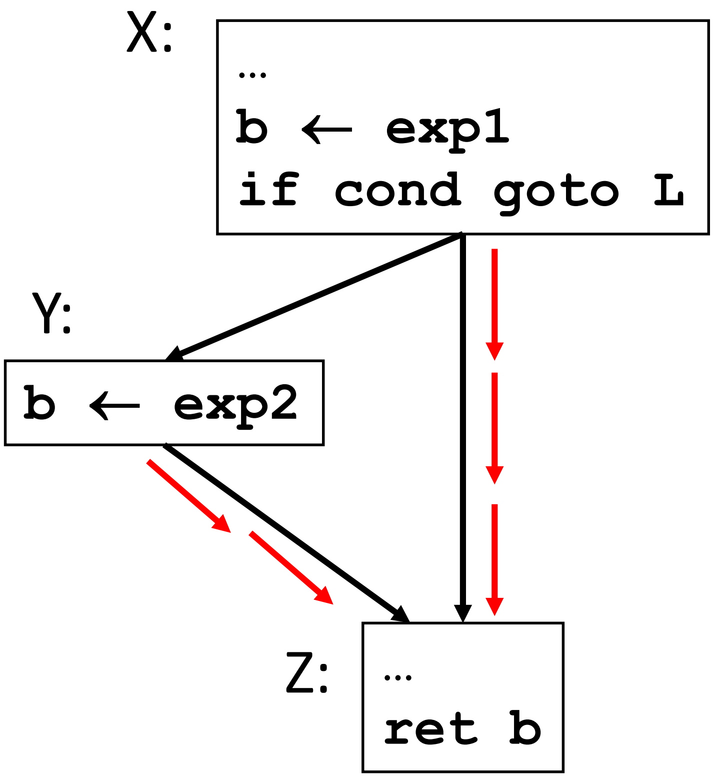

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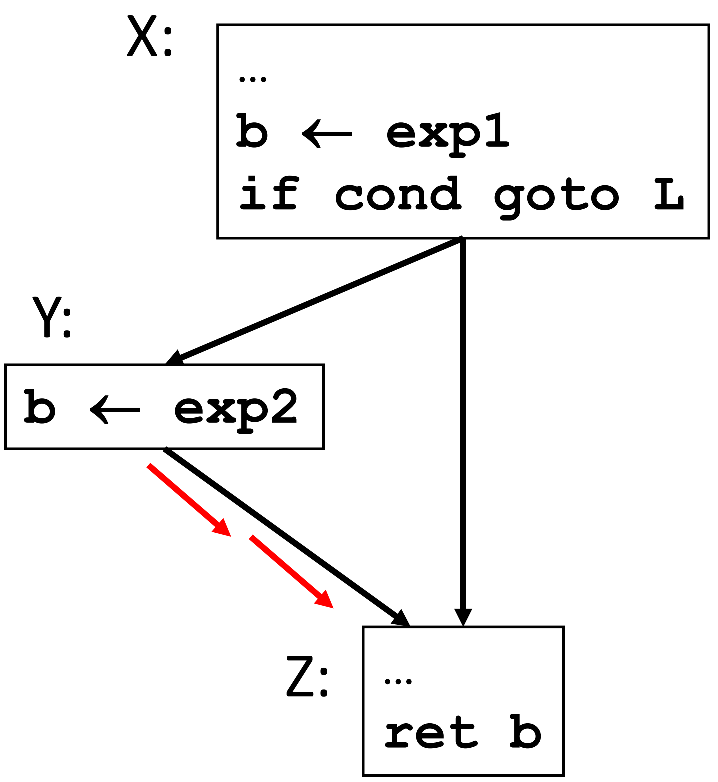

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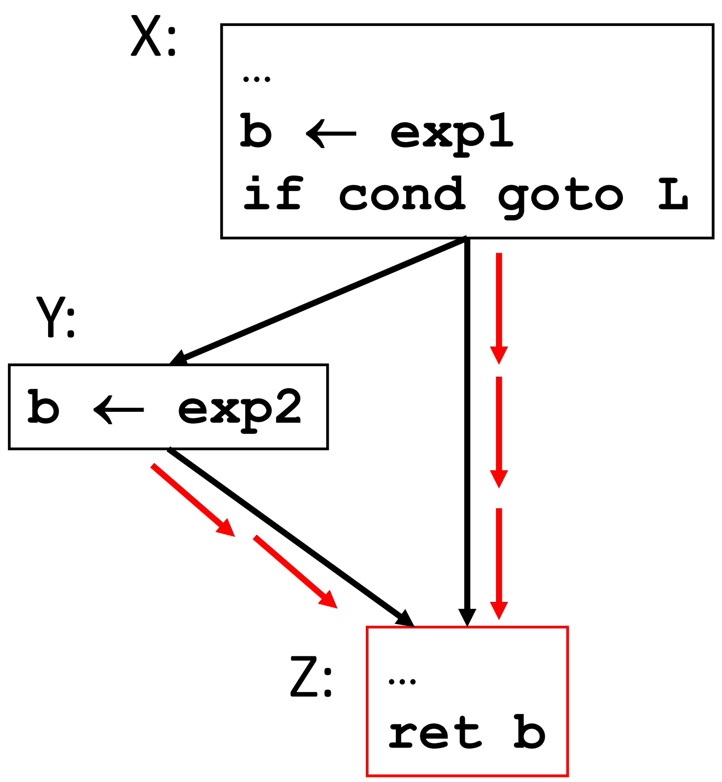

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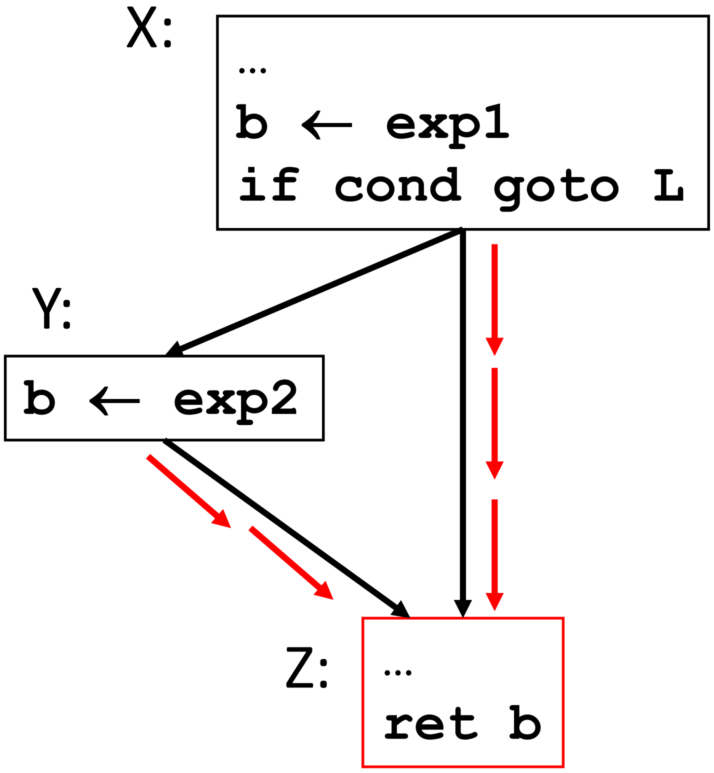


Iterative Insertion

- Implicit def of every variable in start node
- Inserting Φ-function creates new definition
- While there $\exists$ x,y,z that
  - satisfy path-convergence critera
  - and z does not contain Φ-function for b
- do
  - insert $b \leftarrow \Phi(b,b,b,\ldots,b_n)$ at node z, z having n predecessors.

Dominance Property of SSA

- In SSA **definitions dominate uses**.
  - If x_i is used in $x \leftarrow \Phi(\ldots, x_i, \ldots)$, then BB($x_i$) dominates ith pred of BB(PHI)
  - If x is used in $y \leftarrow \ldots x \ldots$, then BB(x) dominates BB(y)
- We can use this for an efficient alg to convert to SSA

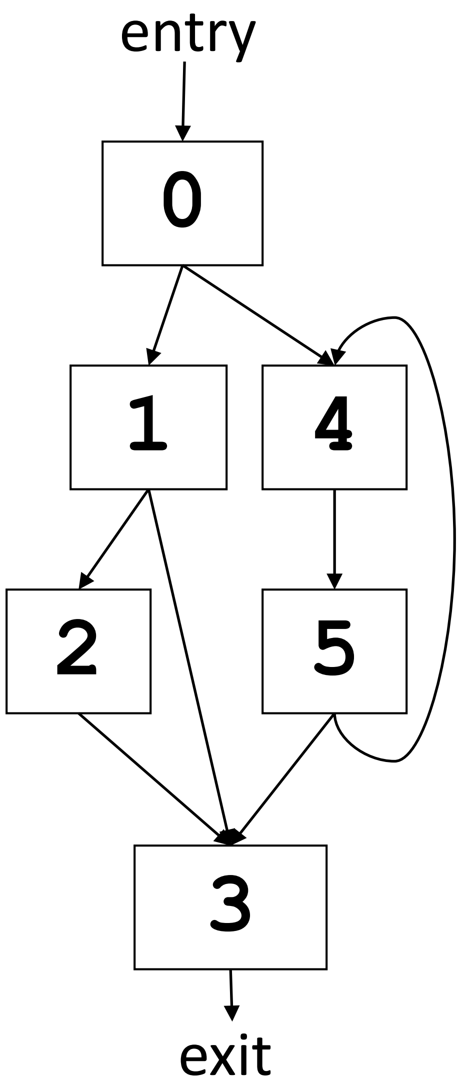
(actually only true for strict SSA, where all variables are defined before they are used.

Side trip: Dominators

Dominators

- *a dom b*
  - block *a dominates* block *b* if every possible execution path from *entry* to *b* includes *a*
    - **entry** dominates everything
    - **0** dominates everything but entry
    - **1** dominates and

Dominators are useful in indentifying “natural” loops



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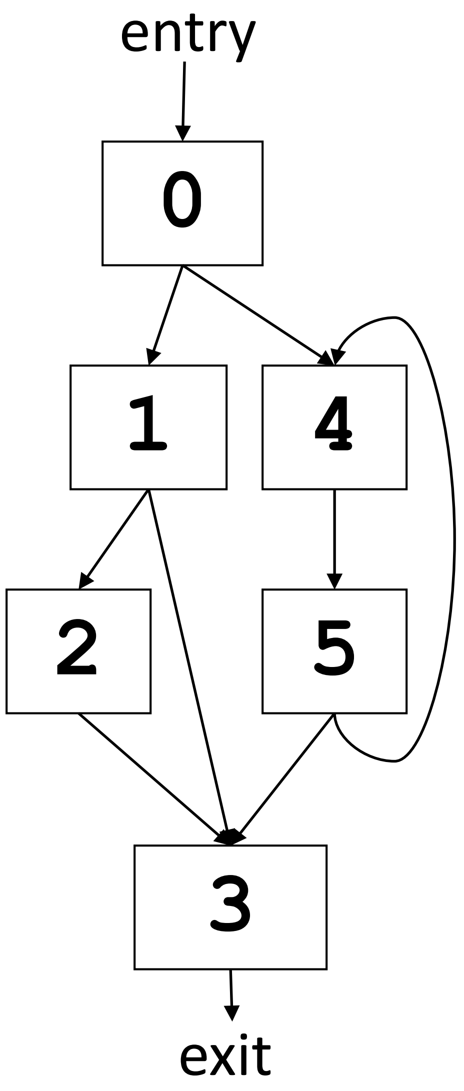
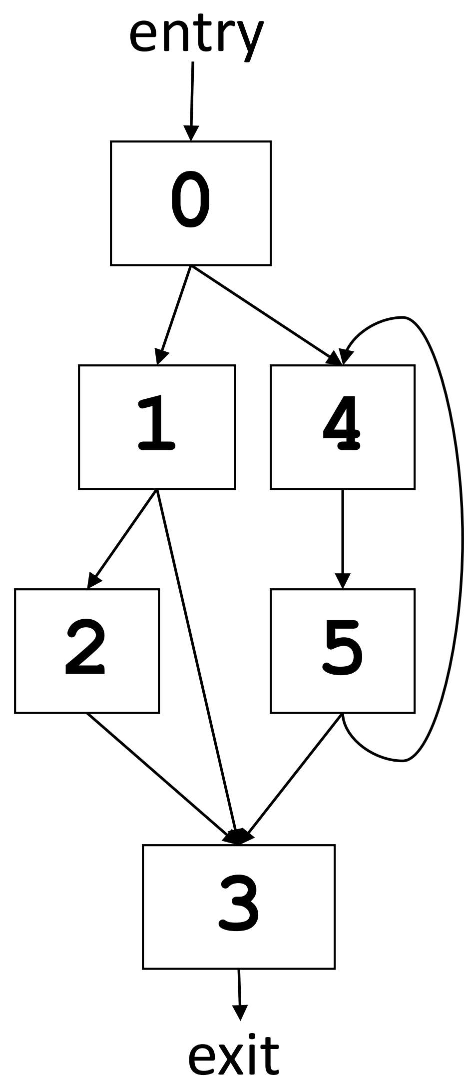


Definitions

- *a sdom b*
  - If *a* and *b* are different blocks and *a dom b*, we say that *a strictly dominates b*
- *a idom b*
  - If *a sdom b*, and there is no *c* such that *a sdom c* and *c sdom b*, we say that *a* is the *immediate dominator* of *b*



Properties of Dom

- Dominance is a partial order on the blocks of the flow graph, i.e.,
  - 1. Reflexivity: a dom a for all a
  - 2. Anti-symmetry: a dom b and b dom a implies a = b
  - 3. Transitivity: a dom b and b dom c implies a dom c
- NOTE: there may be blocks a and b such that neither a dom b or b dom a holds.
- The dominators of each node n are linearly ordered by the dom relation. The dominators of n appear in this linear order on any path from the initial node to n.

Computing dominators

- We want to compute *D*[*n*], the set of blocks that dominate *n*

Initialize each *D*[*n*] (except *D*[entry]) to be the set of all blocks, and then iterate until no *D*[*n*] changes:

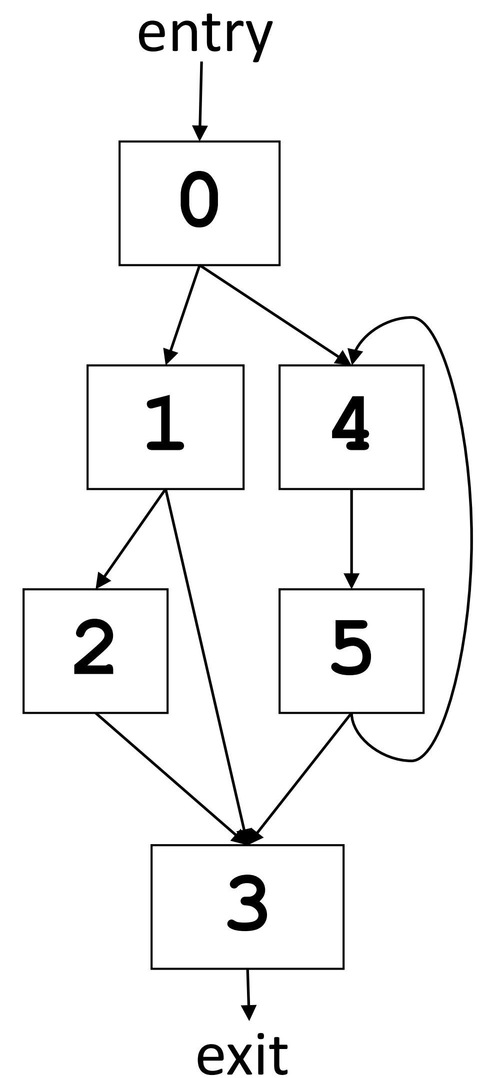
$$D[\text{entry}] = \{\text{entry}\}$$

$$D[n] = \{n\} \cup \left(\bigcap_{p \in \text{pred}(n)} D[p] \right), \quad \text{for} \;\; n \neq \text{entry}$$

Skip example

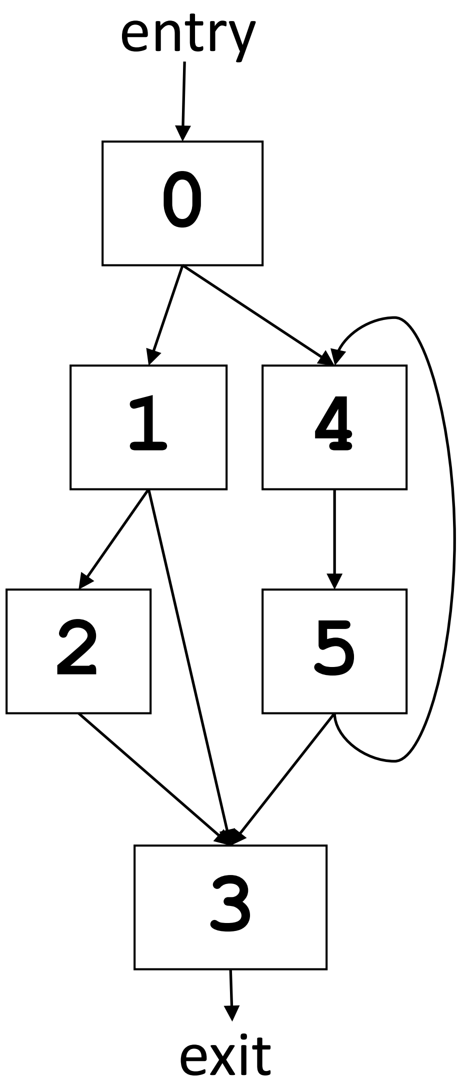
Example

block	Initialization D[n]
entry	{entry}
0	{entry,0,1,2,3,4,5,exit}
1	{entry,0,1,2,3,4,5,exit}
2	{entry,0,1,2,3,4,5,exit}
3	{entry,0,1,2,3,4,5,exit}
4	{entry,0,1,2,3,4,5,exit}
5	{entry,0,1,2,3,4,5,exit}
exit	{entry,0,1,2,3,4,5,exit}



Example

block	Initialization D[n]	First Pass D[n]
entry	{entry}	{entry}
0	{entry,0,1,2,3,4,5,exit}	{0,entry}
1	{entry,0,1,2,3,4,5,exit}	{1,0,entry}
2	{entry,0,1,2,3,4,5,exit}	{2,1,0,entry}
3	{entry,0,1,2,3,4,5,exit}	{3,1,0,entry}
4	{entry,0,1,2,3,4,5,exit}	{4,0,entry}
5	{entry,0,1,2,3,4,5,exit}	{5,4,0,entry}
exit	{entry,0,1,2,3,4,5,exit}	{exit,3,1,0,entry}

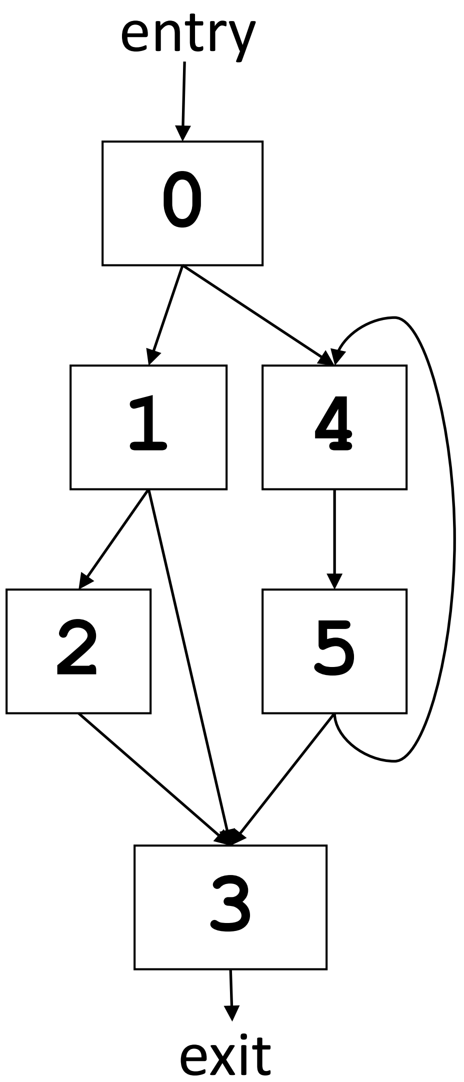


Update rule:

$$D[n] = \{n\} \cup \left(\bigcap_{p \in pred(n)} D[p] \right)$$

Example

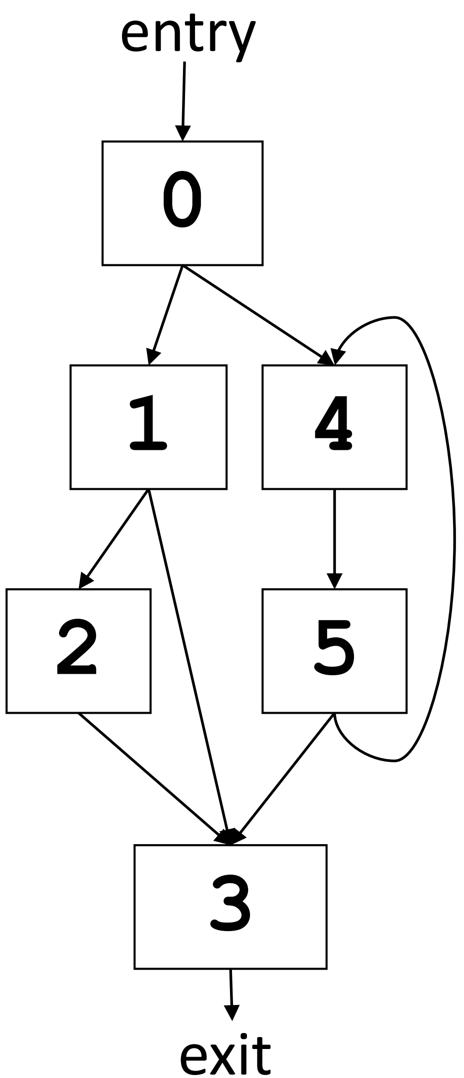
block	First Pass D[n]	Second Pass D[n]
entry	{entry}	{entry}
0	{0,entry}	{0,entry}
1	{1,0,entry}	{1,0,entry}
2	{2,1,0,entry}	{2,1,0,entry}
3	{3,1,0,entry}	{3,0,entry}
4	{4,0,entry}	{4,0,entry}
5	{5,4,0,entry}	{5,4,0,entry}
exit	{exit,3,1,0,entry}	{exit,3,0,entry}



Update rule: $$D[n] = \{n\} \cup \left(\bigcap_{p \in pred(n)} D[p] \right)$$

Example

block	Second Pass D[n]	Third Pass D[n]
entry	{entry}	{entry}
0	{0,entry}	{0,entry}
1	{1,0,entry}	{1,0,entry}
2	{2,1,0,entry}	{2,1,0,entry}
3	{3,0,entry}	{3,0,entry}
4	{4,0,entry}	{4,0,entry}
5	{5,4,0,entry}	{5,4,0,entry}
exit	{exit,3,0,entry}	{exit,3,0,entry}



Update rule: $$D[n] = \{n\} \cup \left(\bigcap_{p \in pred(n)} D[p] \right)$$

Computing dominators

- Iterative algorithm is $O(n^2e)$
  - assuming bit vector sets
- More efficient algorithm due to Lengauer and Tarjan
  - $O(e\cdot\alpha(e,n))$
  - much more complicated
  - your book provides a simple algorithm that is very fast in practice
    - faster than Tarjan algorithm for any realistic CFG

$\alpha(e,n)$ is *inverse Ackermann*

Computing iDom(n)

- Let *sD*[*n*] be the set of blocks that strictly dominate *n*, then

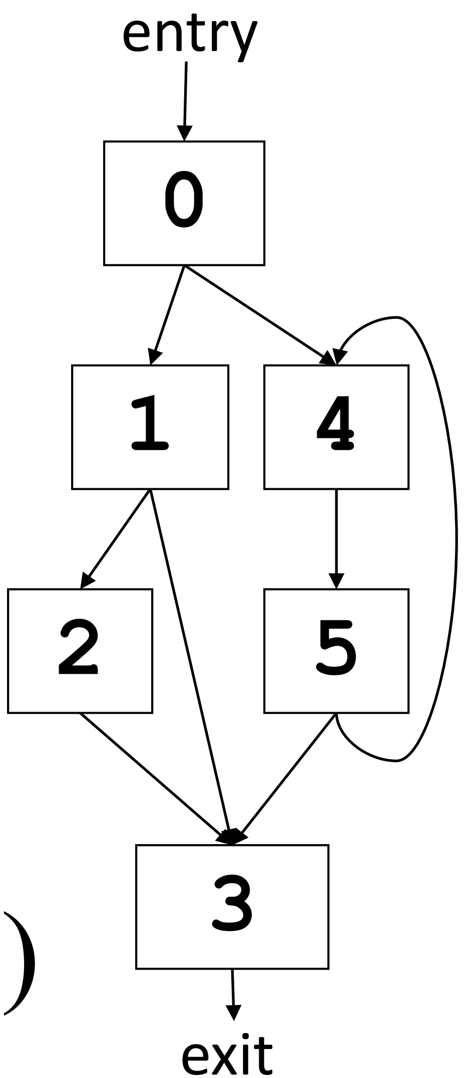
$$sD[n] = D[n] - \{n\}$$

- To compute *iD*[*n*], the set of blocks (size <= 1) that immediately dominate *n*
- Set $iD[n] = sD[n]$
- Repeat until no iD[n] changes:

$$iD[n] = iD[n] - \bigcup_{d \in iD[n]} (sD[d])$$

Example

block	Initialization iD[n]=sD[n]	First Pass iD[n]
entry	{}	{}
0	{entry}	
1	{0,entry}	
2	{1,0,entry}	
3	{0,entry}	
4	{0,entry}	
5	{4,0,entry}	
exit	{3,0,entry}	



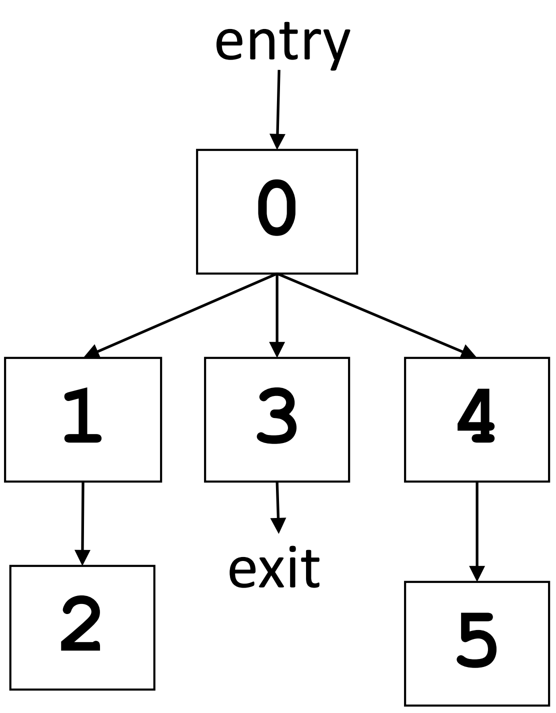
Update rule: $$iD[n] = iD[n] - \bigcup_{d \in iD[n]} (sD[d])$$

Dominator Tree

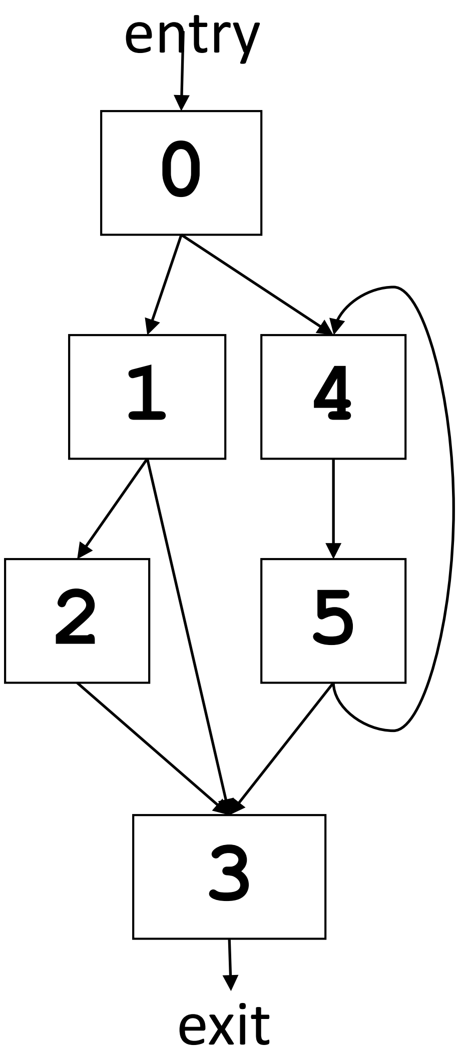
In the dominator tree the initial node is the entry block, and the parent of each other node is its immediate dominator.

block	iD[n]
entry	{}
0	{entry}
1	{0}
2	{1}
3	{0}
4	{0}
5	{4}
exit	{3}

Dominator Tree

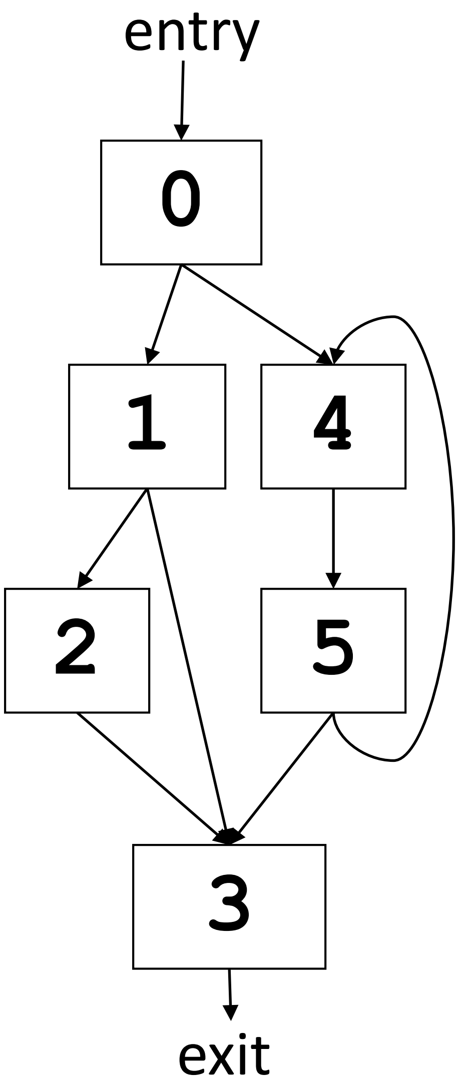


CFG



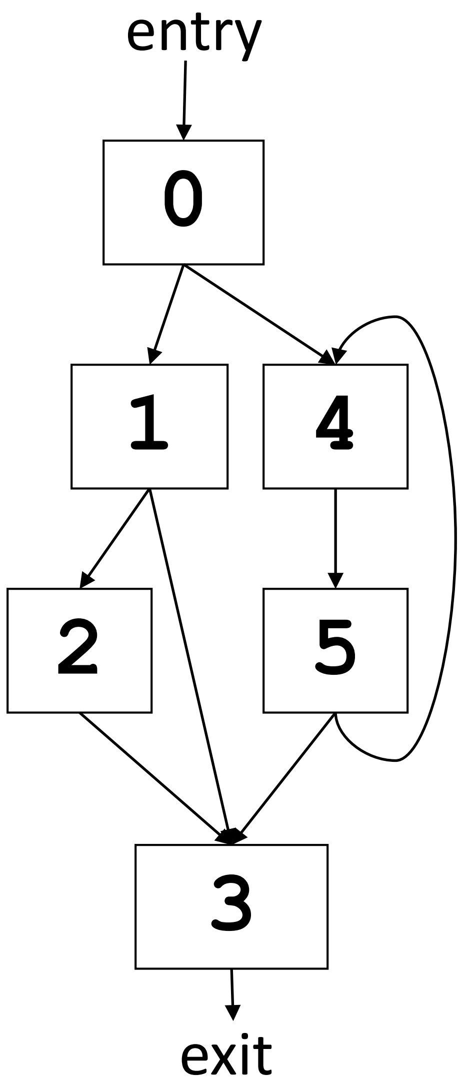
Dominance Frontier

- If *z* is the first node we encounter on the path from *x* which *x* does not *strictly* dominate, *z* is in the dominance frontier of *x*
- For some path from node *x* to *z*,
  $x \rightarrow \ldots \rightarrow y \rightarrow z$
  where *x* dom *y* but not *x* sdom *z*.
- Dominance frontier of **1**?
- Dominance frontier of **2**?
- Dominance frontier of **4**?



Dominance Frontier

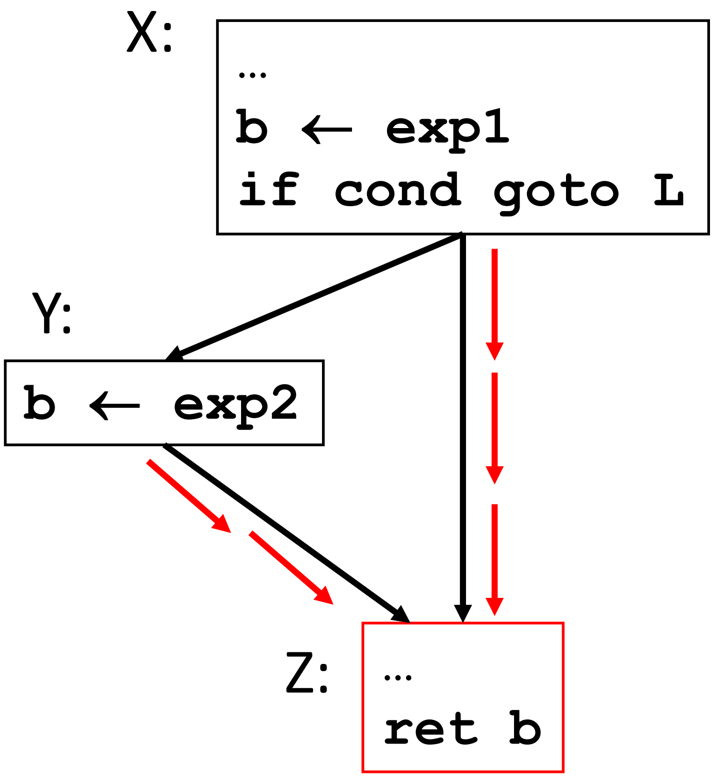
- If z is the first node we encounter on the path from x which x does not *strictly* dominate, z is in the dominance frontier of x
- For some path from node x to z,
  $x \rightarrow \ldots \rightarrow y \rightarrow z$
  where x dom y but not x sdom z.
- Dominance frontier of **1**? {3}
- Dominance frontier of **2**? {3}
- Dominance frontier of **4**? {3,4}



When do we insert Φ?

There should be a Φ-function for variable b at node z of the flow graph exactly when all of the following are true:

- There is a block x containing a def of b
- There is a block y (with y ≠ x) containing a def of b
- There is a nonempty path P_{xz} of edges from x to z
- There is a nonempty path P_{yz} of edges from y to z
- Paths P_{xz} and P_{yz} do not have any node in common other than z, and…
- The node z does not appear within both P_{xz} and P_{yz} prior to the end, though it may appear in one or the other.



IOW, Z ∈ DF(X)

Calculating the Dominance Frontier

- Let *dominates*[n] be the set of all blocks which block n dominates
  - subtree of dominator tree with n as the root
- The dominance frontier of n, *DF*[*n*] is

$$DF[n] = \left(\bigcup_{s \in dominates[n]} succs(s) \right) - \left(dominates[n] - \{n\} \right)$$

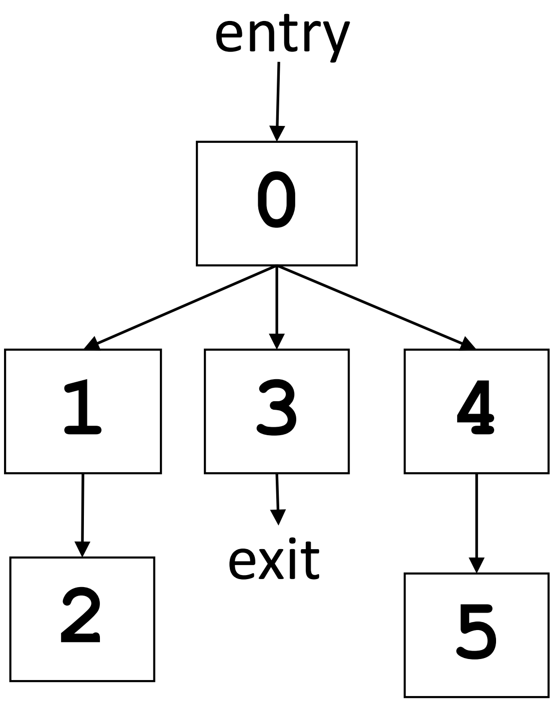
Skip example

Example

First calculate dominates[n] from the dominator tree

block	dominates[n]	
entry	{entry,0,1,2,3,4,5,exit}	
0	{0,1,2,3,4,5,exit}	
1	{1,2}	
2	{2}	
3	{3,exit}	
4	{4,5}	
5	{5}	
exit	{exit}	

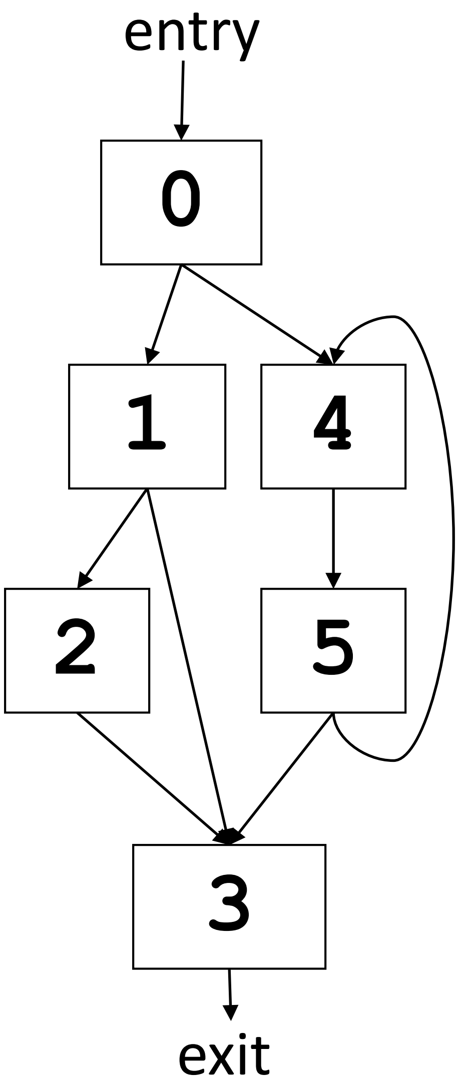
Dominator Tree



Example

Then compute the successor set of dominates[n]

block	dominates[n]	*succ*(dominates[n])
entry	{entry,0,1,2,3,4,5,exit}	
0	{0,1,2,3,4,5,exit}	
1	{1,2}	
2	{2}	
3	{3,exit}	
4	{4,5}	
5	{5}	
exit	{exit}	{}



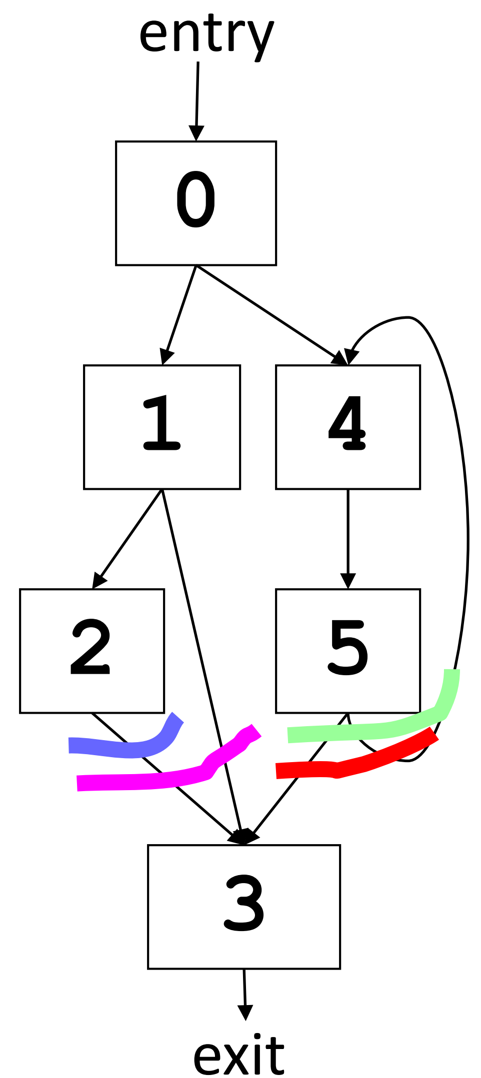
Example

Finally, remove all the blocks from the successor set that are strictly dominated by n to get DF[n]

block	sdominates[n]	*succ*(dominates[n])	DF[n]
entry	{entry,0,1,2,3,4,5,exit}	{0,1,2,3,4,5,exit}	
0	{0,1,2,3,4,5,exit}	{1,2,3,4,5,exit}	
1	{1,2}	{2,3}	
2	{2}	{3}	
3	{3,exit}	{exit}	
4	{4,5}	{3,4,5}	
5	{5}	{3,4}	
exit	{exit}	{}	{}

Example

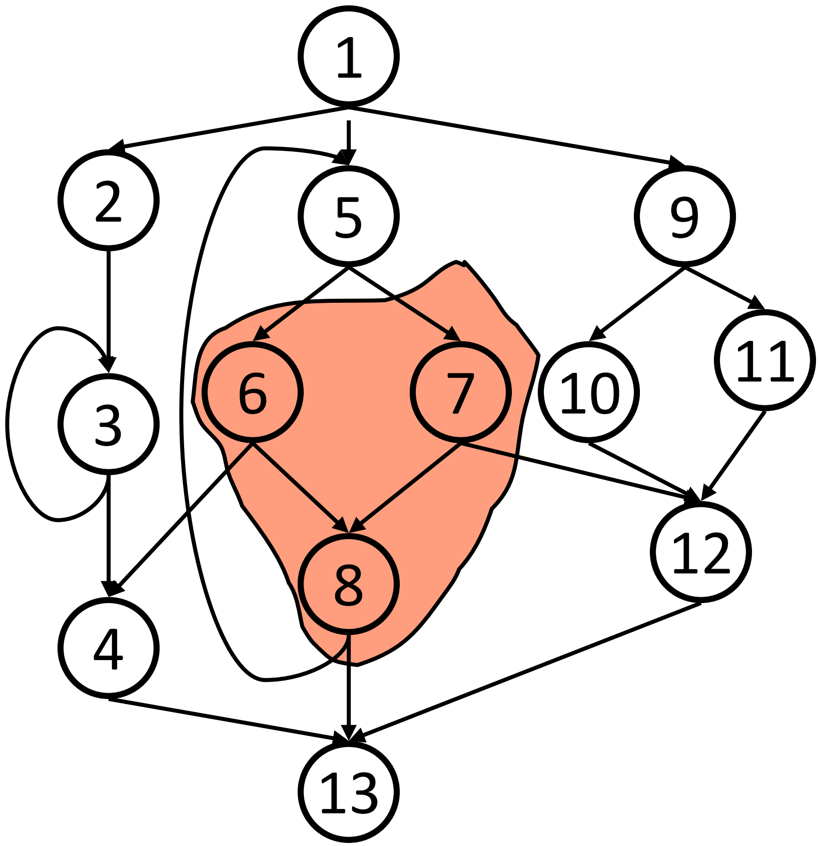
block	DF[n]
entry	{}
0	{}
1	{3}
2	{3}
3	{}
4	{3,4}
5	{3,4}
exit	{}

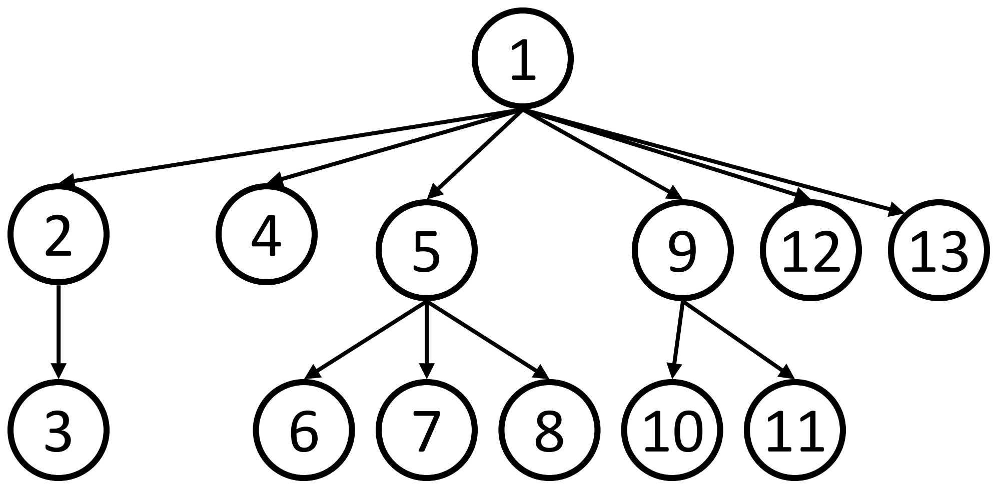


Recap

- *a dom b*
  - every possible execution path from *entry* to *b* includes *a*
- *a sdom b*
  - *a dom b* and *a* != *b*
- *a idom b*
  - *a* is “closest” dominator of *b*
- *a pdom b*
  - every path from *a* to the exit block includes *b*
- Dominator trees
- Dominance frontier

Dominance

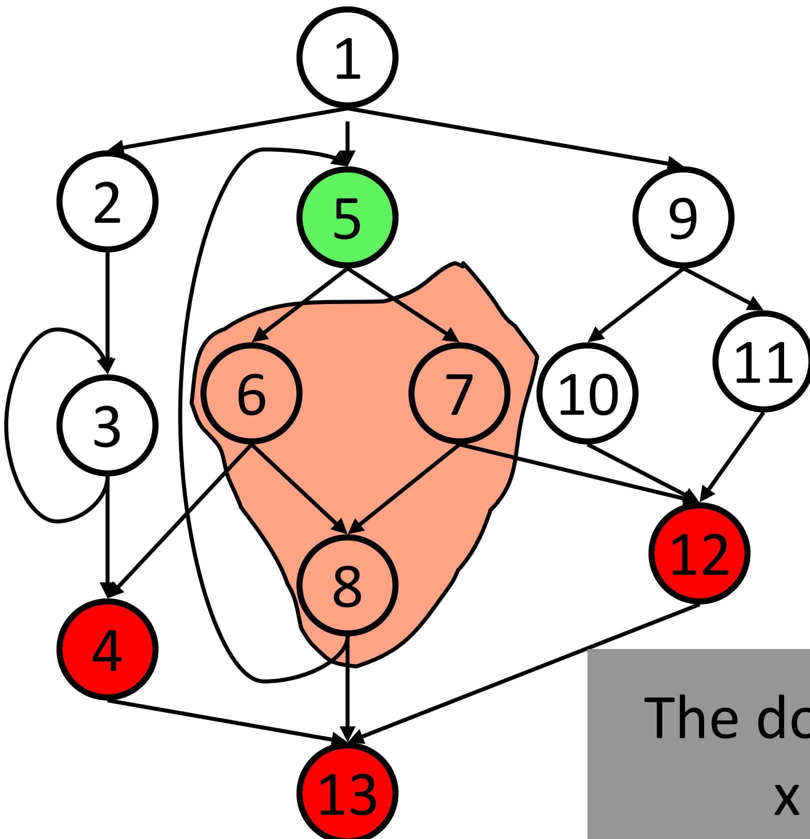




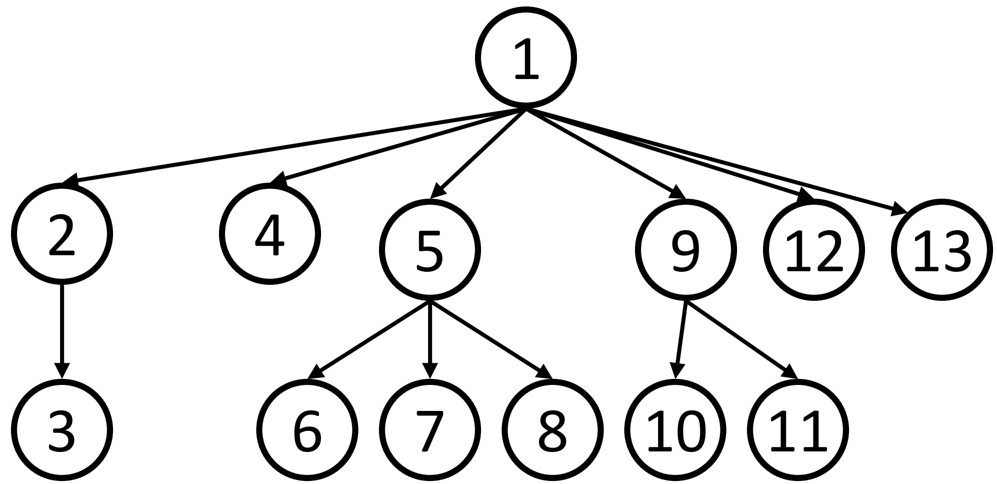
If there is a def of a in block 5, which nodes need a $\Phi()$?

x strictly dominates w (s sdom w) iff x dom w AND $x \neq w$

Dominance Frontier



CFG

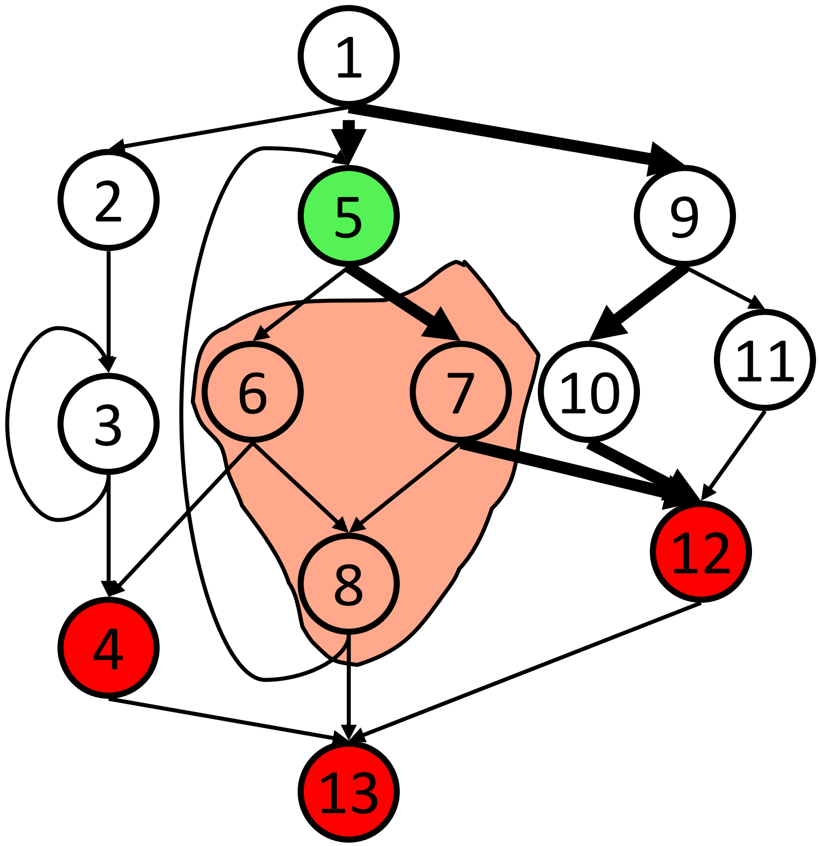


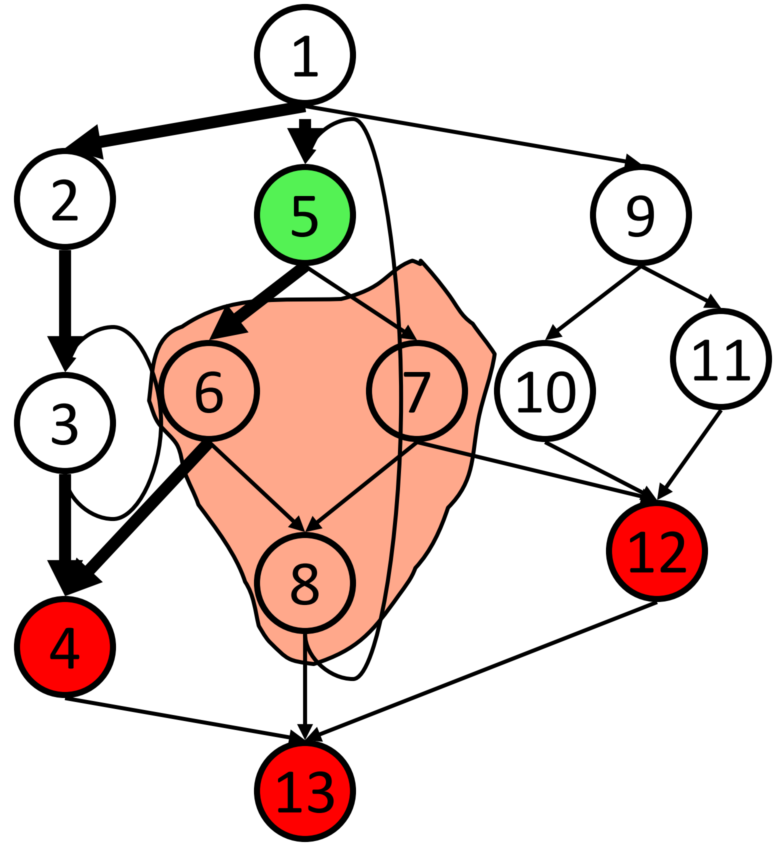
D-Tree

The dominance Frontier of a node x = { w | x dom pred(w) AND !(x sdom w)}

x strictly dominates w (s sdom w) iff x dom w AND x $\neq$ w

Dominance Frontier & path-convergence



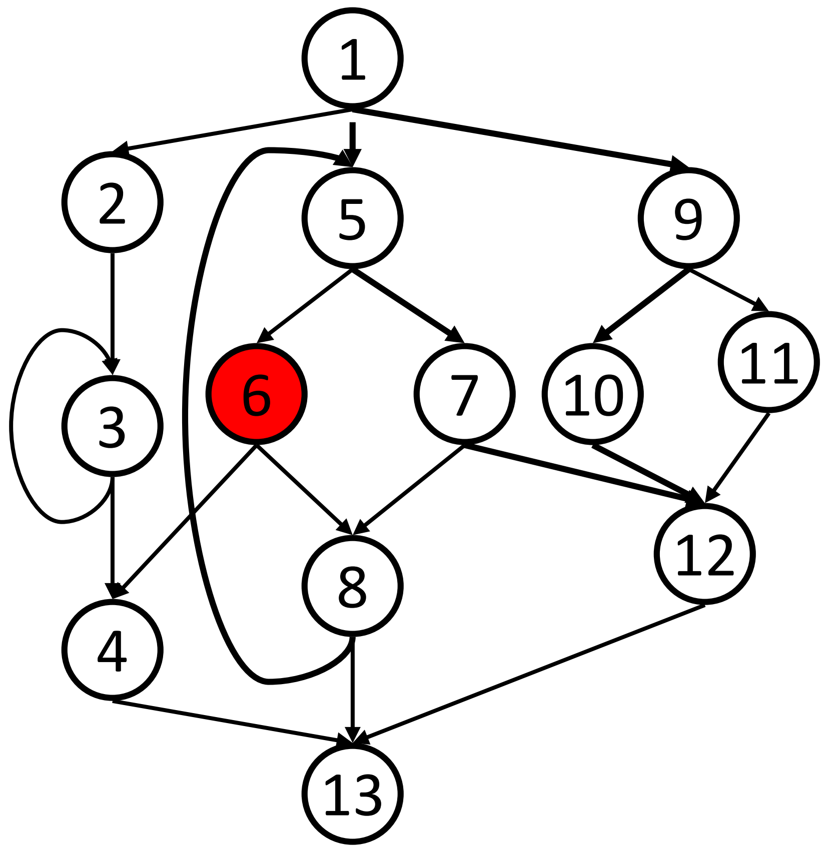


Dominance Frontier Criterion

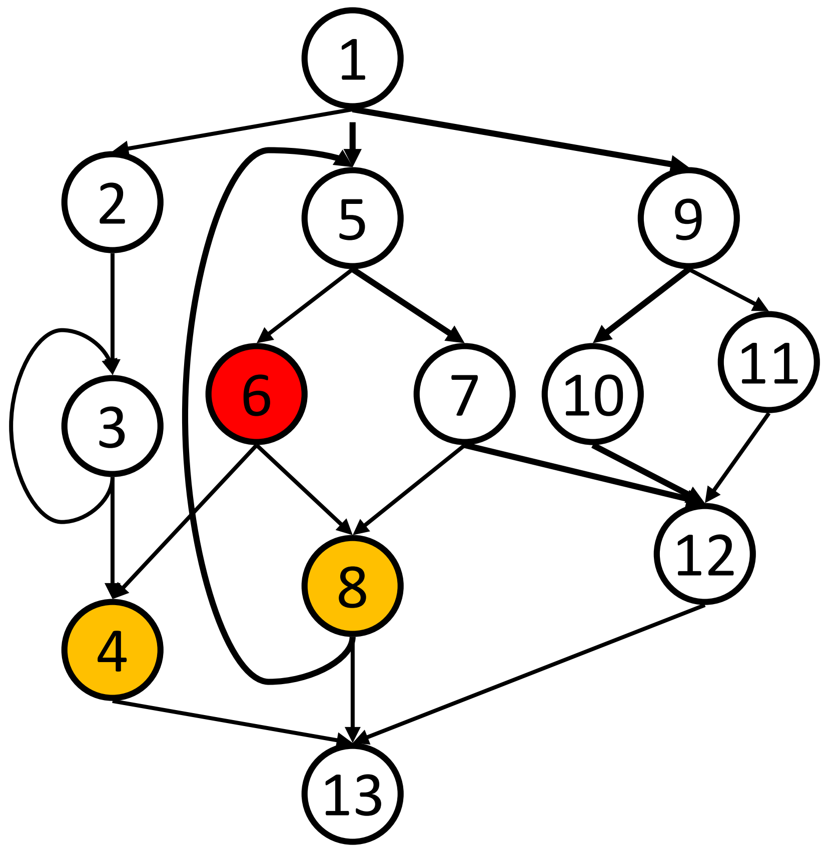
- **Dominance-Frontier Criterion**: Whenever node x contains a definition of some variable a, then any node z $\in$ DF(x), z needs a Φ-function for a.

- **Iterated dominance frontier**: since a Φ-function itself is a definition, we must iterate the dominance-frontier criterion until there are no nodes that need Φ-functions.

Dominance Frontier Criterion

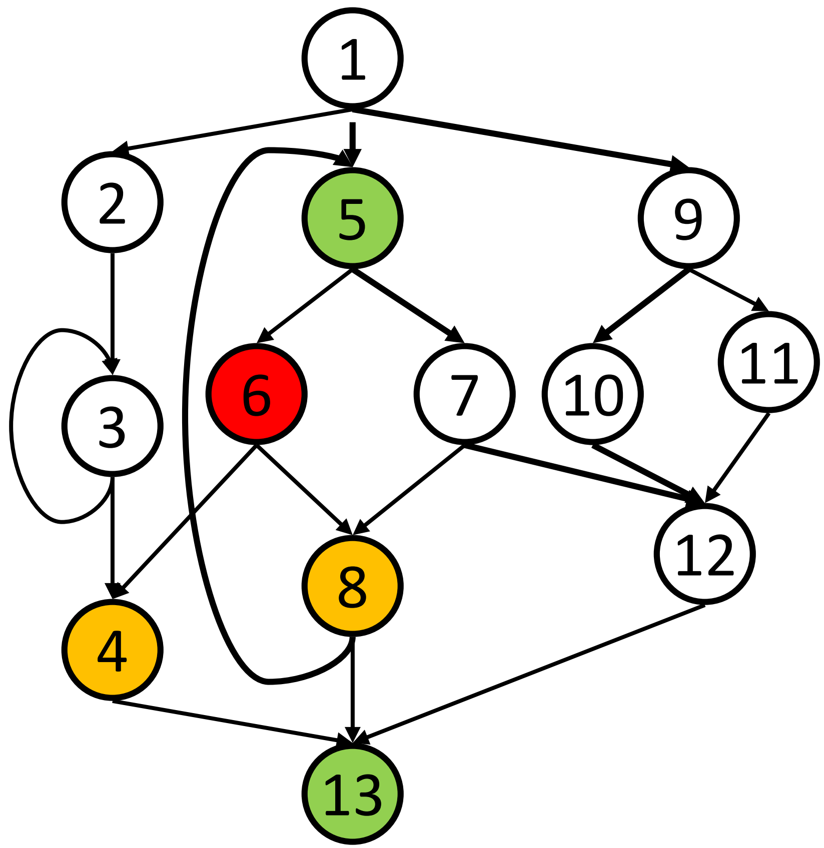


Dominance Frontier Criterion



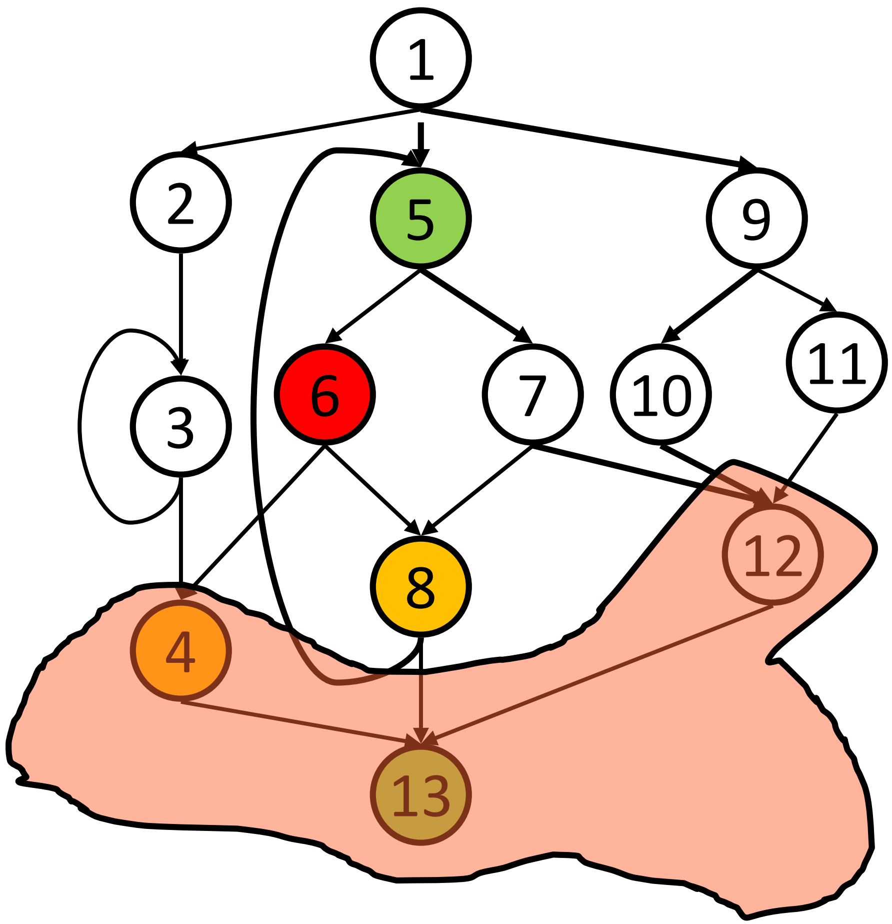
And, Iterating

Dominance Frontier Criterion



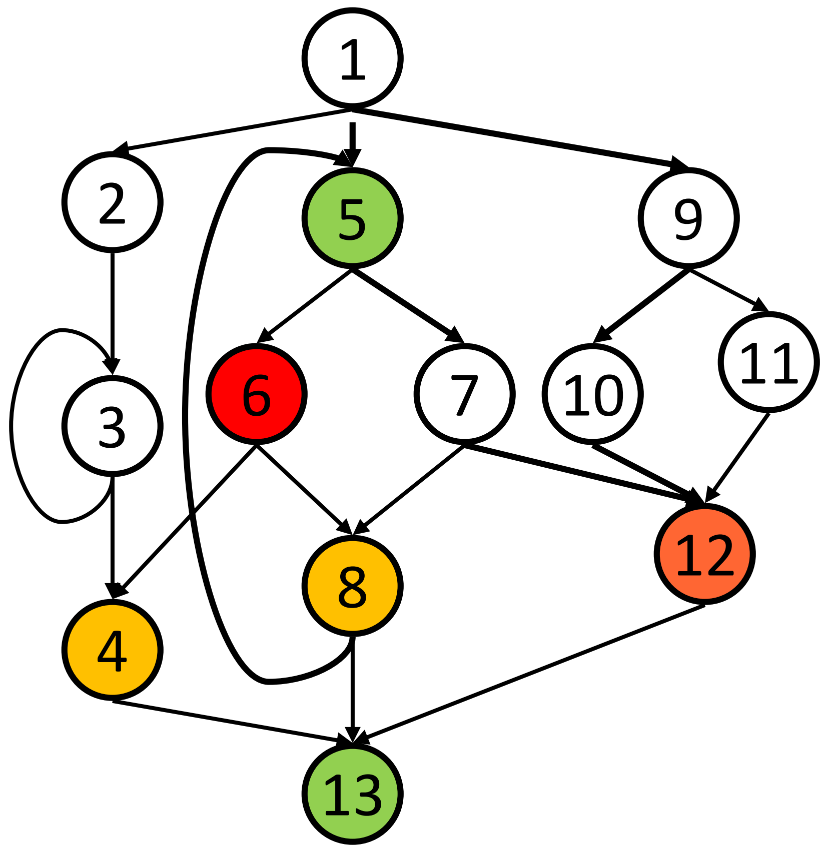
And, Iterating

Dominance Frontier Criterion



And, Iterating

Dominance Frontier Criterion



Done

Using DF to compute minimal SSA

- place all $\Phi()$
- Rename all variables

minimal, but not pruned

Using DF to Place $\Phi()$

- Gather all the defsites of every variable
- Then, for every variable
  - foreach defsite
    - foreach node in DF(defsite)
      - if we haven't put $\Phi()$ in node put one in
      - If this node didn't define the variable before: add this node to the defsites

- This essentially computes the Iterated Dominance Frontier on the fly, creating minimal SSA

Using DF to Place Φ()

```
foreach node n {
  foreach variable v defined in n {
    orig[n] ∪= {v}
    defsites[v] ∪= {n}
  }
  foreach variable v {
    W = defsites[v]
    while W not empty {
      foreach y in DF[n]
      if y ∉ PHI[v] {
        insert "v ← Φ(v,v,…)" at top of y
        PHI[v] = PHI[v] ∪ {y}
        if v ∉ orig[y]: W = W ∪ {y}
      }
    }
  }
}
```

Renaming Variables

- Walk the D-tree, renaming variables as you go
- Replace uses with more recent renamed def
  - For straight-line code this is easy
  - If there are branches and joins?

Renaming for Straight-Line Code

- Need to extend for ϕ-functions.
- Need to maintain property that definitions dominate uses.

for each variable a:
  Count[a] = 0
  Stack[a] = [0]

renameBasicBlock(*B*):
  for each instruction *S* in block *B*:
    for each use of a variable *x* in *S*:
      i = top(Stack[x])
      replace the use of x with x_i
    for each variable *a* that *S* defines
      count[a] = Count[a] + 1
      i = Count[a]
      push i onto Stack[a]
      replace definition of a with a_i

Renaming in CFG

rename(n):

  renameBasicBlock(n)

    for each successor Y of n, **where** n is the j^{th} predecessor of Y:

    for each phi-function f in Y, **where** the operand of f is ‘a’

      i = top(Stack[a])

      replace j^{th} operand with a_i

    for each child of n in D-tree, X:

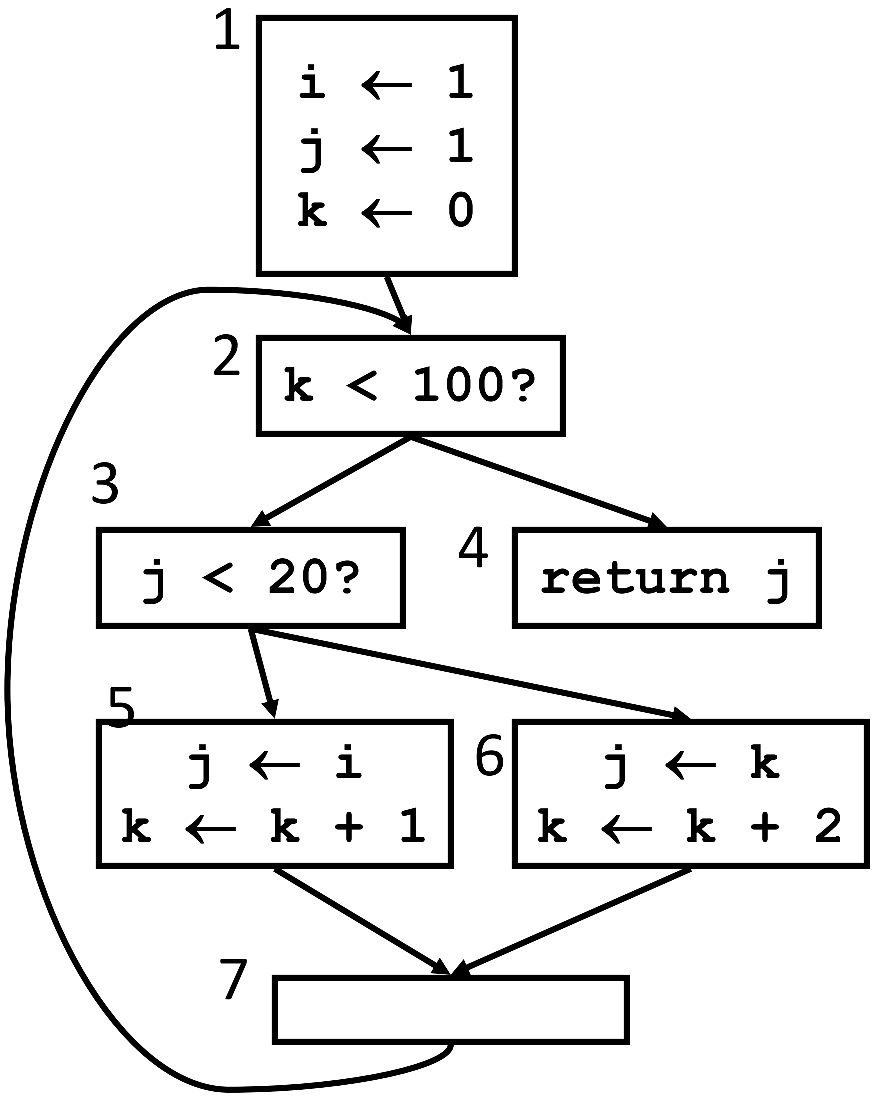
      rename(X)

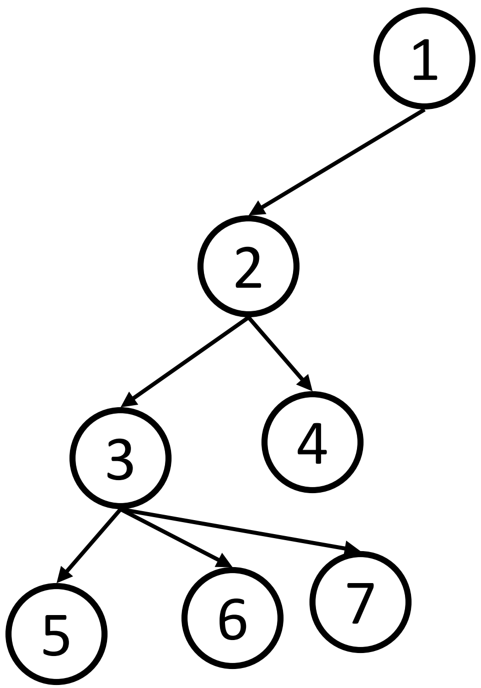
    for each instruction S ∈ n:

      for each variable v that S defines:

        pop Stack[v]

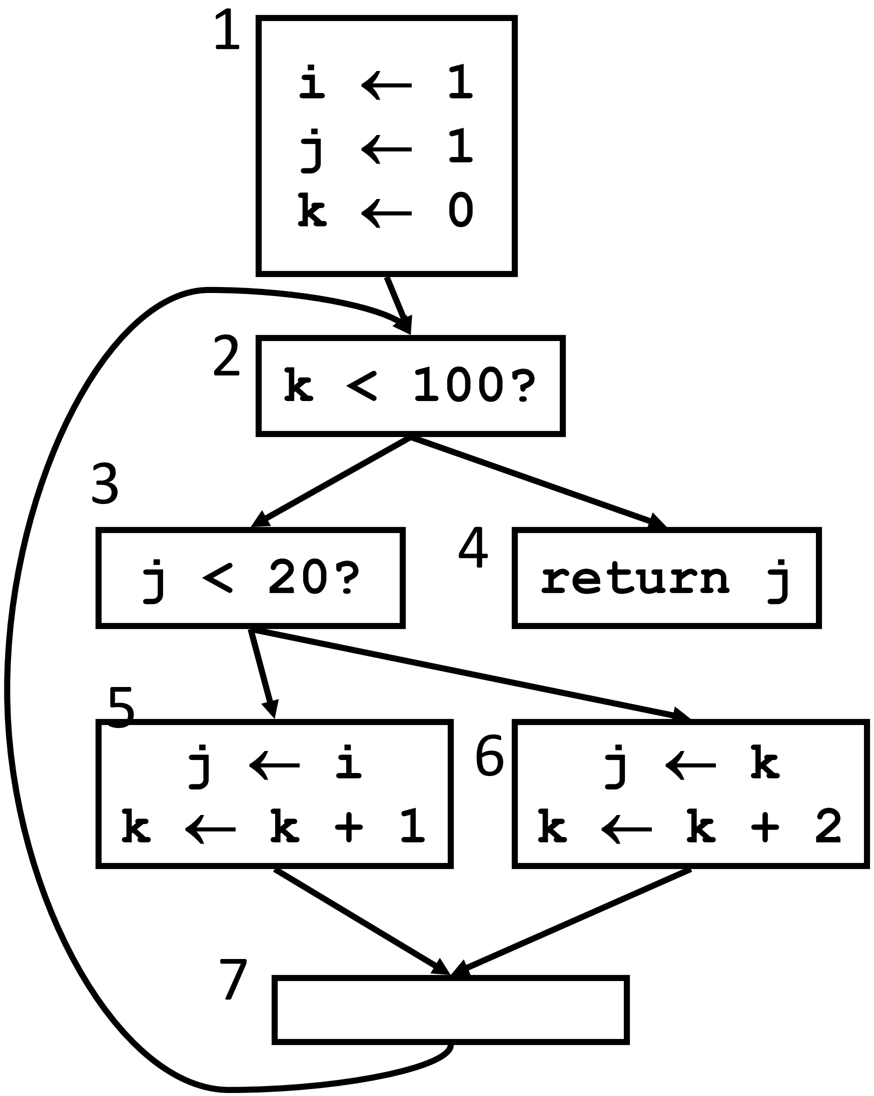
Compute D-tree

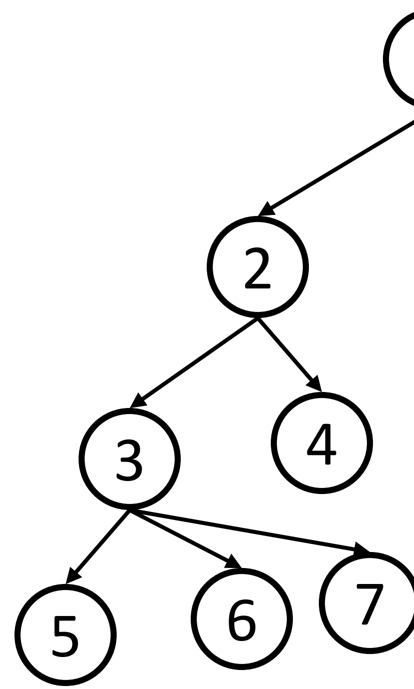




D-tree

Compute Dominance Frontier

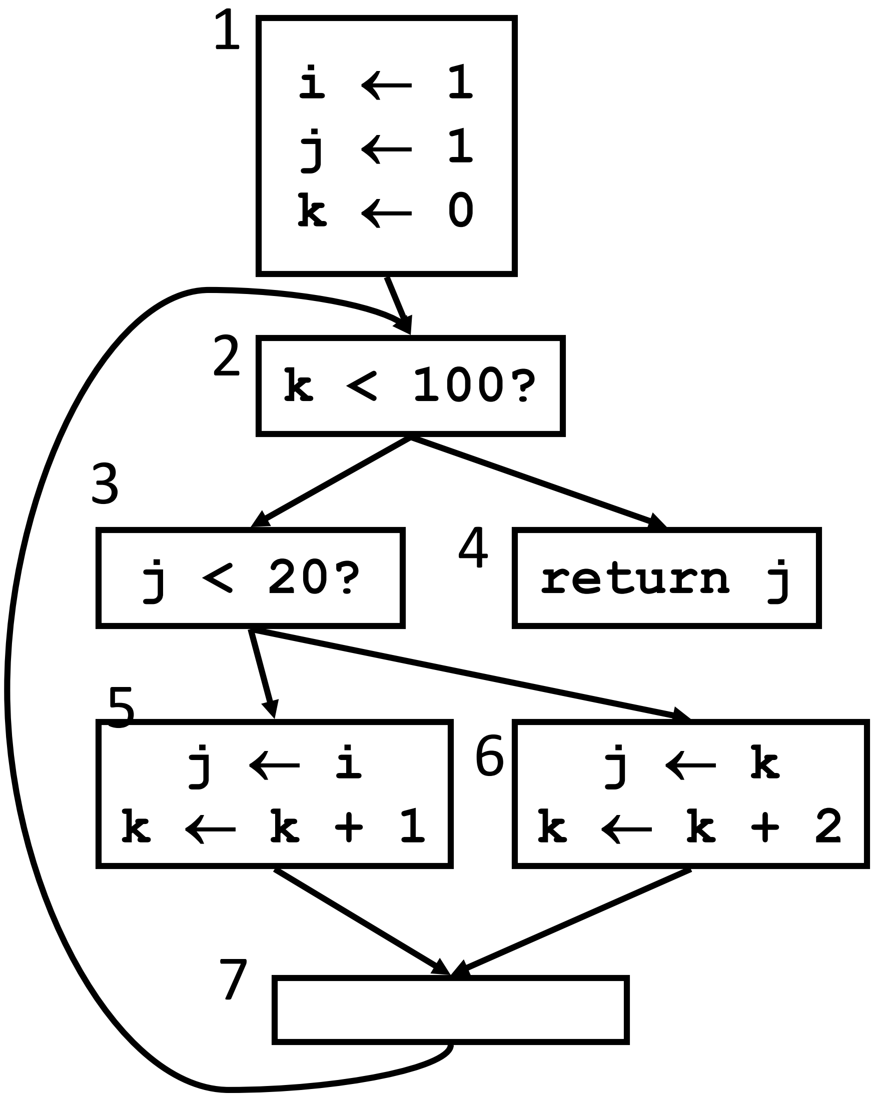




1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

DFs

Insert Φ()



	DFs
1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

	orig[n]
1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

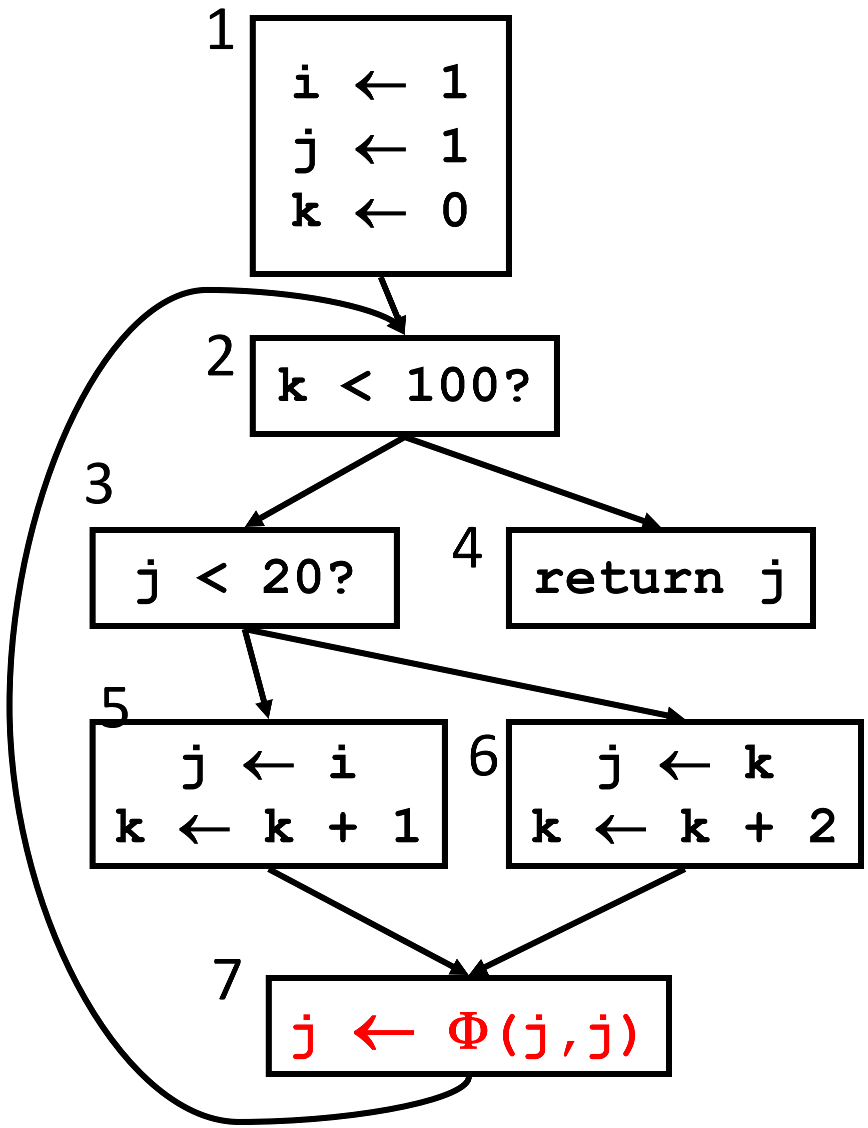
	defsites[v]
i	{1}
j	{1,5,6}
k	{1,5,6}

var i: W={1}

var j: W={1,5,6}

DF{1}, DF{5}

Insert Φ()



1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

DFs

orig[n]

1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

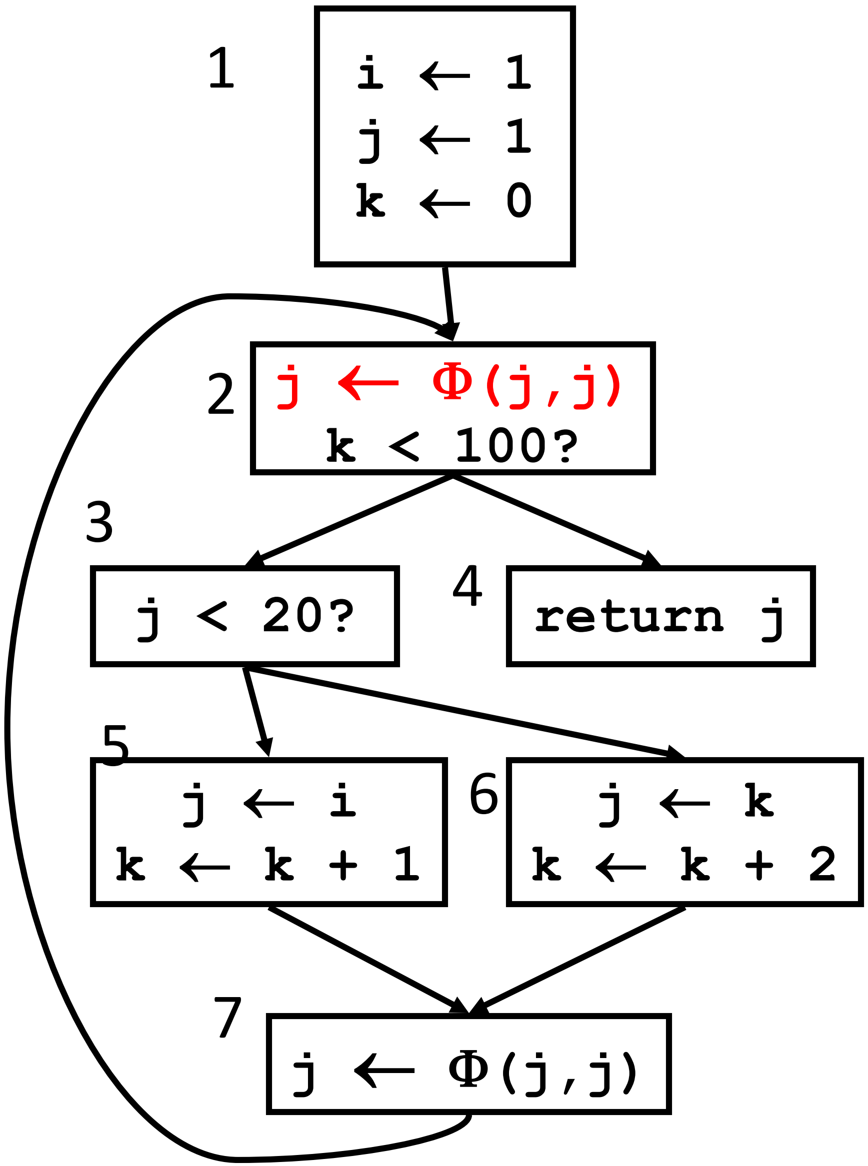
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6}

DF{1}, DF{5}

Insert Φ()



	DFs
1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

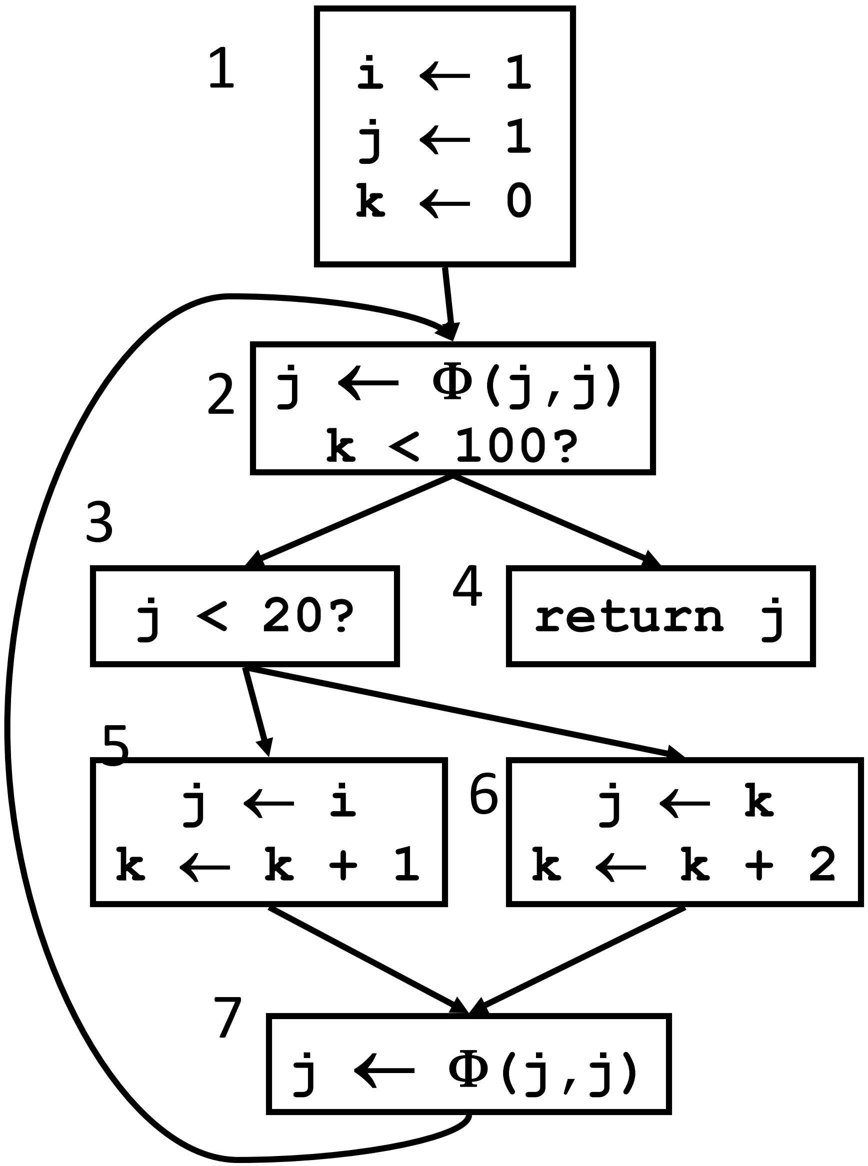
	orig[n]
1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6}

DF{1}, DF{5}

Insert Φ()



1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

DFs

orig[n]

1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

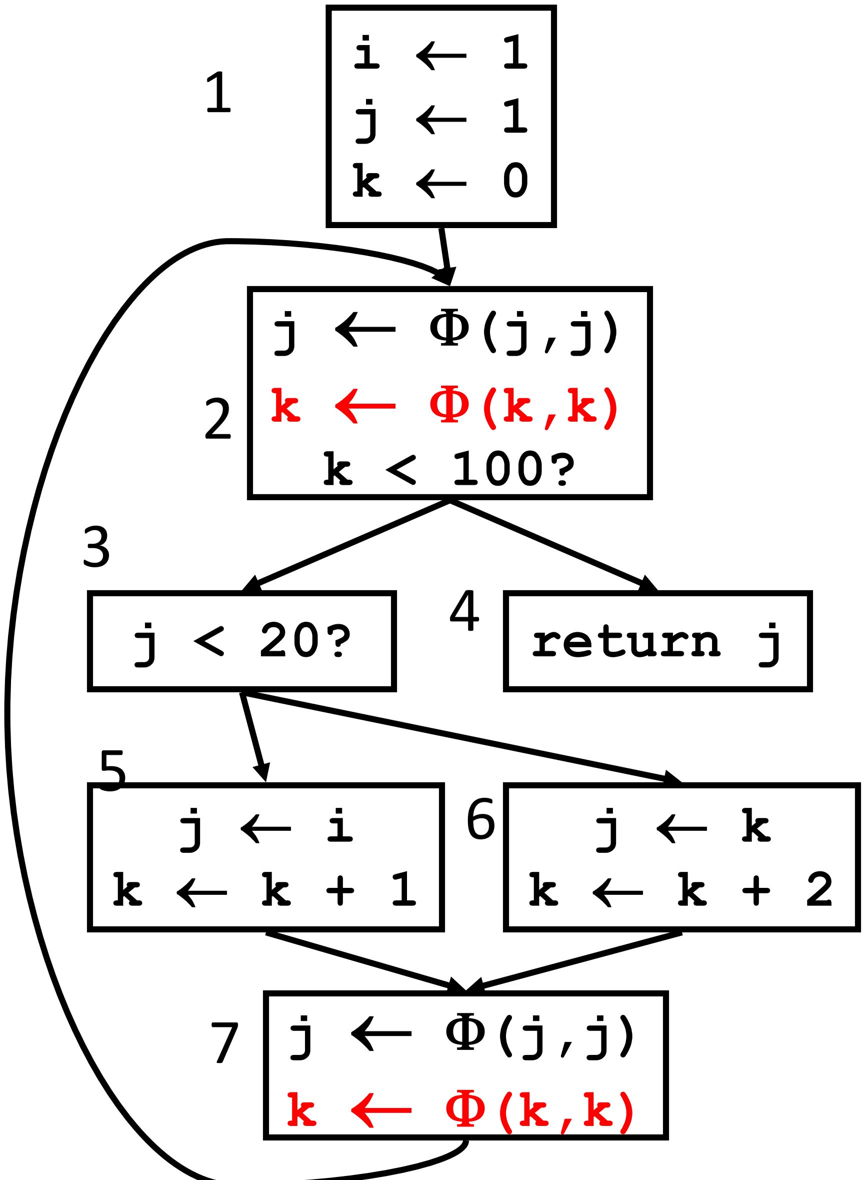
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6}

DF{1}, DF{5}, DF{6}

Insert Φ()



1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

DFs

orig[n]

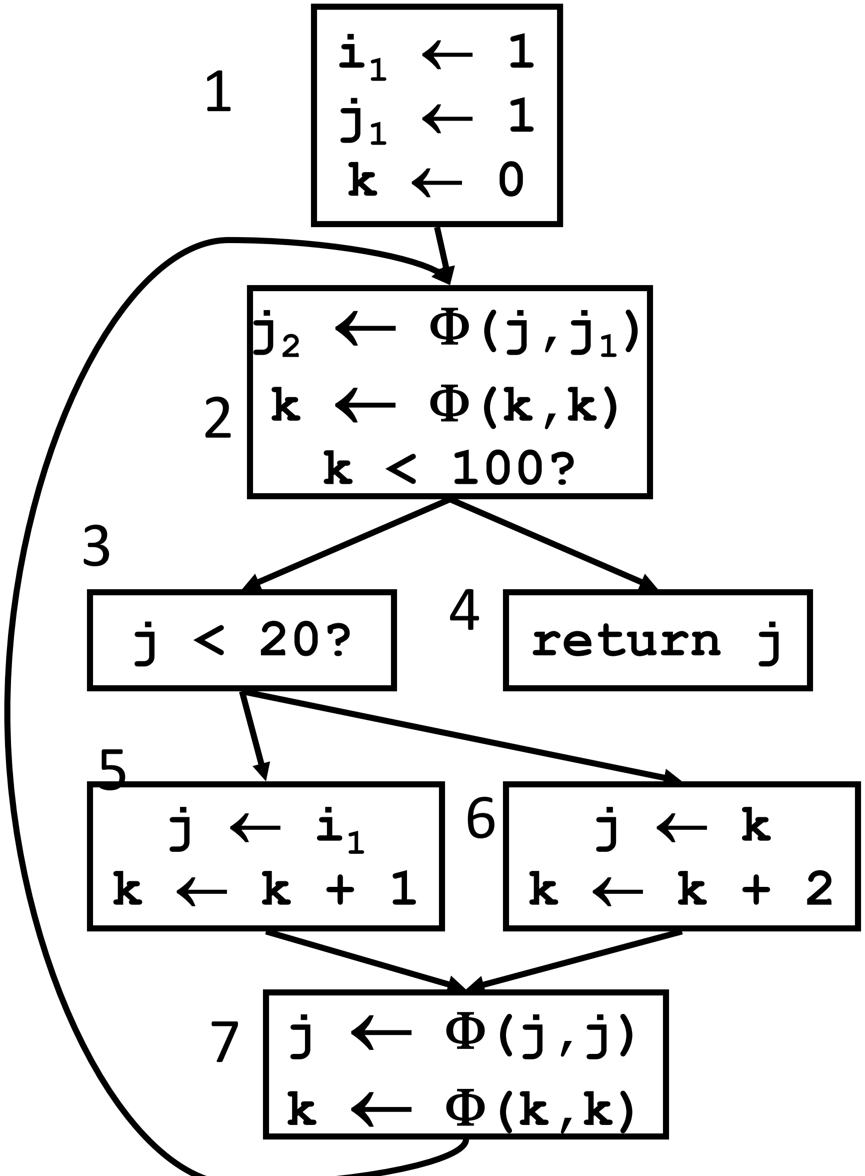
1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]

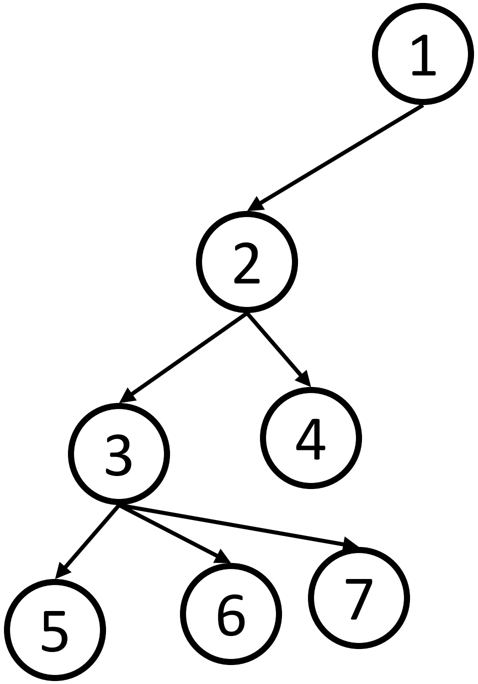
i	{1}
j	{1,5,6}
k	{1,5,6}

var k: W={1,5,6}

Rename Vars

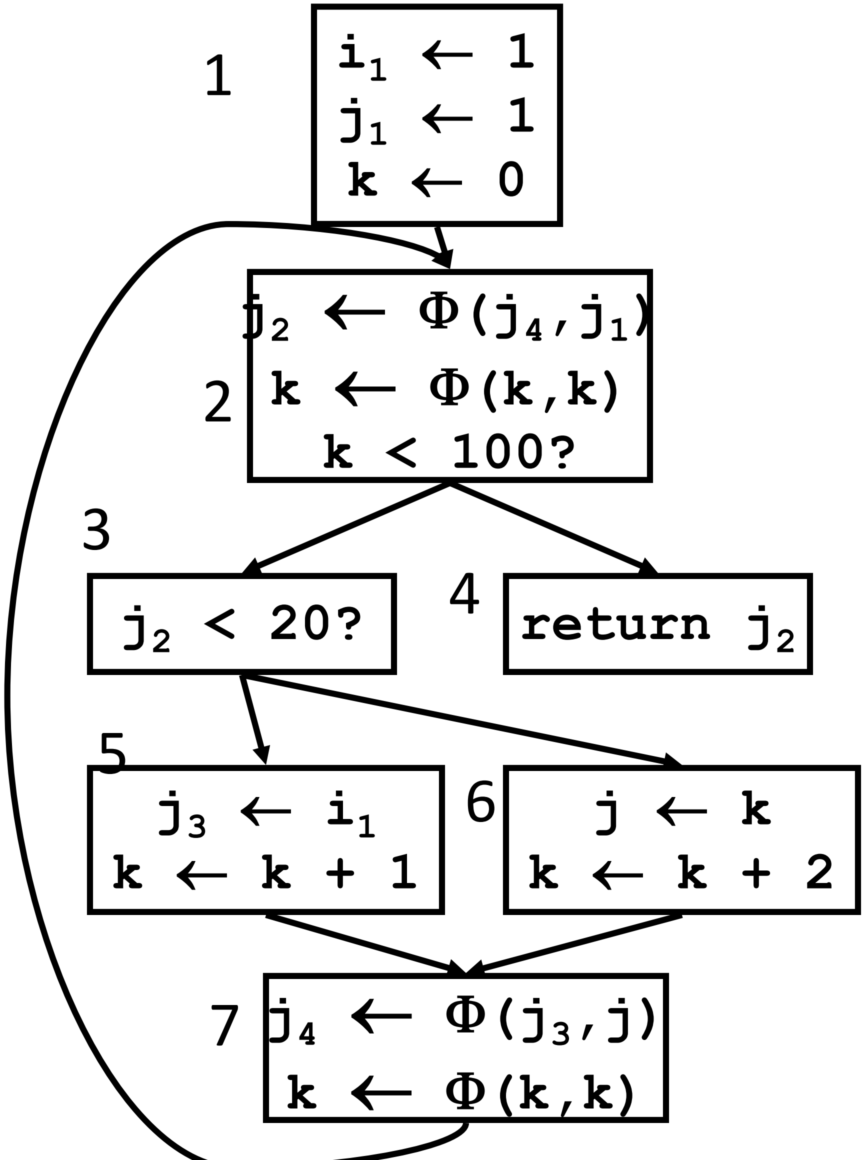


1
i1 ← 1
j1 ← 1
k ← 0
2
j2 ← Φ(j,j1)
k ← Φ(k,k)
k < 100?
3
j < 20?
4
return j
5
j ← i1
k ← k + 1
6
j ← k
k ← k + 2
7
j ← Φ(j,j)
k ← Φ(k,k)

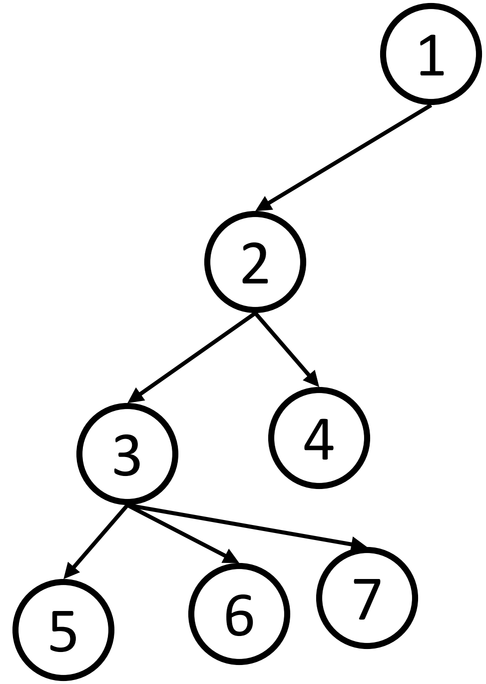


1
2
3
4
5
6
7

Rename Vars



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k \leftarrow 0$
2
$j_2 \leftarrow \Phi(j_4,j_1)$
$k \leftarrow \Phi(k,k)$
k < 100?
3
j_2 < 20?
4
return j_2
5
$j_3 \leftarrow i_1$
$k \leftarrow k + 1$
6
$j \leftarrow k$
$k \leftarrow k + 2$
7
$j_4 \leftarrow \Phi(j_3,j)$
$k \leftarrow \Phi(k,k)$



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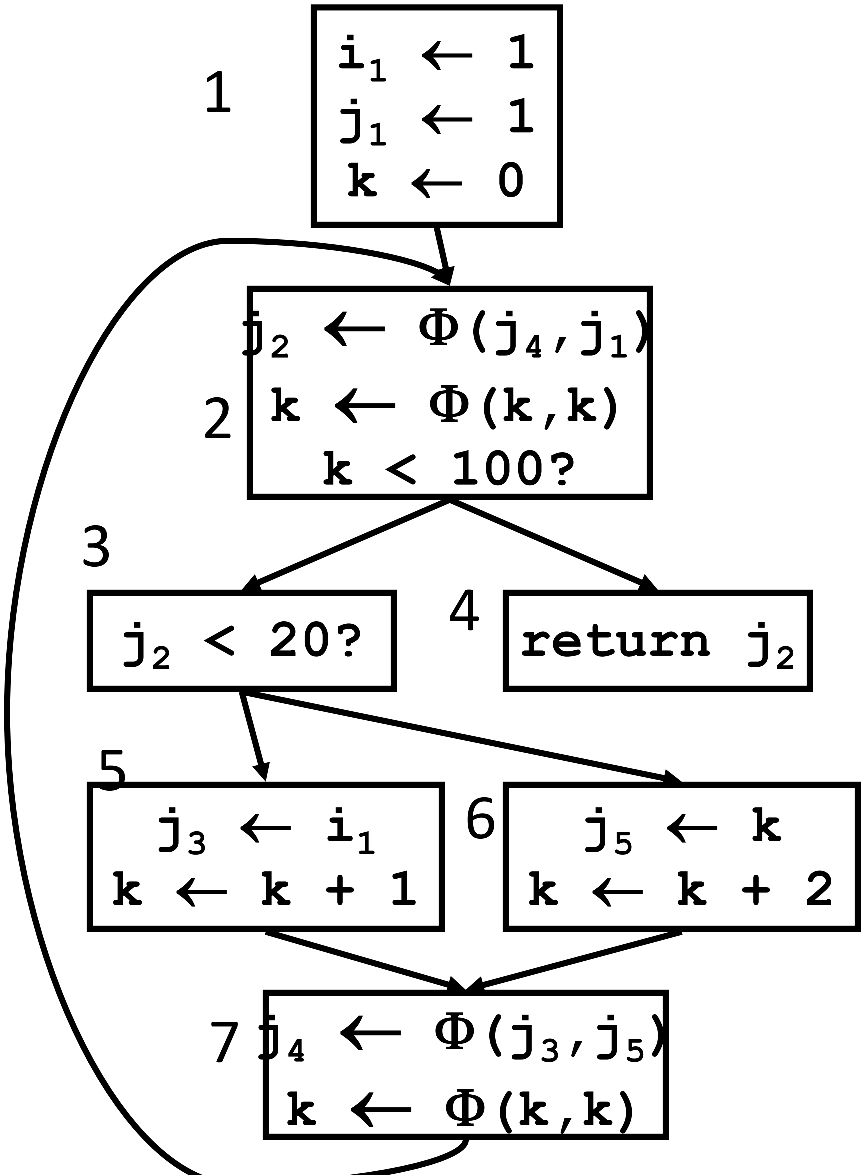
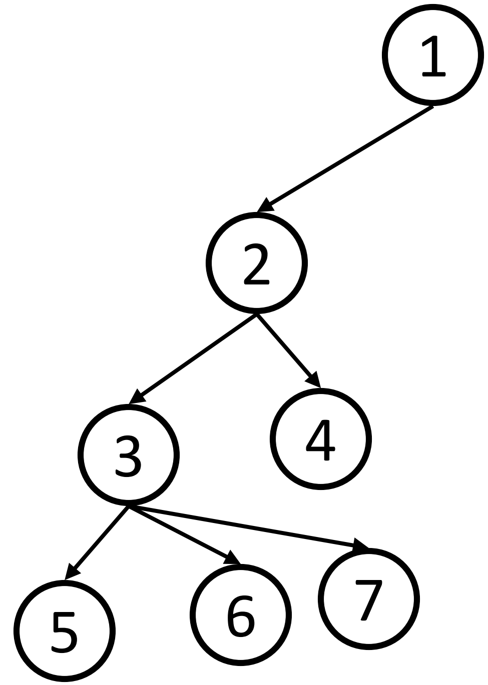
Rename Vars



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k \leftarrow 0$
2
$j_2 \leftarrow \Phi(j_4, j_1)$
$k \leftarrow \Phi(k, k)$
$k < 100?$
3
$j_2 < 20?$
4
return j_2
5
$j_3 \leftarrow i_1$
$k \leftarrow k + 1$
6
$j_5 \leftarrow k$
$k \leftarrow k + 2$
7
$j_4 \leftarrow \Phi(j_3, j_5)$
$k \leftarrow \Phi(k, k)$



1
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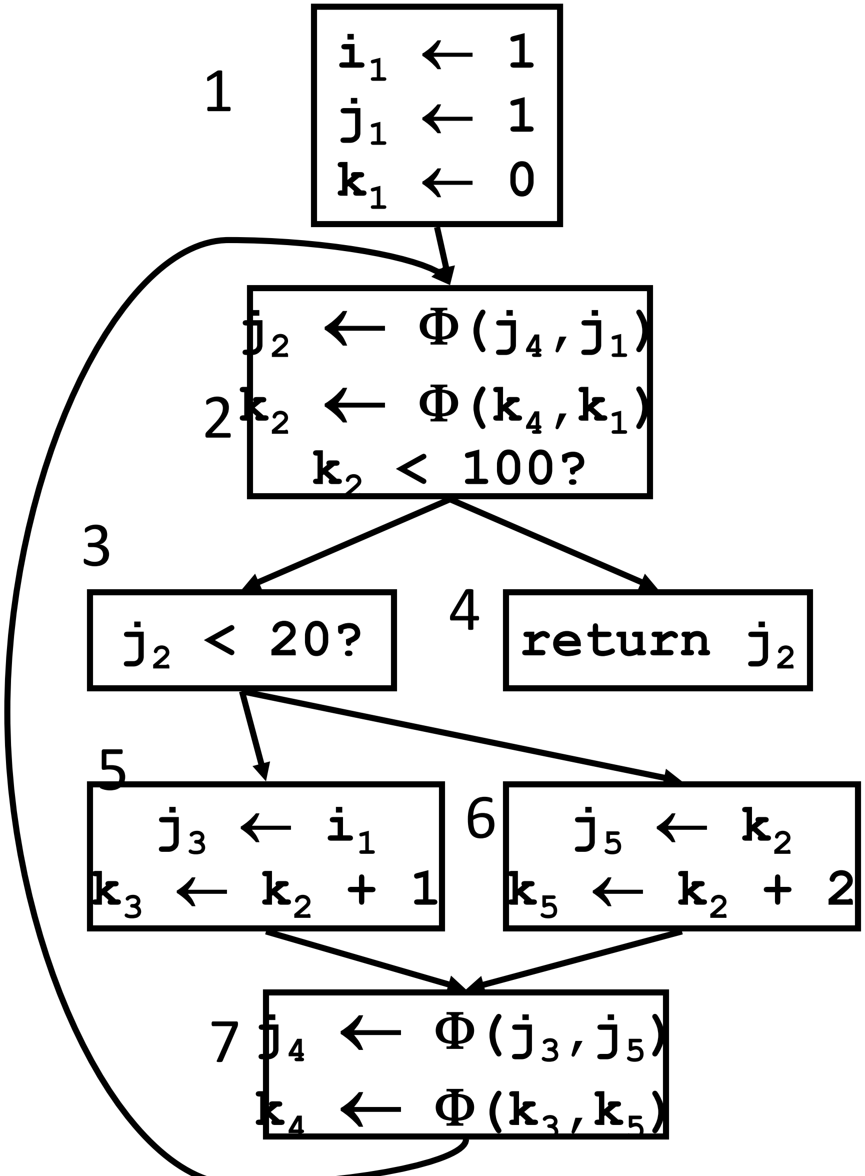
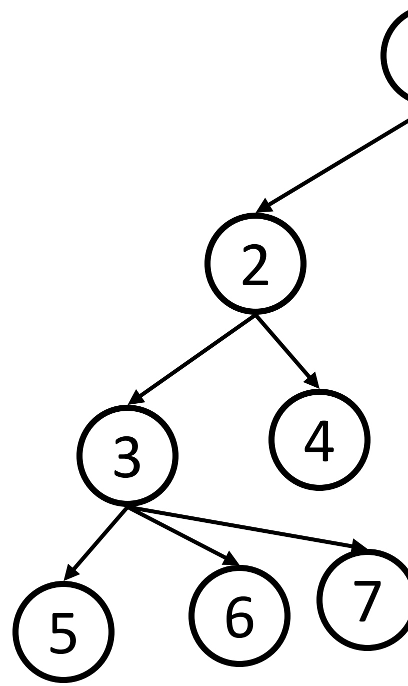
Rename Vars


1
i1 ← 1
j1 ← 1
k1 ← 0
2
j2 ← Φ(j4,j1)
k2 ← Φ(k4,k1)
k2 < 100?
3
j2 < 20?
4
return j2
5
j3 ← i1
k3 ← k2 + 1
6
j5 ← k2
k5 ← k2 + 2
7
j4 ← Φ(j3,j5)
k4 ← Φ(k3,k5)


1
2
3
4
5
6
7

Flavors of SSA

- Minimal SSA
  - at each join point with >1 outstanding definition insert a ϕ-function
  - Some may be dead
- Pruned SSA
  - only add live ϕ-functions
  - must compute LIVEOUT
- Semi-pruned SSA
  - Same as minimal SSA, but only on names live across more than 1 basic block

Deconstructing SSA

- Real machines don't have Φ-functions
- Implement with moves (and swaps) to predecessors.

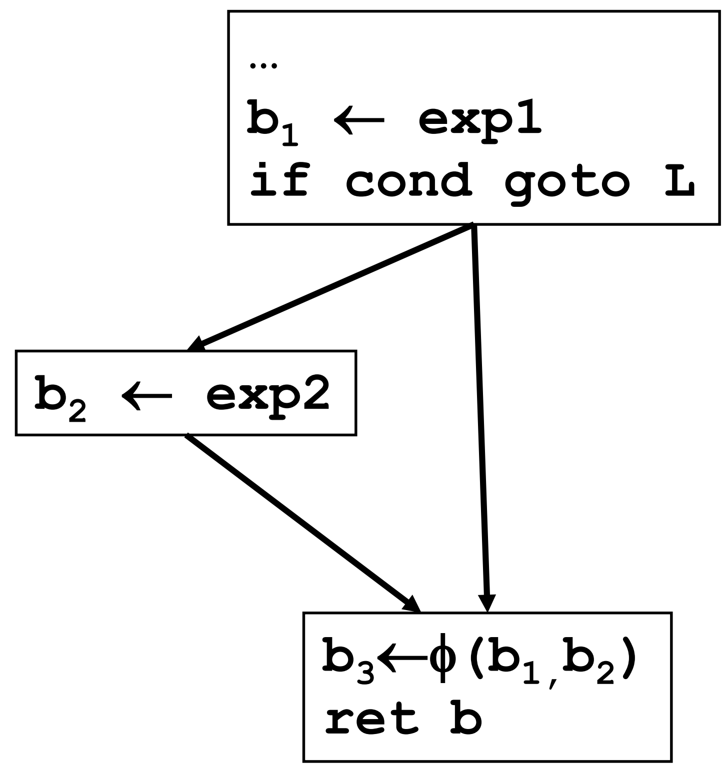
$a_1 \leftarrow x + y$
$b_1 \leftarrow a_1 + x$
$a_3 \leftarrow a_1$
$c_3 \leftarrow c_1$

$a_2 \leftarrow b + 2$
$c_2 \leftarrow y + 1$
$a_3 \leftarrow a_2$
$c_3 \leftarrow c_2$

$\cancel{a_3 \leftarrow \Phi(a_1, a_2)}$
$\cancel{c_3 \leftarrow \Phi(c_1, c_2)}$
$a_4 \leftarrow c_3 + a_3$

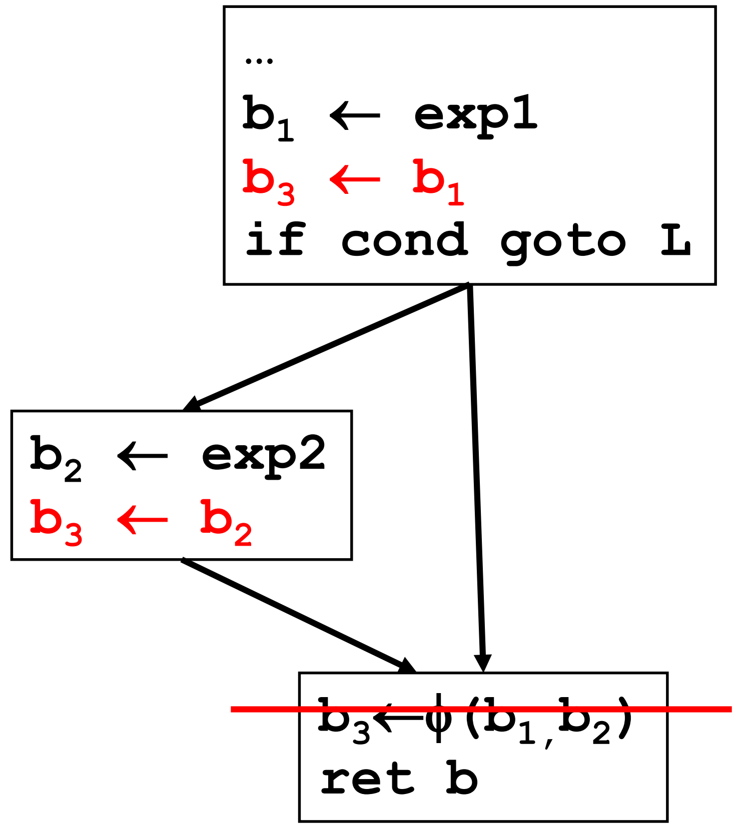
Deconstructing SSA

- Real machines don't have Φ-functions
- Implement with moves (and swaps) to predecessors.
- Issue 1: critical edges



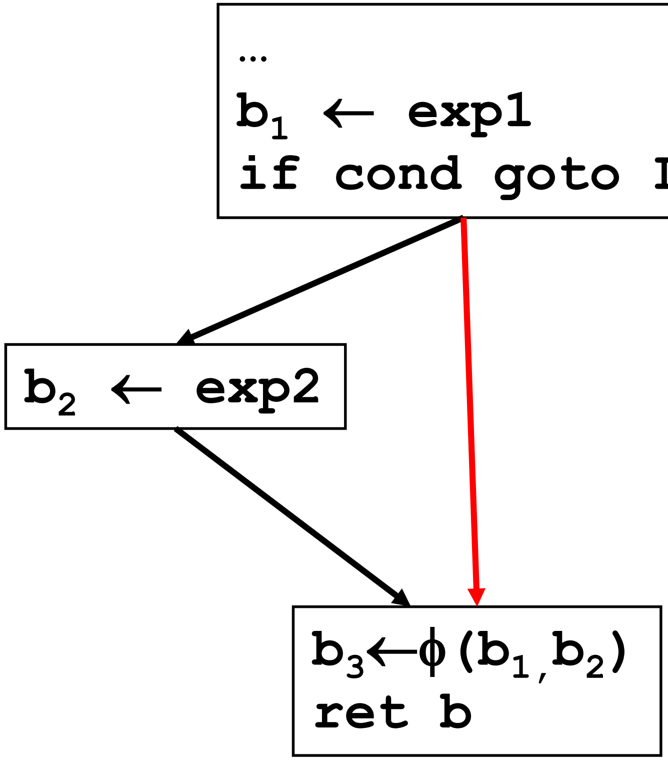
Deconstructing SSA

- Real machines don't have Φ-functions
- Implement with moves (and swaps) to predecessors.
- Issue 1: critical edges
  – unnecc assignment
  – So, first remove critical edges



Removing a Critical Edge

- A critical edge is an edge from a to b when a has > 1 successor and b has > 1 predecessor.
- For each edge (a,b) in CFG
  where a > 1 succ and
  b > 1 pred
  - Insert new block Z
  - replace (a,b) with
    - (a,z) and (z,b)



Removing a Critical Edge

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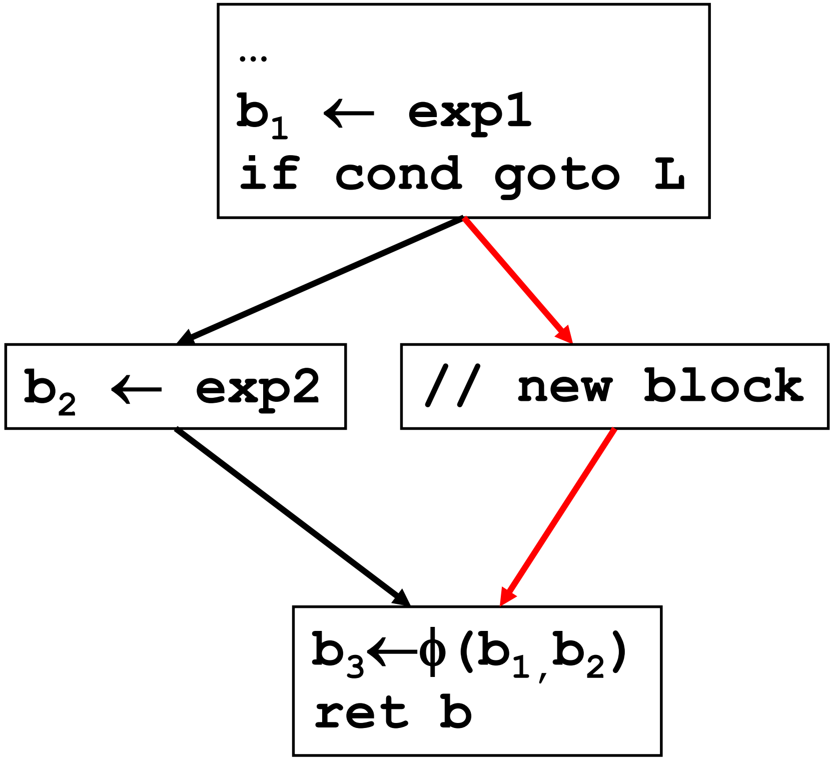
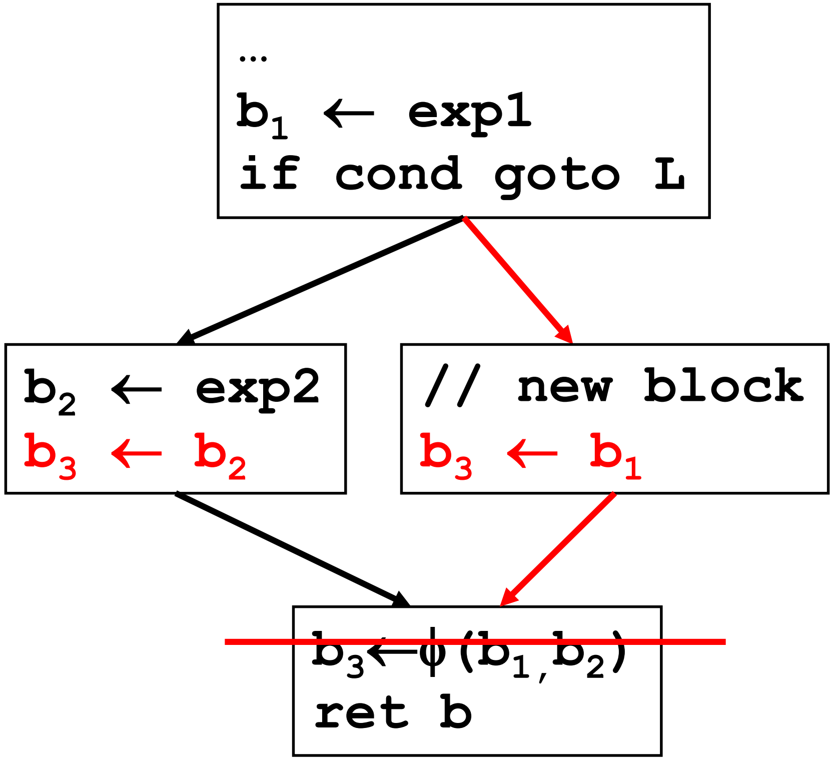


Replacing Φ

- Insert **`b`** ← **`b`**$_i$ in predecessor block for each **`b`**$_i$ in Φ
- remove Φ

- There are still some issues, but only if certain optimizations are performed.



Summary

- SSA is a useful and efficient IR.
- Definitions dominate Uses
- Constructing SSA can be efficient
  (No need to do Lengaur-Tarjan Algorithm, instead see A Simple, Fast Dominance Algorithm by Cooper, Harvey, and Kennedy)

- Don’t do any optimizations yet!