Name: ______________________________________ Andrew Id (Block Capitals): ______________________

ANSWERS TO 15-381 Midterm, Spring 2004

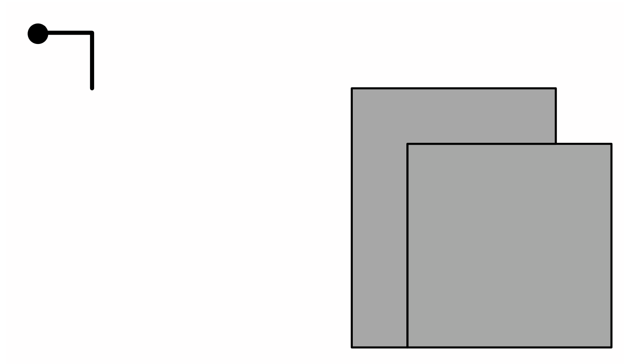
Thursday Mar 4, 2004

1. Place your name and your andrew email address on the front page.
2. You may use any and all notes, as well as the class textbook. Keep in mind, however, that this midterm was designed in full awareness of such.
3. We only require that you provide the answers. We don't need to see your working.
4. The maximum possible score on this exam is 100. You have 80 minutes.
5. The questions are ordered according to our estimate of time it takes to solve each problem. Shortest first.
6. Good luck!

1 Robot Motion Planning (10 points)

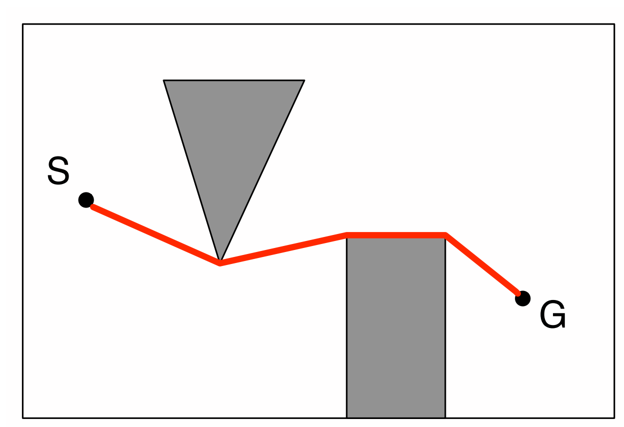
1.1 Configuration Space Transform

Look at this figure. The left one is a robot. The black square is an obstacle. The small robot may translate in 2-D but may not rotate. If we turn the robot into a single point (the black dot), how should we transform the obstacle? Draw the expanded obstacle in the figure.



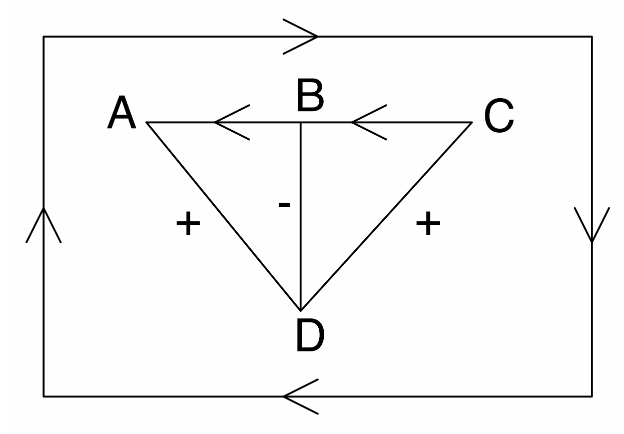
1.2 Visibility Graph

Show the path from S to G that you get using Visibility Graph method. (Only show the path. No need to draw the full visibility graph.)



2 Waltz Algorithm (15 points)

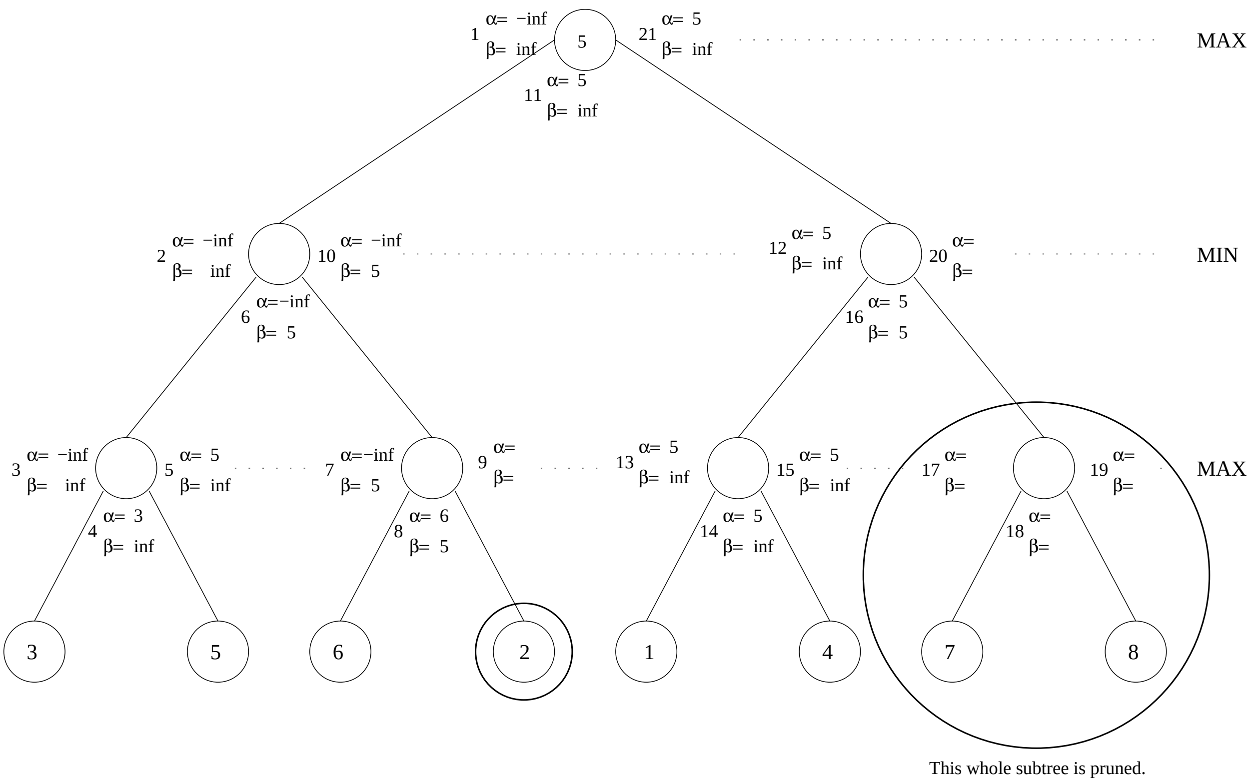
Below is a drawing. Some of the edges have already been labeled. Please show the solution Constraint Propagation would find for the labels for AD, BD and CD. (You don't need to show the steps. Just draw the results directly on the figure.)



3 Alpha-Beta Pruning (15 points)

Perform the alpha-beta algorithm on the following tree, searching left before right:

- Record to the left of a node the alpha and beta values upon first visiting the node.
- Record under a node the alpha and beta values after visiting the left child of the node.
- Record to the right of a node the alpha and beta values upon visiting both children.
- Circle any nodes (including leaves) that are not visited.
- Alpha and beta values need not be written for unvisited nodes nor for leaf nodes.
- Write in the game-theoretic value of the root node in the circle.
- Draw arrows along the path that would be taken if both players made optimal decisions.



4 Auctions (15 points)

Apple's CEO was visiting CMU and personally gave you a 5Gb iPod. But it's of no value to you because you already have a 40Gb one, so you decided to auction it off. Only 3 people are interested in participating in the auction. You know they'll be assigned their values uniformly at random assuming they want to pay $\leq \$50$.

1. What is the expected highest value assigned to the bidders?

   Answer: \$37.5. Let number of bidders be n, then $n = 3$. The expected highest assigned bid is $50 \cdot \frac{n}{n+1} = 50 \cdot \frac{3}{4} = 37.5$

2. What is your expected profit if everybody is playing Nash Equilibria and it's a first-price sealed-bid auction?

   Answer: \$25. According to Nash Equilibrium the winning bid will be (the highest assigned bid)$\cdot(1 - \frac{1}{n}) = 37.5 \cdot \frac{2}{3} = 25$. Since the iPod is of 0 value to you, your profit will be \$25.

3. Suppose the values assigned to the bidders were \$25,\$37 and \$40. What would your profit be if it were a second-price sealed-bid auction assuming NE?

   Answer: \$37. As was mentioned in class, the Nash Equilibrium solution for the second-price sealed-bid auction is to bid truthfully. Hence, the bids will be \$25,\$37 and \$45. The winning bid will be \$45 and the bidder will have to pay \$37, thus your profit will be \$37.

5 Toxic Dumping (15 points)

Four corporations, Enrox, Onren, Exron and Enren, all live round a lake. Company i will discharge x_i tons of toxin into the lake. Each company can individually discharge up to 50 tons, thus for all i, $0 \leq x_i \leq 50$. Discharges are simultaneous. For each ton it dumps a company saves $10. If the lake fills up with *strictly* more than 100 tons the lake explodes and fatally poisons all the inhabitants of the adjacent towns. The resulting loss of employees and public relations issues and lobbying issues cost each company $10,000,000 if this happens, regardless of who is to blame. Note that if there are exactly 100 tons of toxin in the lake it does not explode.

In this scenario, a strategy is described by giving the number of tons of toxin dumped by each company.

(a) Is there a pure strategy Nash Equilibrium in which all companies discharge equal amounts and in which the lake remains safe? If so, give an example of such an equilibrium.

Answer: Yes. 25, 25, 25, 25 is the only such pure strategy Nash Equilibrium.

(b) Is there a pure strategy Nash Equilibrium in which all companies discharge equal amounts and in which the lake explodes? If so, give an example of such an equilibrium.

Answer: Yes. 50, 50, 50, 50 is the only such pure strategy Nash Equilibrium.

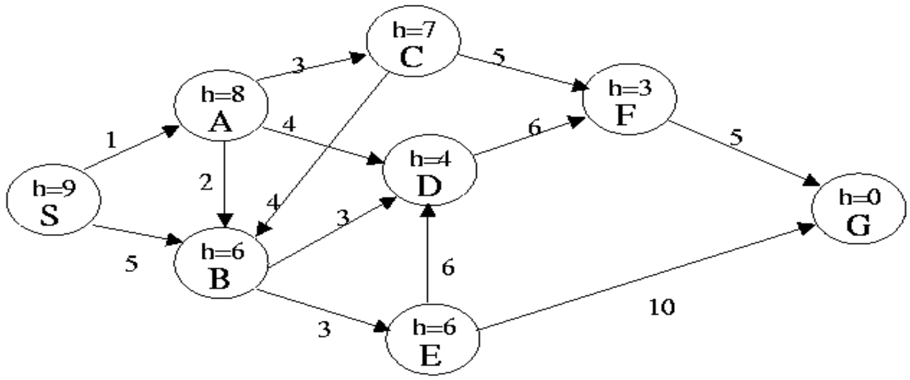
(c) Is there a pure strategy Nash Equilibrium in which not all the discharges are the same number of tons and in which the lake remains safe? If so, give an example of such an equilibrium.

Answer: Yes. Any strategy in which the discharges are not the same, and in which the sum of all of the discharges is equal to 100 is a pure strategy Nash Equilibrium in which the lake remains safe. 50, 50, 0, 0 is one example, 33, 33, 33, 1 is another.

(d) Is there a pure strategy Nash Equilibrium in which not all the discharges are the same number of tons and in which the lake explodes? If so, give an example of such an equilibrium.

Answer: No.

6 Search (15 points)



In this problem the start state is **S**, and the goal state is **G**. The transition costs are next to the edges, and the heuristic estimate, h, of the distance from the state to the goal is in the state's node. Assume ties are always broken by choosing the state which comes first alphabetically.

1. What is the order of states expanded using Depth First Search? Assume DFS terminates as soon as it reaches the goal state.

   Answer: S, A, B, D, F ,G

2. What is the order of states expanded using Breadth First Search?

   Answer: S, A, B, C, D, E, F, G

3. What is the order of states expanded using Best First Search? Assume BFS terminates as soon as it reaches the goal state.

   Answer: S, B, D, F, G

4. What is the order of states expanded using A* search?

   Answer: S, A, B, D, C, E, F, G

5. What is a least cost path from S to G?

   Answer: S, A, C, F, G