15-381 Spring 2007 Assignment 3 Solutions: Robot Motion Planning and Game Theory

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1. Robot Motion Planning

   (a) (5 points) Draw (approximately) the Voronoi edges in this space assuming that A, B, C, and D are obstacles (ignoring S and G). Now draw the path from start (S) to goal (G) found by using the Voronoi diagram. `There was no need to consider the walls as obstacles, but we also accepted solutions that had considered the walls.`

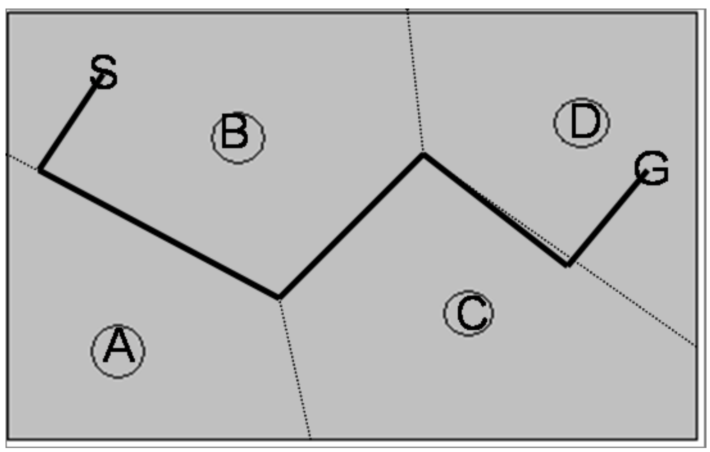

Figure 1: Voronoi solution

   (b) (5 points) Draw (approximately) the path found by using the visibility graph technique from start (S) to goal (G). Draw the initial visibility graph.

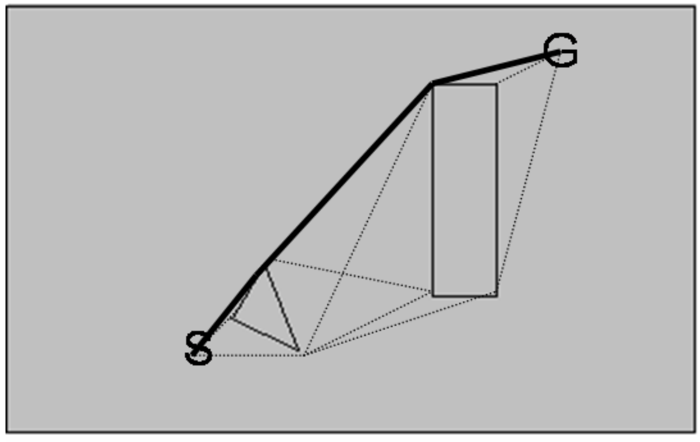

Figure 2: Visibility graph solution

(c) (5 points) Draw the path found by using approximate cell decomposition from start (S) to goal (G). Begin with an initial grid spacing of (approximately) 3 cm. Draw the cells that partition this space. `Solutions varied depending on the initial size of the image.`

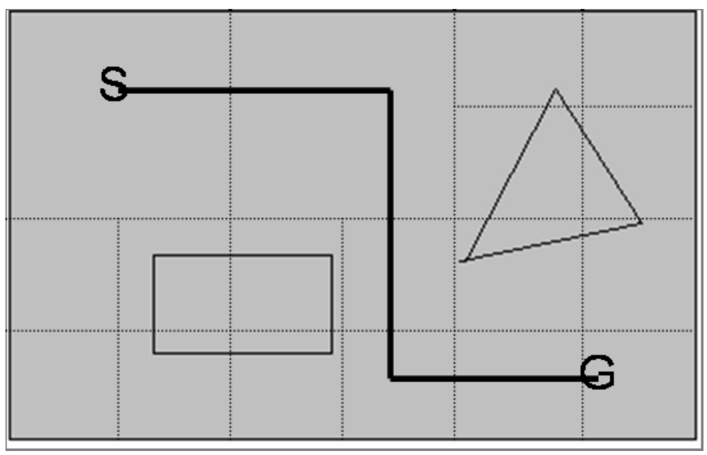

Figure 3: Approximate Cell Decomposition solution

(d) (5 points) Draw (approximately) the edges created with Rapidly Exploring Random Trees when the random new sample nodes appear in the order specified on the diagram. The initial tree is just the node S. Use the second way of determining ϵ as discussed in class. That is: if the line between q_{near} and q_{rand} intersects an obstacle, q_{new} is the closest point to q_{rand} in front of (on the q_{near} side of) the obstacle. Otherwise, q_{new} is q_{rand}.

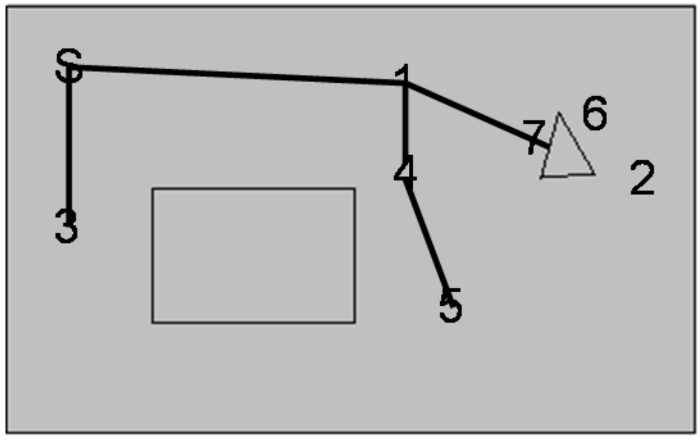

Figure 4: RRT solution

2. Probabilistic Roadmap

(a) (40 points) For this question, you will implement A-star search on a probabilistic roadmap using the samples provided. Your roadmap will use only samples in the free space specified by the map and consider the $K = 4$ nearest neighbors of each sample for creating edges. Of the $K = 4$ possible edges, only create those that do not intersect obstacles. The start and goal points can effectively be considered samples when creating edges.

(b) (2 pts) Describe the advantages and disadvantages of using sampling.

`The advantage of sampling is that it finds reasonable solutions in domains too large (whether size-wise or dimension-wise) for optimal search techniques like A* to be feasible. The disadvantage is that it may not find a solution even if one exists.`

(c) (6 pts) Report the results of running each of the three map (map1, map2, map3) scenarios with the two sample (samples1, samples2) files. Be sure to report whether or not a solution was found and the number of expansions. Include all the output requested when running the executable. Also discuss the effectiveness of the two different sampling strategies used on the map scenarios.

```
./findPath map1.txt samples1.txt
Number of expansions:  6
Path not found

./findPath map2.txt samples1.txt
Number of expansions:  6
(2,1)
(2,5)
(4,6)
(6,6)
Total distance:  8.23607

./findPath map3.txt samples1.txt
Number of expansions:  7
Path not found

./findPath map1.txt samples2.txt
Number of expansions:  4
Path not found

./findPath map2.txt samples2.txt
```

```
Number of expansions:  5
(2,1)
(5,1)
(6,4)
(6,6)
Total distance:  8.16228

./findPath map3.txt samples2.txt
Number of expansions:  6
(2,6)
(1,4)
(2,2)
(5,1)
(4,4)
Total distance:  10.7967
```

Samples2.txt worked better since the samples were spread more uniformly.

(d) (2 pts) Describe how you detected the intersection of an edge of your roadmap with an obstacle. Also describe the heuristic you used with A-star.

One way to detect the intersection of an edge of your roadmap with an obstacle is to start at the center of the cell at one end of the edge and walk along the edge in small increments to the center of the cell at the other end of the edge. At each increment, check if that point along the edge passes into the boundaries of an obstacle cell.

An easy heuristic to use with A-star is straight line distance.

3. (5 points) Chomp is a 2-player game played on a rectangular "chocolate bar" made up of smaller rectangular cells. The players take it in turns to choose one block and "eat it" (remove from the board), together with those that are below it and to its right. The top left block is "poisoned" and the player who eats this loses. Please see http://en.wikipedia.org/wiki/Chomp for more details about this game. This game is an example of a two-player zero-sum deterministic game of perfect information.

Any two-player zero-sum deterministic game can be represented by the following quintuple: $(S, I, Succ, T, V)$, where S is the entire space of game states, I is the initial state, $Succ$ is the successor function, T are the terminal states, and V maps from terminal states to its payoff/utility. Please describe each element of the quintuple in the game of Chomp.

There are multiple ways to express this. Here is one:
S: A rectangular grid with an asterisk (*) at the top left cell for ''poison'', with 1s for ''chocolate'' and 0s for ''empty space'' such that there is never a 1 below a 0 and never a 1 directly to the right of a 0. Each state also includes a number I or II denoting which player will select a piece to eat next.
I: The rectangular grid with an asterisk (*) at the top left cell and 1s everywhere else, with I or II to indicate the first player.
$Succ$: Select a single cell that has value 1 or $*$ on the board. Convert that cell and all cells to the right and below to 0s.
T: One player makes a move that results in the top left cell being eaten (converting the $*$ to 0).
V: The player who eats the top left cell gets -1. The other player gets +1.

4. (5 points) A vendor at a street carnival offers the following games that involve rolling a single 6-sided die:

Game 1: You pay 3 dollars and win the dollar amount on the roll.
Game 2: You pay 1 dollar and win 2 dollars if the roll is odd.
Game 3: You pay 2 dollars and win 8 dollars if you roll at least a 5.

Which of these games would you play? Rank the games in order of which you would play them. Explain your reasoning in terms of expected payoff.

Expected payoff for Game 1: $\frac{1+2+3+4+5+6}{6} - 3 = \frac{1}{2}$ dollars
Expected payoff for Game 2: $\frac{1}{2} \times 2 + \frac{1}{2} \times 0 - 1 = 0$ dollars
Expected payoff for Game 3: $\frac{1}{3} \times 8 + \frac{2}{3} \times 0 - 2 = \frac{2}{3}$ dollars

To maximize my expected payoff, I would rank the games in the order of 3 (best), 1, 2. I may or may not play Game 2 since my expected payoff is 0 dollars.

5. The Even versus Odd game

A game is played as follows: The two players, A and B, simultaneously hold up one or two sticks. A wins if the total number of sticks is odd while B wins otherwise. The amount won is the total number of sticks held. It is paid to the winner by the loser.

(a) (5 points) Write down the matrix form of the game. Is there a pure strategy solution in this game? Explain why or why not.

		B	
		I	II
A	I	-2	+3
	II	+3	-4

Figure 5: Matrix form of Even versus Odd game

There is no pure strategy solution to this game. None of the four game states are in equilibrium. If we start at state (I,I), A will switch to strategy II to get a higher payoff, but if A switches to strategy II, B will then switch to strategy II to get a higher payoff, and the cycle continues without convergence.

(b) (5 points) Assume B holds up one stick $\frac{1}{2}$ the time and two sticks the other $\frac{1}{2}$ of the time. What is the expected payoff for player A if A also chooses one stick $\frac{1}{2}$ the time and two sticks the other $\frac{1}{2}$ of the time? What is the expected payoff for player A if A chooses one stick $\frac{3}{4}$ths of the time and two sticks the remaining $\frac{1}{4}$th?

A's expected payoff: Prob(A chooses I) $\times$ [Prob(B chooses I) $\times$ reward at (I,I) $+$ Prob(B chooses II) $\times$ reward at (I,II)] $+$ Prob(A chooses II) $\times$ [Prob(B chooses I) $\times$ reward at (II,I) $+$ Prob(B chooses II) $\times$ reward at (II,II)]

If B holds up one stick $\frac{1}{2}$ the time and two sticks the other $\frac{1}{2}$ of the time, and A also chooses one stick $\frac{1}{2}$ the time and two sticks the other $\frac{1}{2}$ of the time, then A's expected payoff is: $\frac{1}{2} \times [\frac{1}{2} \times -2 + \frac{1}{2} \times 3] + \frac{1}{2} \times [\frac{1}{2} \times 3 + \frac{1}{2} \times -4] = 0$.

If B holds up one stick $\frac{1}{2}$ the time and two sticks the other $\frac{1}{2}$ of the time, and A also chooses one stick $\frac{3}{4}$ the time and two sticks the other $\frac{1}{4}$ of the time, then A's expected payoff is: $\frac{3}{4} \times [\frac{1}{2} \times -2 + \frac{1}{2} \times 3] + \frac{1}{4} \times [\frac{1}{2} \times 3 + \frac{1}{2} \times -4] = \frac{1}{4}$.

6. Cing and Veri were best friends at CMU, but Cing recently graduated and moved to another state. Wanting to stay connected, Cing and Veri both decide to sign up for cell phone service. However, they both have different preferences:

- Cing travels a lot and likes Cingular above all else for its extensive network.
- Veri likes Verizon above all else because of its cell tower conveniently placed above Wean Hall.
- If at least one of them gets T-mobile, the two could talk for free with the T-mobile Fave-5 plan.
- If both of them get Cingular, the two could talk for free with Cingular's in-Network plan.
- If both of them get Verizon, the two could talk for free with Verizon's in-Network plan.
- If they cannot talk for free, they would rather not have cell phone service at all.

(a) (5 points) We can model their preferences as a non-zero-sum game. Fill in the payoff matrix for the two of them that reflects their preferences (there are many ways to do this). State your assumptions (e.g. No cell phones pays 0).

`There are multiple solutions. Here is one:`

		Veri		
		Cingular	Verizon	T-mobile
Cing	Cingular	2,1	0,0	2,1
	Verizon	0,0	1,2	1,1
	T-mobile	1,1	1,2	1,1

Figure 6: Payoff matrix for Cing, Veri

```
Assumptions:
Getting your favored network pays 2
Not getting your favored network but getting to talk to the other pays 1
No cell phones pays 0
```

(b) (2 points) Are there any strictly dominating strategies for either of them? If so, which are they?

`No.  There are no strictly dominating strategies for either of them.`

(c) (3 points) Which strategy should they take? Please explain your reasoning.

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There are multiple equal Nash equilibria here.  According to our payoff matrix, the
nash equilibria are at (Cingular, Cingular), (Cingular, T-mobile), (Verizon, Verizon),
and (T-mobile, Verizon).  In any of these scenarios, neither party prefers a different
solution given that the strategy of the other is fixed, and there are no solutions
which are better for both players, so they should choose one of these by any definition
of optimality.
```