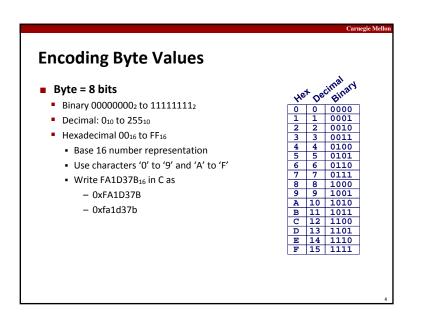
Bits, Bytes, and Integers

15-213: Introduction to Computer Systems
2<sup>nd</sup> and 3<sup>rd</sup> Lectures, Jan 19 and Jan 24, 2012

Instructors:
Todd C. Mowry & Anthony Rowe

# Binary Representations Base 2 Number Representation Represent 15213<sub>10</sub> as 11101101101101<sub>2</sub> Represent 1.20<sub>10</sub> as 1.0011001100110011[0011]...<sub>2</sub> Represent 1.5213 X 10<sup>4</sup> as 1.1101101101101<sub>2</sub> X 2<sup>13</sup> Electronic Implementation Easy to store with bistable elements Reliably transmitted on noisy and inaccurate wires 3.3V 2.8V 0.5V 0.0V

### Today: Bits, Bytes, and Integers Representing information as bits Bit-level manipulations Integers Representation: unsigned and signed Conversion, casting Expanding, truncating Addition, negation, multiplication, shifting Summary Representations in memory, pointers, strings



### **Data Representations**

C Data Type	Typical 32-bit	Intel IA32	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	4	8
long long	8	8	8
float	4	4	4
double	8	8	8
long double	8	10/12	10/16
pointer	4	4	8

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- Bit-level manipulations
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### **General Boolean Algebras**

- Operate on Bit Vectors
- Operations applied bitwise

All of the Properties of Boolean Algebra Apply

**Example: Representing & Manipulating Sets** Representation ■ Width w bit vector represents subsets of {0, ..., w-1} ■ aj = 1 if j ∈ A 01101001  $\{0, 3, 5, 6\}$ 76543210 01010101 {0, 2, 4, 6} 76543210 Operations 01000001 {0,6} & Intersection Union 01111101 { 0, 2, 3, 4, 5, 6 } Symmetric difference 00111100 { 2, 3, 4, 5 } Complement 10101010 { 1, 3, 5, 7 }

## Bit-Level Operations in C ■ Operations &, |, ~, ^ Available in C ■ Apply to any "integral" data type ■ long, int, short, char, unsigned ■ View arguments as bit vectors ■ Arguments applied bit-wise ■ Examples (Char data type) ■ ~0x41 → 0xBE ■ ~010000012 → 1011111102 ■ ~0x00 → 0xFF ■ ~000000002 → 1111111112 ■ 0x69 & 0x55 → 0x41 ■ 011010012 & 010101012 → 010000012 ■ 0x69 | 0x55 → 0x7D ■ 011010012 | 010101012 → 011111012

Contrast: Logic Operations in C

Contrast to Logical Operators

&&, ||,!

View 0 as "False"

Anything nonzero as "True"

Always return 0 or 1

Early termination

Examples (char data type)

10x41 → 0x00

10x00 → 0x01

110x41 → 0x01

0x69 && 0x55 → 0x01

0x69 || 0x55 → 0x01

0x69 || 0x55 → 0x01

p && \*p (avoids null pointer access)

**Contrast: Logic Operations in C** Contrast to Logical Operators **&** &&, ||, ! View 0 as "False" Anything nonzero as "True" Watch out for && vs. & (and || vs. |)... one of the more common oopsies in Always return 0 or 1 C programming Early termination Examples (char data type) ■  $!0x41 \rightarrow 0x00$ •  $!0x00 \rightarrow 0x01$ ■ !!0x41 → 0x01 ■ 0x69 && 0x55 → 0x01 ■ 0x69 || 0x55 → 0x01 p && \*p (avoids null pointer access)

### **Shift Operations**

- Left Shift: x << y
- Shift bit-vector x left y positions
  - Throw away extra bits on left
  - Fill with 0's on right
- Right Shift: x >> y
- Shift bit-vector x right y positions
  - Throw away extra bits on right
- Logical shift
  - Fill with 0's on left
- Arithmetic shift
  - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ word size</p>

• 0 for nonnegative

• 1 for negative

Argument x	01100010
<< 3	00010 <i>000</i>
Log. >> 2	00011000
<b>Arith.</b> >> 2	00011000

Argument x	10100010
<< 3	00010 <i>000</i>
Log. >> 2	00101000
Arith. >> 2	11101000

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**Encoding Integers** Unsigned **Two's Complement**  $B2U(X) = \sum_{i=1}^{w-1} x_i \cdot 2^i$ short int x = 15213;short int y = -15213;Sign Bit ■ C short 2 bytes long Decimal Hex Binary 3B 6D 00111011 01101101 C4 93 11000100 10010011 -15213 Sign Bit • For 2's complement, most significant bit indicates sign

**Encoding Example (Cont.)** 15213: 00111011 01101101 -15213: 11000100 10010011 y = Weight 15213 -15213 1 1 0 0 0 64 0 128 1 128 256 256 0 512 512 0 1024 1 1024 2048 2048 0 4096 4096 0 8192 8192 0 16384 16384 -32768 -32768 Sum 15213 -15213 Numeric Ranges
Unsigned Values

■ *UMin* = 0 000...0

■  $UMax = 2^w - 1$ 111...1 ■ Two's Complement Values

■  $TMin = -2^{w-1}$ 100...0

■  $TMax = 2^{w-1} - 1$ 011...1

Other Values

Minus 1111...1

Values for W = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

**Values for Different Word Sizes** 

			W	
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

Observations

■ | *TMin* | = *TMax* + 1

Asymmetric range

UMax = 2 \* TMax + 1

C Programming

#include limits.h>

Declares constants, e.g.,

ULONG\_MAX

LONG\_MAX

LONG\_MIN

Values platform specific

**Unsigned & Signed Numeric Values** 

Х	B2U(X)	B2T(X)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	<b>-</b> 5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

Equivalence

Same encodings for nonnegative values

Uniqueness

 Every bit pattern represents unique integer value

 Each representable integer has unique bit encoding

■ ⇒ Can Invert Mappings

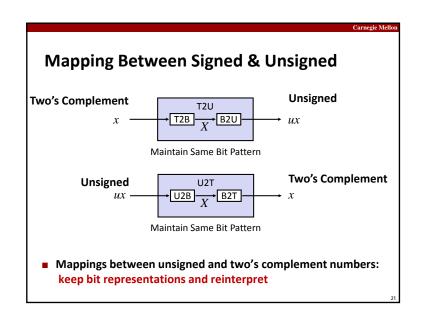
• U2B(x) = B2U<sup>-1</sup>(x)

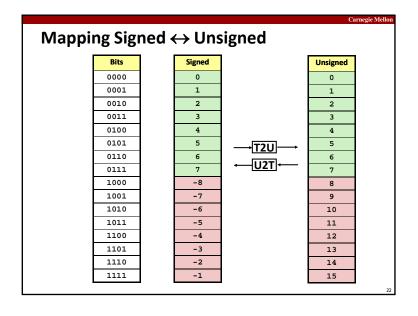
 Bit pattern for unsigned integer

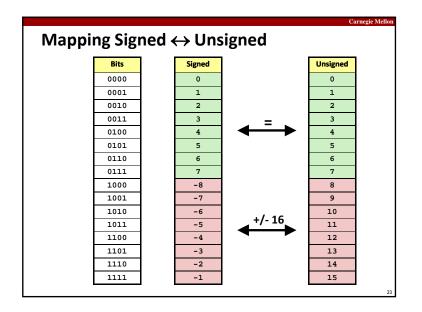
■  $T2B(x) = B2T^{-1}(x)$ 

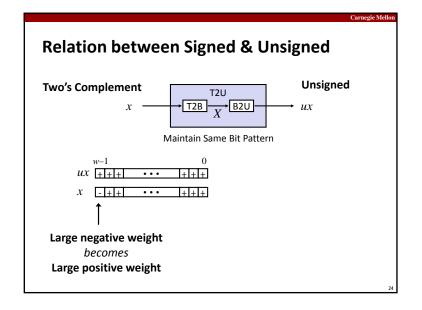
 Bit pattern for two's comp integer **Today: Bits, Bytes, and Integers** 

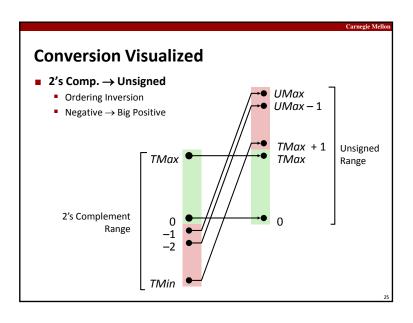
- Representing information as bits
- Bit-level manipulations
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Signed vs. Unsigned in C

Constants

By default are considered to be signed integers

Unsigned if have "U" as suffix

U, 4294967259U

Casting

Explicit casting between signed & unsigned same as U2T and T2U int tx, ty;

unsigned ux, uy;

tx = (int) ux;

uy = (unsigned) ty;

Implicit casting also occurs via assignments and procedure calls

tx = ux;

uy = ty;

### **Casting Surprises**

- **■** Expression Evaluation
  - If there is a mix of unsigned and signed in single expression, signed values implicitly cast to unsigned
  - Including comparison operations <, >, ==, <=, >=
  - Examples for W = 32: TMIN = -2,147,483,648, TMAX = 2,147,483,647

■ Constant <sub>1</sub>	Constant <sub>2</sub>	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

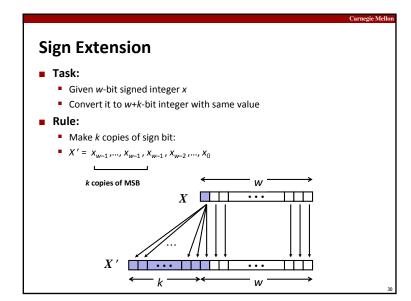
- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2<sup>w</sup>
- Expression containing signed and unsigned int
  - int is cast to unsigned!!

Carnegie Mello

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**Sign Extension Example** 

short int x = 15213;
int ix = (int) x;
short int y = -15213;
int iy = (int) y;

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
У	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	1111111 11111111 11000100 10010011

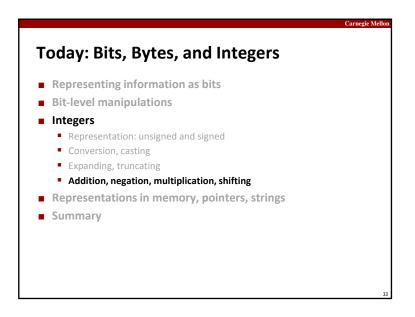
- Converting from smaller to larger integer data type
- C automatically performs sign extension

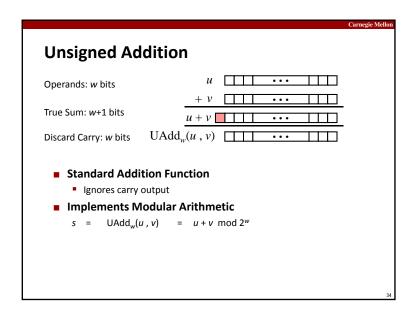
Summary:

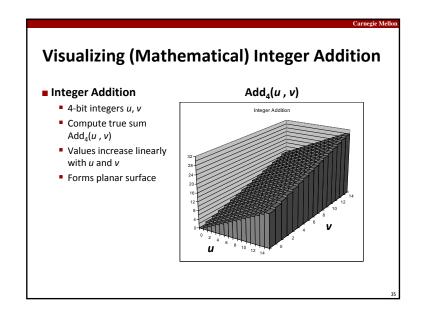
### **Expanding, Truncating: Basic Rules**

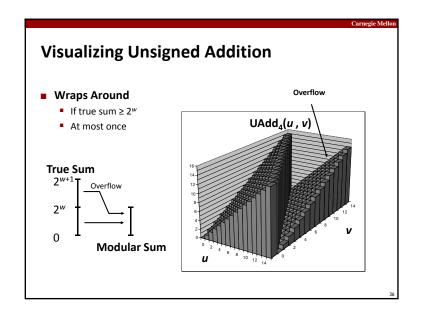
- Expanding (e.g., short int to int)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result
- Truncating (e.g., unsigned to unsigned short)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers yields expected behavour

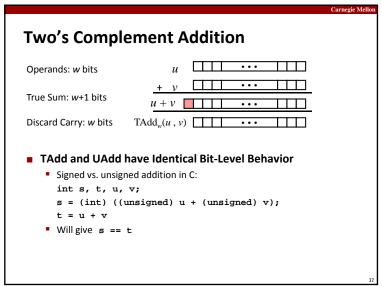
Carnegie Mel

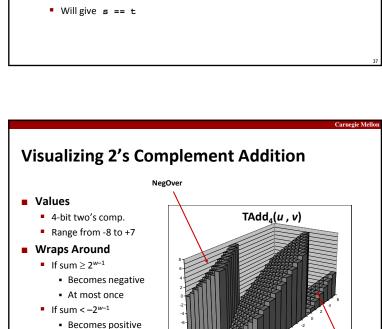






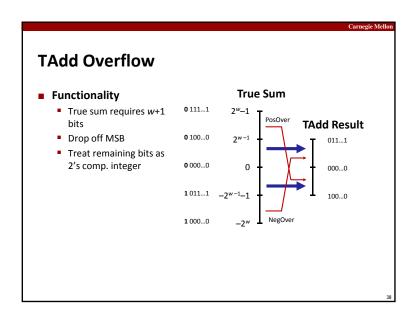






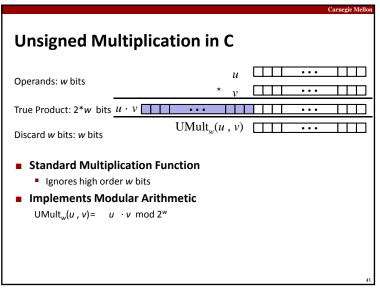
PosOver

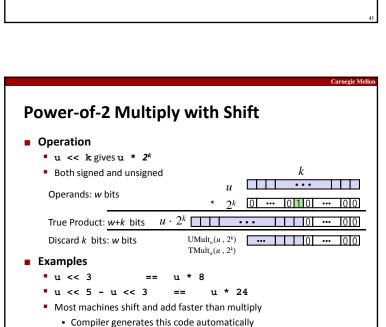
At most once

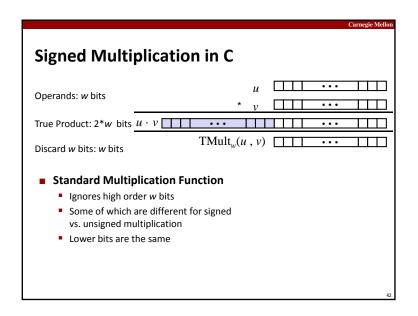


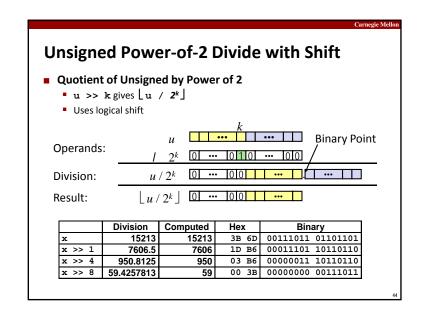
### Multiplication

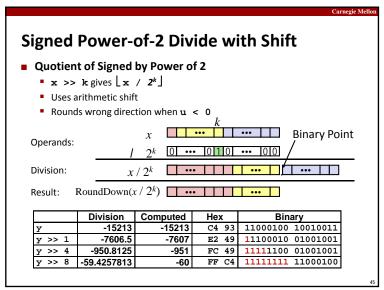
- Goal: Computing Product of w-bit numbers x, y
  - Either signed or unsigned
- But, exact results can be bigger than w bits
  - Unsigned: up to 2w bits
    - Result range:  $0 \le x * y \le (2^w 1)^2 = 2^{2w} 2^{w+1} + 1$
  - Two's complement min (negative): Up to 2w-1 bits
    - Result range:  $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
  - Two's complement max (positive): Up to 2w bits, but only for (TMin<sub>w</sub>)<sup>2</sup>
    - Result range:  $x * y \le (-2^{w-1})^2 = 2^{2w-2}$
- So, maintaining exact results...
  - would need to keep expanding word size with each product computed
  - is done in software, if needed
    - e.g., by "arbitrary precision" arithmetic packages

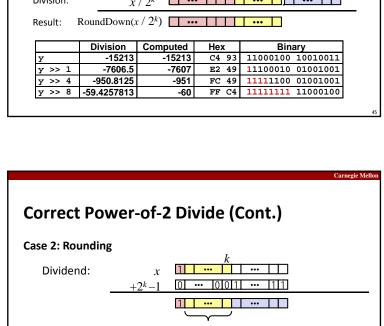


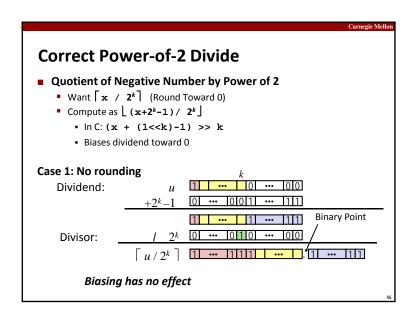


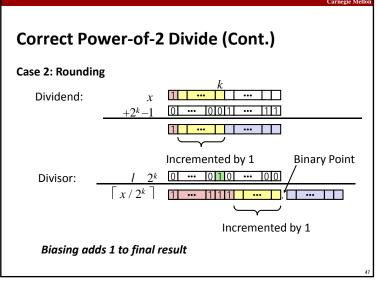


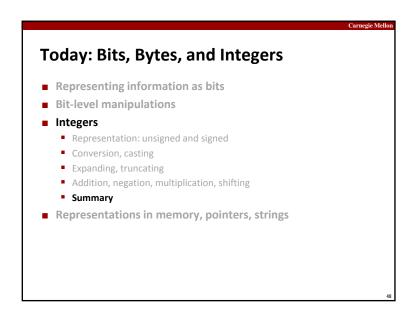












### **Arithmetic: Basic Rules**

### Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod 2<sup>w</sup>
  - Mathematical addition + possible subtraction of 2<sup>w</sup>
- Signed: modified addition mod 2<sup>w</sup> (result in proper range)
  - Mathematical addition + possible addition or subtraction of 2<sup>w</sup>

### Multiplication:

- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod 2<sup>w</sup>
- Signed: modified multiplication mod 2<sup>w</sup> (result in proper range)

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### Why Should I Use Unsigned?

■ Don't Use Just Because Number Nonnegative

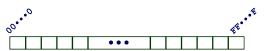
```
Easy to make mistakes
   unsigned i;
   for (i = cnt-2; i >= 0; i--)
     a[i] += a[i+1];
```

Can be very subtle

#define DELTA sizeof(int) int i: for (i = CNT; i-DELTA >= 0; i-= DELTA)

- Do Use When Performing Modular Arithmetic
  - Multiprecision arithmetic
- Do Use When Using Bits to Represent Sets
  - Logical right shift, no sign extension

### **Byte-Oriented Memory Organization**



- Programs refer to data by address
  - Conceptually, envision it as a very large array of bytes
    - In reality, it's not, but can think of it that way
  - An address is like an index into that array
    - and, a pointer variable stores an address
- Note: system provides private address spaces to each "process"
- Think of a process as a program being executed
- So, a program can clobber its own data, but not that of others

# Machine Words Any given computer has a "Word Size" Nominal size of integer-valued data and of addresses Most current machines use 32 bits (4 bytes) as word size Limits addresses to 4GB (2<sup>32</sup> bytes) Becoming too small for memory-intensive applications leading to emergence of computers with 64-bit word size Machines still support multiple data formats Fractions or multiples of word size Always integral number of bytes

Addresses Specify Byte	32-bit Words	64-bit Words	Bytes	Addı
Locations				0000
<ul> <li>Address of first byte in word</li> </ul>	Addr =			0001
<ul> <li>Addresses of successive words differ</li> </ul>	0000			0002
by 4 (32-bit) or 8 (64-bit)		Addr =		0003
by 4 (32-bit) of 8 (04-bit)		0000		0004
	Addr =			000
	0004			0000
				000
				0008
	Addr =			000
	0008	Addr		0010
		=		001
		8000		0012
	Addr =			0013
	0012			0014
				001

r other data representations too					
C Data Type	Typical 32-bit	Intel IA32	x86-64		
char	1	1	1		
short	2	2	2		
int	4	4	4		
long	4	4	8		
long long	8	8	8		
float	4	4	4		
double	8	8	8		
long double	8	10/12	10/16		
pointer	4	4	8		

