## 10-601 Machine Learning

## Computational biology: Sequence alignment and profile HMMs

## Central dogma



## Central dogma

Can be measured using sequencing techniques


## Comparison of Different Organisms

|  | Genome size | Num. of genes |
| :--- | :--- | :--- |
| E. coli | $.05^{\star} 10^{8}$ | 4,200 |
| Yeast | $.15^{\star} 10^{8}$ | 6,000 |
| Worm | $1^{\star} 10^{8}$ | 18,400 |
| Fly | $1.8^{*} 10^{8}$ | 13,600 |
| Human | $30^{*} 10^{8}$ | 25,000 |
| Plant | $1.3^{*} 10^{8}$ | 25,000 |

## Growth in biological data

Growth of GenBank


Growth of Gene Expression Omnibus


## Assigning function to proteins

- One of the main goals of molecular (and computational) biology.
- There are 25000 human genes and the vast majority of their functions is still unknown
- Several ways to determine function
- Direct experiments (knockout, overexpression)
- Interacting partners
- 3D structures
- Sequence homology



## Function from sequence homology

- We have a query gene: ACTGGTGTACCGAT
- Given a database containing genes with known function, our goal is to find similar genes from this database (possibly in another organism)
- When we find such gene we predict the function of the query gene to be similar to the resulting database gene
- Problems
- How do we determine similarity?


## Sequence analysis techniques

- A major area of research within computational biology.
- Initially, based on deterministic or heuristic alignment methods
- More recently, based on probabilistic inference methods


## Sequence analysis

- Traditional
- Dynamic programming
- Probabilsitic
- Profile HMMs


## Pairwise sequence alignment

ACATTG
AACATT

AGCCTT AGCATT

$$
\begin{aligned}
& \text { A G C } \overline{C_{1}} \stackrel{-1}{1} \\
& \begin{array}{lllll} 
& & \left|\begin{array}{lll}
1 & 1 \\
1 & 1 \\
A & G & 1 \\
1 & A & 1
\end{array}\right|
\end{array}
\end{aligned}
$$

## Pairwise sequence alignment

AGCCTT ACCATT

- We cannot expect the alignments to be perfect.
- Major reasons include insertion, deletion and substitutions.
- We need to allow gaps in the resulting alignment.


## Scoring Alignments

- Alignments can be scored by comparing the resulting alignment to a background (random) model.

$$
\begin{gathered}
\text { Independent } \\
P(x, y \mid I)=\prod_{i} q_{x_{i}} \prod_{j} q_{x_{j}}
\end{gathered}
$$

Related

$$
P(x, y \mid M)=\prod_{i} p_{x_{i} y_{i}}
$$

Score for alignment:

$$
S=\sum_{i} s\left(x_{i}, y_{i}\right) \quad \text { where: } s(a, b)=\log \left(\frac{p_{a, b}}{q_{a} q_{b}}\right)
$$

Can be computed for each pair of letters

## Scoring Alignments

- Alignments can be scored by comparing the resulting alignment to a background (random) model.

In other words, we are trying to find an alignment that maximizes the likelihood ratio of the aligned pair compared to the background model

Score for alignment:

$$
S=\sum_{i} s\left(x_{i}, y_{i}\right) \quad \text { where: } s(a, b)=\log \left(\frac{p_{a, b}}{q_{a} q_{b}}\right)
$$

## Computing optimal alignment:

The Needham-Wuncsh algorithm

$$
F(i, j)=\max \left\{\begin{array}{l}
F(i-1, j-1)+s\left(x_{i} x_{j}\right) \\
F(i-1, j)+d \\
F(i, j-1)+d
\end{array}\right.
$$

$d$ is a penalty for a gap


| $F(i-1, j-1)$ | $F(i-1, j)$ |
| :--- | :--- |
| $F(i, j-1)$ | $F(i, j)$ |

## Example

Assume a simple model where $S(a, b)=1$ if $a=b$ and -5 otherwise.
Also, assume that $\mathrm{d}=-1$

|  |  | A | G | C | C | T | T |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 |
| A | -1 |  |  |  |  |  |  |
| C | -2 |  |  |  |  |  |  |
| C | -3 |  |  |  |  |  |  |
| A | -4 |  |  |  |  |  |  |
| T | -5 |  |  |  |  |  |  |
| T | -6 |  |  |  |  |  |  |

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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 |
| A | -1 | 1 |  |  |  |  |  |
| C | -2 |  |  |  |  |  |  |
| C | -3 |  |  |  |  |  |  |
| A | -4 |  |  |  |  |  |  |
| T | -5 |  |  |  |  |  |  |
| T | -6 |  |  |  |  |  |  |

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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 |
| A | -1 | 1 | 0 |  |  |  |  |
| C | -2 | 0 |  |  |  |  |  |
| C | -3 |  |  |  |  |  |  |
| A | -4 |  |  |  |  |  |  |
| T | -5 |  |  |  |  |  |  |
| T | -6 |  |  |  |  |  |  |

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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 |
| A | -1 | 1 | 0 | -1 | -2 | -3 | -4 |
| C | -2 | 0 | -1 |  |  |  |  |
| C | -3 | -1 |  |  |  |  |  |
| A | -4 | -2 |  |  |  |  |  |
| T | -5 | -3 |  |  |  |  |  |
| T | -6 | -4 |  |  |  |  |  |

## Example

Assume a simple model where $S(a, b)=1$ if $a=b$ and -5 otherwise.
Also, assume that $d=-1$

|  |  | A | G | C | C | T | T |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 |
| A | -1 | 1 | 0 | -1 | -2 | -3 | -4 |
| C | -2 | 0 | -1 | 1 | 0 | -1 | -2 |
| C | -3 | -1 | -2 | 0 | 2 | 1 | 0 |
| A | -4 | -2 | -3 | -1 | 1 | 0 | -1 |
| T | -5 | -3 | -4 | -2 | 0 | 2 | 1 |
| T | -6 | -4 | -5 | -3 | -1 | 1 | 3 |

## Example

Assume a simple model where $S(a, b)=1$ if $a=b$ and -5 otherwise.
Also, assume that $d=-1$

|  |  | A | G | C | C | T | T |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 |
| A | -1 | 1 | 0 | -1 | -2 | -3 | -4 |
| C | -2 | 0 | -1 | 1 | 0 | -1 | -2 |
| C | -3 | -1 | -2 | 0 | 2 | 1 | 0 |
| A | -4 | -2 | -3 | -1 | 1 | 0 | -1 |
| T | -5 | -3 | -4 | -2 | 0 | 2 | 1 |
| T | -6 | -4 | -5 | -3 | -1 | 1 | $\mathbf{3}$ |

## Example

Assume a simple model where $S(a, b)=1$ if $a=b$ and -5 otherwise.
Also, assume that $\mathrm{d}=-1$


|  |  | A | G | C | C | T | T |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 |
| A | -1 | 1 | 0 | -1 | -2 | -3 | -4 |
| C | -2 | 0 | -1 | 1 | 0 | -1 | -2 |
| C | -3 | -1 | -2 | 0 | 2 | 1 | 0 |
| A | -4 | -2 | -3 | -1 | $\mathbf{4}$ | 0 | -1 |
| T | -5 | -3 | -4 | -2 | 0 | 2 | 1 |
| T | -6 | -4 | -5 | -3 | -1 | 1 | $\mathbf{3}$ |

## Running time

- The running time of an alignment algorithms if $\mathrm{O}\left(\mathrm{n}^{2}\right)$
- This doesn't sound too bad, or is it?
- The time requirement for doing global sequence alignment is too high in many cases.
- Consider a database with tens of thousands of sequences. Looking through all these sequences for the best alignment is too time consuming.
- In many cases, a much faster heuristic approach can achieve equally good results.


## Sequence analysis

- Traditional
- Dynamic programming $\sqrt{ }$
- Probabilsitic
- Profile HMMs


## Protein families

- Proteins can be classified into families (and further into sub families etc.)
- A specific family includes proteins with similar high level functions
- For example:
- Transcription factors
- Receptors
- Etc.


## Family assignment is an important first <br> step towards function prediction

# Methods for Characterizing a Protein Family 

- Objective: Given a number of related sequences, encapsulate what they have in common in such a way that we can recognize other members of the family.
- Some standard methods for characterization:
- Multiple Alignments
- Regular Expressions
- Consensus Sequences
- Hidden Markov Models


## Multiple Alignment Process

- Process of aligning three or more sequences with each other
- We can determine such alignment by generalizing the algorithm to align two sequences
- Running time exponential in the number of sequences


## Training a HMM from an existing alignment

- Start with a predetermined number of states accounting for matches, insertions and deletions.

MLE
estimates

- For each position in the model, assign a column in the multiple alignment that is relatively conserved
- Emission probabilities are set according to amino acid counts in columns.
- Transition probabilities are set according to how many sequences make use of a given delete or insert state.


## Remember the simple example



- Chose six positions in model.
- Highlighted area was selected to be modeled by an insert due to variability.
- Can also do neat tricks for picking length of model, such as model pruning.


## So... what do we do with a model?

- Given a query protein:
- Design statistical tests to determine how likely it is to get this score from a random (gene) sequence
- Use several protein family models for classifying new proteins, assign protein to most highly scoring family.


## Choosing the best model: Aligning sequences to a models

- Compute the likelihood of the best set of states for this sequence
- We know how to do this: The Viterbi algortthm
- Time: O(N*M)


## Scoring our simple HMM



$$
\begin{aligned}
& \text { ACA-- ATG } \\
& \text { TCAACTATC } \\
& \text { ACAC--AGC } \\
& \text { AGA-- ATC } \\
& \text { ACCG--ATC }
\end{aligned}
$$

- \#1 - "T G C T A G G" vrs: \#2 - "A C A C A T C"
- HMM:
\#1 = Score of -0.97 \#2 Score of 6.7 (Log odds)


## Training from unaligned sequences

- Baum-Welch algorithm
- Start with a model whose length matches the average length of the sequences and with random emission and transition probabilities.
- Align all the sequences to the model.
- Use the alignment to alter the emission and transition probabilities
- Repeat. Continue until the model stops changing
- By-product: It produces a multiple alignment


## Multiple Alignment: Reasons for differences



## Designing HMMs: Consensus (match) states

We first include states to output the consensus sequence

$$
\begin{array}{lllllll}
A & C & A & - & -A & T \\
T & C & A & A & C & T & A \\
A & C & A & C & -A & G \\
A & G & A & - & -A & T \\
A & C & C & G--A & A
\end{array}
$$

| $\mathrm{A}: 0.8$ |
| :---: |
| $\mathrm{~T}: 0.2$ |$\longrightarrow$| $\mathrm{C}: 0.8$ |
| :--- |
| $\mathrm{G}: 0.2$ |$\longrightarrow$| $\mathrm{A}: 0.8$ |
| :--- |
| $\mathrm{C}: 0.2$ |$\longrightarrow$| $\mathrm{T}: 0.8$ |
| :--- |
| $\mathrm{G}: 0.2$ |

## Designing HMMs: Insertions

We next add states to allow insertions

| $A$ | $C$ | $A$ | - | $-A T$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $T$ | $C$ | $A$ | $A$ | $C$ | $T$ | $A$ |
| $A$ | $C$ |  |  |  |  |  |
| $A$ | $C$ | $A$ | $C$ | - | $A$ | $G$ |
| $A$ | $G$ | $A$ | - | $-A$ | $T$ |  |
| $A$ | $C$ | $C$ | $G--A T$ |  |  |  |

## Designing HMMs: Deletions

Finally we add states with no output to allow for deletions

$$
\begin{array}{lllllll}
A & C & A & - & -A & T \\
T & C & A & A & C & T & A \\
\hline & C \\
A & C & A & C & - & -A & G \\
A & G & A & - & -A & T \\
A & C & C & G--A &
\end{array}
$$



## Training from unaligned continued

- Advantages:
- You take full advantage of the expressiveness of your HMM.
- You might not have a multiple alignment on hand.
- Disadvantages:
- HMM training methods are local optimizers, you may not get the best alignment or the best model unless you're very careful.
- Can be alleviated by starting from a logical model instead of a random one.


## Summary

- Initial methods for sequence alignment relied on combinatorial and dynamic programming methods.
- These methods do not generalize well for multiple sequence alignment and for searching large databases.
- State of the art methods rely on Al techniques, primarily variants of HMMs to overcome this problem.

