15-453 Formal Languages, Automata, and Computation

Homework Assignment 4

Due Friday, February 18

Problem 1 (30 points)

Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$. [This is a modified version of Exercise 2.4 a–e in the textbook.]

a. $\{w \mid w$ constains at least three 1s$\}$.

b. $\{w \mid w$ starts and ends with the same symbol$\}$.

c. $\{w \mid$ the length of w is odd$\}$.

d. $\{w \mid$ the length of w is odd and its middle symbols is a 0$\}$.

e. $\{w \mid w$ contains more 1s than 0s$\}$.

Indicate for each language if it is regular or not. For those languages which are **not** regular, prove that they are not regular.

Problem 2 (30 points)

Give informal descriptions and state diagrams of pushdown automata for the languages in Problem 1. Note that even if the language is regular, you may be able to give a smaller PDA than NFA recognizing the language. [This is Exercise 2.5 a–e in the textbook.]

Problem 3 (20 points)

This problem illustrates a technique for proving that languages are not context-free without directly using the pumping lemma. [This is Problem 2.17 in the textbook.]

a. Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free.

b. Use part (a) and one of examples 2.20, 2.21, or 2.22 in the textbook to show that the language $L = \{w \mid w \in \{a, b, c\}^*$ and contains equal numbers of a's, b's and c's$\}$ is not context-free.

Problem 4 (20 points)

Use the pumping lemma for context-free languages to show that $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$ is not context-free. Carry out your argument in detail and make clear which conditions of the pumping lemma you use and where. [This is adapted from Problem 2.18 (a) in the textbook.]

Problem 5 (20 points extra credit)

Let $C = \{x\#y \mid x, y \in \{0,1\}^* \text{ and } x \neq y\}$. Show that C is a context-free language. [This is Problem 2.26 in the textbook.]