15-453 Formal Languages, Automata, and Computation

Assignment 2

Due: Friday, February 4

Problem 1 (15 points)

1. Construct a nondeterministic automaton M to recognize the language $((ab \cup aba)^* a)^*$.
2. Convert M to a deterministic automaton recognizing the same language.

Problem 2 (15 points)

Give regular expressions for each of the following subsets of $\{a, b\}^*$:

1. $\{x \mid x$ contains an even number of a's$\}$
2. $\{x \mid x$ contains an odd number of b's$\}$
3. $\{x \mid x$ contains an even number of a's or an odd number of b's$\}$
4. $\{x \mid x$ contains an even number of a's and an odd number of b's$\}$

Try to give regular expressions as simple as possible (you get 5pt. extra credit if you give the shortest possible in all cases).

Problem 3 (20 points)

Prove the following facts about the regular languages L_1 and L_2 over some alphabet Σ:

1. $L_1 \cap L_2$ is regular.
2. L_1^C is regular. (We denote the complement of a language L by $L^C = \Sigma^* \setminus L$).

Problem 4 (25 points)

You are given two finite automata M_1 and M_2. Using the results of Problem 3, describe algorithms to check for the following properties:

1. $L(M_1)$ is empty.
2. $L(M_1)$ and $L(M_2)$ are disjoint. **Hint:** express this fact using the emptiness of some language, and use 1.
3. $L(M_1) \subseteq L(M_2)$. **Hint:** use 2.
4. $L(M_1) = L(M_2)$. **Hint:** use 3, twice.

Problem 5 (25 points)

If A is any language, let $A_{\frac{1}{2}}$ be the set of the last halves of strings in A, i.e.

$$A_{\frac{1}{2}} = \{x \mid \exists y \text{ s.t. } |x| = |y| \text{ and } yx \in A\}$$

Prove that if A is regular, then so is $A_{\frac{1}{2}}$.